Financial Market Imperfections and Self-fulfilling Beliefs *

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Abstract

We develop a model in which a strategic complementarity in saving decisions arises due to financial market imperfection. We explore the role of self-fulling beliefs in determining the long run dynamics. The model exhibits a wide range of dynamic phenomena such as a poverty trap, a big push and a sunspot equilibrium depending on the level of financial market imperfection. They account for excessive volatility and a sudden change in the saving rate and the macroeconomic consequences without any shocks to fundamentals.

JEL Classification: E21, E32, E44, O11, O16

Keywords: Financial market imperfection, strategic complementarity, saving rate, self-fulfilling belief, sunspot, endogenous fluctuation, multiple equilibria

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1 Introduction

Countries sometimes experience a sudden change and excessive volatility in the saving rate. The sudden change may be persistent lasting over many years. When shocks to fundamentals such as technology or preference are not the apparent cause, we require an alternative approach. There is a strand of literature that focus on financial market imperfections as a cause for multiple steady states (e.g. Banerjee and Newman 1993, Piketty 1997, Mookherjee and Ray 2003, Matsuyama 2004). In these models, the long-run dynamics is determined by initial conditions of the economies. However, countries may experience changes in macroeconomic climate which are hard to be predicted by historical paths.

The alternative approach we present in this paper is based on a strategic complementarity in saving decisions that arise due to financial market imperfections. Strategic complementarity arises when the optimal decision of an agent depends positively on the decisions of the other agents (c.f. Bulow et al. 1985). When saving decisions are strategic complements, the saving rate changes suddenly whenever agents change their beliefs about other's beliefs; self-fulfilling beliefs can change the course of the economy at any point in time without any shocks to fundamentals.[1] Even in the presence of multiple steady states our model shows that self-fulfilling beliefs play a crucial role in determining the long run dynamics.

We take the structure of Matsuyama's (2004) two-period overlapping generation model with financial market imperfection. In his model, however, the young do not consume and therefore save the wage inelastically; the autarky economy converges monotonically to a unique steady state.[2] Once the young make intertemporal saving decisions, the implications are far reaching. In a model with financial market imperfection savings play a dual role. On the one hand, higher savings lead to higher investment, which lowers the return on capital and therefore the interest rate. On the other hand,

[1] This is a typical case of a sunspot equilibrium—sunspots coordinate individual decisions—where extrinsic uncertainty matters even in the long run. The technique to use sunspots as coordination devices to choose one equilibrium out of many is presented in Matsuyama (2002). We make use of the theory of the so called random iterated function systems to obtain well defined forward orbits when multiple equilibria exist (e.g. Mitra, Montrucchio and Privileggi 2004, Mitra and Privileggi 2004, Gardini et al. 2009, Mitra and Privileggi 2009), and derive the stationary probability distribution over all the possible states.

[2] Matsuyama (2004) analyzes a world economy which consists of identical small open economies, which differ only in their levels of capital stock. In the small open economy, the interest rate does not adjust to equate domestic savings to domestic investment. The financial market imperfection then generates multiple steady states.

higher savings make investors less dependent on borrowing, which increases the interest rate. When the latter effect is stronger, the interest rate increases with savings generating a strategic complementarity in saving decisions. The strategic complementarity causes multiplicity of both steady states *and* equilibria. While external economies in production has been studied as the cause of multiple equilibria (e.g. Murphy et al. 1989, Krugman 1991, Matsuyama 1991) we are not aware of any study that analyzes how financial market imperfection generates a strategic complementarity in saving decisions causing multiplicity of both steady states and equilibria.

We fully characterize all possible dynamics of the economy. We show how the dynamics of the saving rate depends on the level of financial marketimperfection. The saving rate asymptotically displays no volatility when financial market imperfection is either low or high; the economy converges to a steady state. The complementarity in saving decisions arises when financial market imperfection is at an intermediate level. When multiple equilibria and multiple steady state exist, the economy either falls into a poverty trap or fluctuates through self-fulfilling beliefs. Multiple equilibria imply that agents find it rational to save at a level with a "virtuous" or a "vicious" consequence if everyone else does so. Under this circumstance we identify a threshold of financial market imperfection that divides the dynamics into two cases. Below the threshold the economy falls into a poverty trap for any initial conditions when agents coordinate their saving behaviors at a low level. Once in the trap the wage is too low for agents to save at a high level to escape the trap. Above the threshold the economy fluctuates whenever agents coordinate their saving behaviors at a different level. Escape from a poverty trap in a manner of "big push" is possible but the economy can also fluctuate indefinitely as in a "sunspot" equilibrium. In the latter case, self-fulfilling beliefs may cause sudden shifts and excessive volatility of the saving rate at any time. This result agrees with findings that economies, which are not financially developed, are subject to large changes in the growth rate both up and down (cf. Acemoglu and Zilibotti 1997).

Our model also has implications on income inequality. The borrowing constraint keeps the return on lending below the marginal product of capital and therefore creates a wedge of income between lenders and borrowers as in Banerjee and Newman (1993), Galor and Zeira (1993) and Aghion and Bolton (1997). On the other hand, no income inequality between lenders and borrowers exists when the borrowing constraint is not binding. When financial market imperfection is low the economy converges to a steady state where the borrowing constraint is not binding. Hence, the model predicts that financial development eventually eliminates inequality.[3]

[3] See Levine (1995) for discussions on financial development and income distribution.

The structure of the paper is as follows. Section 2 formulates the model and characterizes the equilibrium. Section 3 defines and analyzes the dynamics. Section 4 discusses the implications of the model. Remaining proofs can be found in Appendix.

2 The model

The economy is populated by overlapping generations of agents who live for two periods. Each agent supplies one unit of labor in the first period and consumes in both periods. Successive generations have unit mass. In period t, production combines the current stock of capital k_t supplied by the old with the unit quantity of labor supplied by the young.[4] The resulting per-capita output is $f(k_t)$ where $f\colon \mathbb{R}_+ \to \mathbb{R}_+$ is continuous, strictly increasing, strictly concave and twice continuously differentiable on $\mathbb{R}_{++}$, and satisfies the following boundary conditions: $f(0) = 0$, $\lim_{k\downarrow 0} f'(k) = \infty$ and $\lim_{k\uparrow\infty} f'(k) = 0$. Let $\rho(k) = kf'(k)$. We assume that $\rho\colon \mathbb{R}_+ \to \mathbb{R}_+$ is strictly increasing with $\lim_{k\downarrow 0} \rho(k) = 0$ and $\lim_{k\uparrow\infty} \rho(k) = \infty$. Factor markets are competitive paying the young the wage $w(k_t) := f(k_t) - k_t f'(k_t)$ and the old the return on capital given by $f'(k_t)$. After the distribution of factor payments, the old consume the return on capital, while the young take the wage and make saving and investment decisions.

2.1 Saving behavior

The young in period t either lend in the credit market for a return r_{t+1} or run a discrete indivisible project. The project takes one unit of the consumption good in period t and returns $R > 0$ units of the capital good in period $t+1$. Let s_t denote the aggregate saving in the economy. Saving must be equal to investment and thus capital stock is determined by $k_{t+1} = Rs_t$. We will later restrict our parameters such that the young agent i who starts a project must borrow $1 - s_t^i$. The restriction implies that not everyone, even if they would like to, can obtain credit. Given that all agents are ex ante homogenous—everyone earns the wage $w(k_t)$—we consider an equilibrium in which the market allocates the credit to everyone with the same probability and everyone chooses the same level of saving.[5] The assumption is internally consistent as the market must allocate the credit randomly if everyone has the same level of saving.

[4] Capital depreciates fully between periods, so capital stock is equal to investment.

[5] This is a symmetric Nash equilibrium. Brueckner, Kikuchi and Vachadze (2013) consider an alternative equilibrium, in which some agents save more than others and pay a higher interest rate to obtain credit. The strategic complementarity does not arise in that economy.

Given the probability s_t to obtain the credit and an additively separable utility function $u(c_t) + \beta u(c_{t+1})$ where $\beta \in (0,1]$, the young maximize the expected utility by choosing individual saving:

$$\max_{s_t^i \in [0, w(k_t)]} u[w(k_t) - s_t^i] + \beta[s_t u(Rf'(Rs_t) - (1 - s_t^i)r_{t+1}) + (1 - s_t)u(s_t^i r_{t+1})].$$

The first order condition is

$$\beta r_{t+1} \left[s_t u'(Rf'(Rs_t) - (1 - s_t^i)r_{t+1}) + (1 - s_t)u'(r_{t+1}s_t^i) \right] = u'(w - s_t^i). \tag{1}$$

When the project return $Rf'(Rs_t)$ is equal to r_{t+1}, the first order condition reduces to the standard Euler equation: $\beta r_{t+1} u'(r_{t+1}s_t^i) = u'(w - s_t^i)$.

2.2 Investment behavior

The young are willing to start the project whenever

$$r_{t+1} \leq Rf'(Rs_t). \tag{2}$$

We refer to this inequality as the profitability constraint. Given that all the young save the same amount, those who start a project must borrow $1 - s_t$ and repay $r_{t+1}(1 - s_t)$ when old. We assume that the young can only credibly pledge a fraction $\lambda \in (0,1]$ of the project return $Rf'(Rs_t)$. The borrowing constraint is therefore

$$r_{t+1}(1 - s_t) \leq \lambda Rf'(Rs_t). \tag{3}$$

The parameter λ can be interpreted as a measure of financial market imperfection, with higher values corresponding to lower imperfection. Equilibrium implies that the mass of agents who start projects (and hence s_t) increases until one of the constraints, either (2) or (3), binds. From this reasoning we obtain

$$r_{t+1} = r(s_t) := \begin{cases} \frac{\lambda}{1-s_t} Rf'(Rs_t) & \text{if } s_t < 1 - \lambda \\ Rf'(Rs_t) & \text{if } s_t \geq 1 - \lambda. \end{cases} \tag{4}$$

We refer to the interest rate consistent with the optimal investment behavior in (4) as the investment curve. It follows from (4) that the investment curve decreases with the aggregate saving for $s_t > 1 - \lambda$ as $f'' < 0$. For $s_t < 1 - \lambda$, when the borrowing constraint is binding, the investment curve may increase with the aggregate saving. This is because savings in our model play a dual role. On the one hand, higher savings lead to higher investment, which lowers the return on capital and therefore the interest rate. On the other hand, higher savings allow more agents to start investment projects by mitigating the borrowing constraint. If the latter effect is stronger, the interest rate increases with savings.

2.3 Strategic complementarity

This section explains how financial market imperfection causes a strategic complementarity in individual saving decisions. We start by characterizing the investment curve. To simplify the analysis we make some assumptions on the elasticity of the production function $\varepsilon(k) := -kf''(k)/f'(k)$.

Assumption 2.1. (a) $\varepsilon(k) \in (0,1)$; (b) $\varepsilon'(k) \leq 0$.

Many standard production functions satisfy the assumptions.[6] Let

$$H(s,\lambda,R) := \frac{\lambda R f'(Rs)}{1-s}, \tag{5}$$

which describes the investment curve under the borrowing constraint.

Lemma 2.1. *If Assumption 2.1 is satisfied, then for any $(\lambda, R) > 0$, $H(s,\lambda,R)$ obtains its minimum at $s^c = \phi(R) \in (0,1)$ where ϕ is a differentiable, non-increasing function.*

Figure 1 shows an example of the non-monotonic investment curve for a Cobb-Douglas production function. Existence of the unique interior minimum implies that the investment curve is a non-monotonic function of the aggregate saving if and only if $\phi(R) < 1-\lambda$ as shown in the figure.[7] The non-monotonicity of the investment curve is

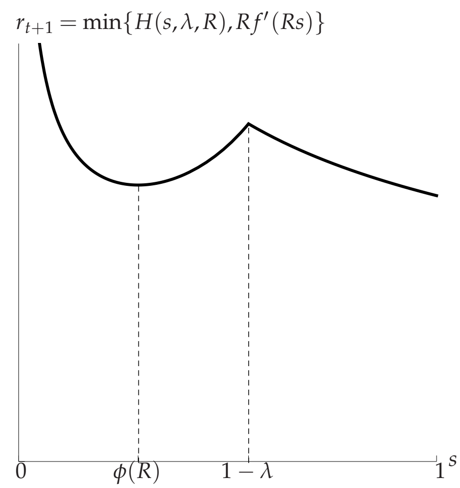


Figure 1: Investment curve

[6]In fact, Assumption (a) follows directly from $\rho'(k) > 0$ and $f'(k) > 0 > f''(k)$. For the Cobb-Douglas function $f(k) = k^\alpha$ with $\alpha \in (0,1)$, $\varepsilon(k) = 1-\alpha$ and $\varepsilon'(k) = 0$.

[7]For the Cobb-Douglas production function, $\phi(R) = \frac{1-\alpha}{2-\alpha}$ and thus $\phi(R) < 1-\lambda \Leftrightarrow \lambda < \frac{1}{2-\alpha}$.

a consequence of the borrowing constraint and the indivisibility of investment.[8] Without either of them, our model reduces essentially to the Diamond model, in which saving decisions are always strategic substitutes and the economy would monotonically converge to a unique steady state.

Strategic complementarity arises when the optimal decision of an agent depends positively on the decisions of the other agents. It is conceivable from (1) that individual saving may be a non-monotonic function of the aggregate saving for any standard utility function if the investment curve is a non-monotonic function of the aggregate saving. If that is the case, then at least for some range of the aggregate saving, the optimal individual saving may be an increasing function of the aggregate saving: individual saving decisions are strategic complements. The complications that arise in characterizing the equilibrium for a general utility function come from the interaction between the curvature of the utility function and financial market imperfection and are of a second order. (For example a complete characterization of all possible equilibria would be much more involved.)

We can avoid the complications and focus on the first order effects by taking a linear utility function $u(c) = c$. The first order condition (1) then yields the saving function

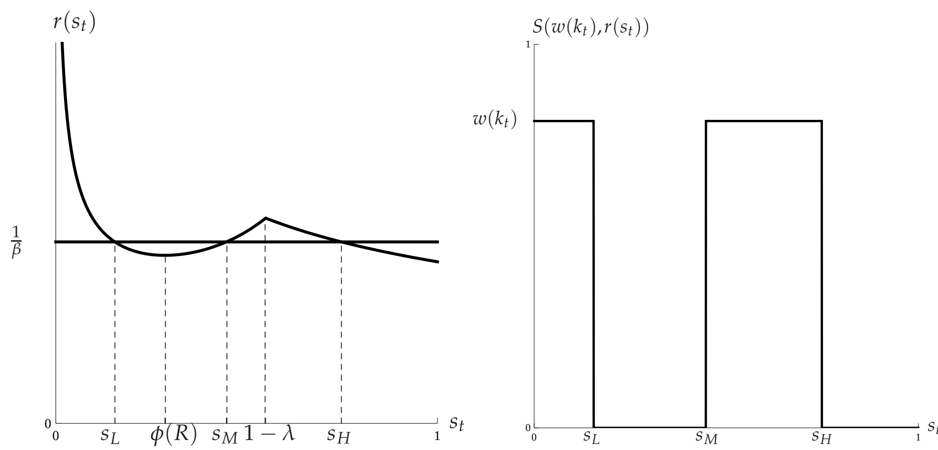
$$s_t^i = S(w(k_t), r_{t+1}) = \begin{cases} 0 & \text{if } 1 > \beta r_{t+1} \\ [0, w(k_t)] & \text{if } 1 = \beta r_{t+1} \\ w(k_t) & \text{if } 1 < \beta r_{t+1}. \end{cases} \tag{6}$$

We know from Lemma 2.1 that r_{t+1} can increase with s_t when the borrowing constraint is binding. The saving function (6) shows that individual saving increases from 0 to $w(k_t)$ when r_{t+1} crosses $1/\beta$ from below as s_t increases and therefore individual decisions are strategic complements. (The infinite elasticity of saving with respect to the interest rate at $1/\beta$ simplifies the analysis but is not necessary for the results in the paper. Numerical simulation indicates that the structure of the model remains essentially the same for the constant relative risk aversion (CRRA) utility function.)

[8]The model can be generalized to allow agents to run more than one project. The project size would then expand until the borrowing constraint binds. Critical is the minimum project size of one unit of consumption goods. If this assumption is relaxed, competition would force agents to reduce the project size until the borrowing constraint is not binding.

2.4 Equilibrium

It is well known from Cooper and John (1988) that a strategic complementarity is a necessary condition for multiple equilibria. This section derives the exact conditions when the complementarity arises and generates multiple equilibria. To determine the equilibrium saving we start by investigating the solution to $r(s) = 1/\beta$. Let R^+ be the solution to $Rf'(R) = 1/\beta$.[9] We assume that $Rf'(R) < 1/\beta$. In other words, we consider only $R \in (0, R^+)$. This ensures that $r(s) = 1/\beta$ has at least one solution and $s_t < 1$ so that the young need to borrow $1 - s_t > 0$ to start the investment project. There are three solutions if $\phi(R) < 1 - \lambda$ and $H(\phi(R), \lambda, R) < 1/\beta < H(1 - \lambda, \lambda, R)$, and one solution otherwise. Let s_L and s_M denote the solutions to $H(s, \lambda, R) = 1/\beta$ and s_H denote the solution to $Rf'(Rs) = 1/\beta$. By construction $s_L < \phi(R) < s_M < 1 - \lambda$ and $s_H < 1$ for all $R \in (0, R^+)$. Since H is increasing in λ, s_L and s_M depend on λ while s_H does not. Figure 2(a) shows when there are three solutions. Substituting (4) into (6) we obtain individual saving as a function of aggregate saving $s_t^i = S(w(k_t), r(s_t))$. Figure 2(b) shows the corresponding saving function to Figure 2(a).



(a) Three solutions to $r(s) = 1/\beta$ (b) Individual saving

Figure 2: Complementarity in saving decisions

When there is one solution to $r(s) = 1/\beta$, individual saving is always decreasing in aggregate saving: saving decisions are strategic substitutes. If there are three solutions, individual saving is non-monotonic in aggregate saving: a strategic complementarity in saving decisions arises.

Next we identify the parameter conditions for which a strategic complementarity arises. Let $\lambda = \psi_1(R)$ be the solution to $H(1 - \lambda, \lambda, R) = 1/\beta$ and $\lambda = \psi_2(R)$ be the solution to

[9] Properties of ρ implies the existence and uniqueness of R^+.

$H(\phi(R),\lambda,R) = 1/\beta$. Figure 3 shows the parameter values when the saving function takes the three possible cases in Proposition 2.1 (a), (b) and (c). In the figure (λ^c, R^c) denotes a unique pair such that $\lambda^c = \psi_1(R^c) = \psi_2(R^c) = 1 - \phi(R^c)$.

Proposition 2.1. *(a) If and only if $\lambda \in (0, \psi_1(R))$,*

$$S(w(k_t), r(s_t)) = \begin{cases} w(k_t) & \text{for } s_t < s_L \\ 0 & \text{for } s_t \geq s_L. \end{cases}$$

(b) If and only if $R \in (R^c, R^+)$ and $\lambda \in (\psi_1(R), \psi_2(R))$,

$$S(w(k_t), r(s_t)) = \begin{cases} w(k_t) & \text{for } s_t < s_L \text{ and } s_M < s_t < s_H \\ 0 & \text{for } s_L \leq s_t \leq s_M \text{ and } s_t \geq s_H\ . \end{cases}$$

(c) If and only if either $R \in (0, R^c)$ and $\lambda \in (\psi_1(R), 1]$ or $R \in (R^c, R^+)$ and $\lambda \in (\psi_2(R), 1]$,

$$S(w(k_t), r(s_t)) = \begin{cases} w(k_t) & \text{for } s_t < s_H \\ 0 & \text{for } s_t \geq s_H. \end{cases}$$

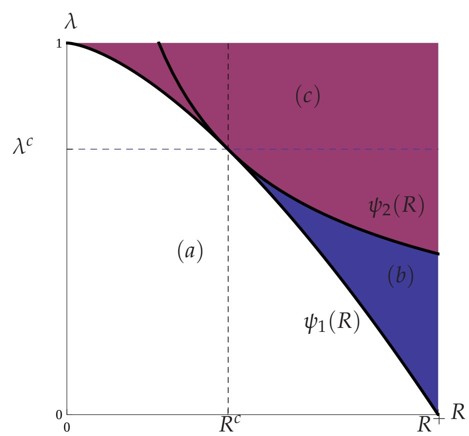


Figure 3: Solutions to $r(s) = 1/\beta$

Proposition 2.1 shows that a strategic complementarity in saving decisions arises only in the case (b) where the optimal decision of an agent depends positively upon the decisions of other agents: when s_t crosses s_M from below. Following Proposition 2.1 we can now characterize the equilibrium saving. In equilibrium, the young save the same amount: $s_t^i = s_t$. The equilibrium aggregate saving is then a solution to $s_t = S(w(Rs_{t-1}), r(s_t))$. Let $\Psi(s) := w(Rs)$. The case (a), (b) and (c) in Proposition 2.1 defines the equilibrium saving (a), (b) and (c) in the next proposition.

Proposition 2.2. *Let $R \in (0, R^+)$, $\lambda \in (0,1]$, $\beta \in (0,1]$. The equilibrium saving is given by*

(a) $s_t = \min\{\Psi(s_{t-1}), s_L\}$ *if and only if* $\lambda \in (0, \psi_1(R))$.

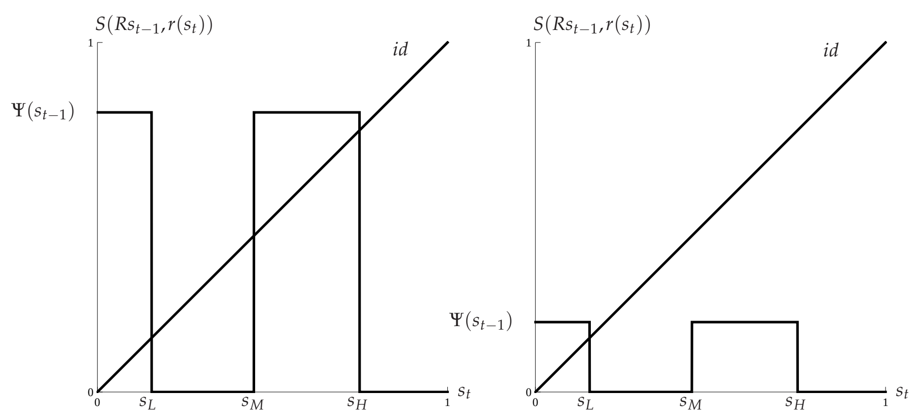
(b)

$$s_t \in \begin{cases} \min\{\Psi(s_{t-1}), s_L\} & \text{for } \Psi(s_{t-1}) < s_M \\ \{s_L, s_M, \min\{\Psi(s_{t-1}), s_H\}\} & \text{for } \Psi(s_{t-1}) \geq s_M \end{cases}$$

if and only if $R \in (R^c, R^+)$ *and* $\lambda \in (\psi_1(R), \psi_2(R))$.

(c) $s_t = \min\{\Psi(s_{t-1}), s_H\}$ *if and only if either* $R \in (0, R^c)$ *and* $\lambda \in (\psi_1(R), 1]$ *or* $R \in (R^c, R^+)$ *and* $\lambda \in (\psi_2(R), 1]$.

The cases (a) and (c) in Proposition 2.2 show that the equilibrium saving for a given investment technology is low (high) if financial market imperfection is high (low). The case (b) shows that multiple equilibria arise for $\Psi(s_{t-1}) > s_M$ if the investment technology is highly productive ($R > R^c$) but financial market imperfection is at an intermediate level. This case is shown in Figure 4 (a). When the wage is lower than s_M, i.e., $\Psi(s_{t-1}) < s_M$ there exists a unique equilibrium as shown in Figure 4 (b).



(a) Three equilibria: $\Psi(s_{t-1}) > s_M$

(b) Unique equilibrium: $\Psi(s_{t-1}) < s_M$

Figure 4: Multiple equilibria

3 Dynamics

This section examines the dynamics of savings where the state space is taken to be $(0,1)$. It is well-known that OLG models can exhibit multiple steady states even with neoclassical technology. To avoid unnecessary complications not related to the financial market imperfection, we assume that $\lim_{k\downarrow 0} w'(k) = \infty$ and w is strictly concave, which are satisfied by the Cobb-Douglas function $f(k) = k^{\alpha}$. By examining the cases (a), (b) and (c) in Proposition 2.2 it is clear that the possible steady states are s_L, s_M, s_H and s^* where s^* is the unique solution to $s = \Psi(s)$.[10] Notice that $s_L, s_M, s_H < 1$ implies that $s_t < 1$ for all t as it was previously assumed.

In both cases (a) and (c), the economy converges monotonically to a unique steady state for any $s_0 \in (0,1)$. In the case (a) financial market imperfection is high. The economy is in a low saving/low investment equilibrium and converges to $\min\{s^*, s_L\}$ where the borrowing constraint is binding.[11] In the case (c), financial market imperfection is low. The economy is in a high saving/high investment equilibrium and converges to $\min\{s^*, s_H\}$ where the borrowing constraint is not binding.[12]

The remainder of this section focuses on the case (b) where financial market imperfection is at an intermediate level. When multiple equilibria exist, we assume that the choice of equilibrium, as in a complementary game, is made outside the market mechanism. In particular, we define a forward orbit of the economy by employing an iterate function system (IFS) as the set of maps and probabilities (c.f. Mitra, Montrucchio and Privileggi 2004, Mitra and Privileggi 2004, Mitra and Privileggi 2009, Gardini et al. 2009)

$$\{s_L, s_M, \min\{\Psi(s_{t-1}), s_H\}; p_L, p_M, p_H\}. \tag{7}$$

The three maps are the constants s_L and s_M and the map $\min\{\Psi(s_{t-1}), s_H\}$, which is non-constant only for $s_{t-1} \in (\Psi^{-1}(s_M), \Psi^{-1}(s_H))$. The probability p_L, p_M, p_H is attached to each map.[13]

3.1 Multiple steady states

Monotonicity of Ψ implies that multiple steady states exist only if there exist multiple equilibria. We would like to define the parameters for which multiple steady states exist. Proposition 2.2 (b) defines the parameter set for which multiple equilibria exist:

$$\mathcal{E} := \{(\beta, \lambda, R) \mid (\beta, \lambda, R) \in (0,1] \times (\psi_1(R), \psi_2(R)) \times (R^c, R^+)\}. \tag{8}$$

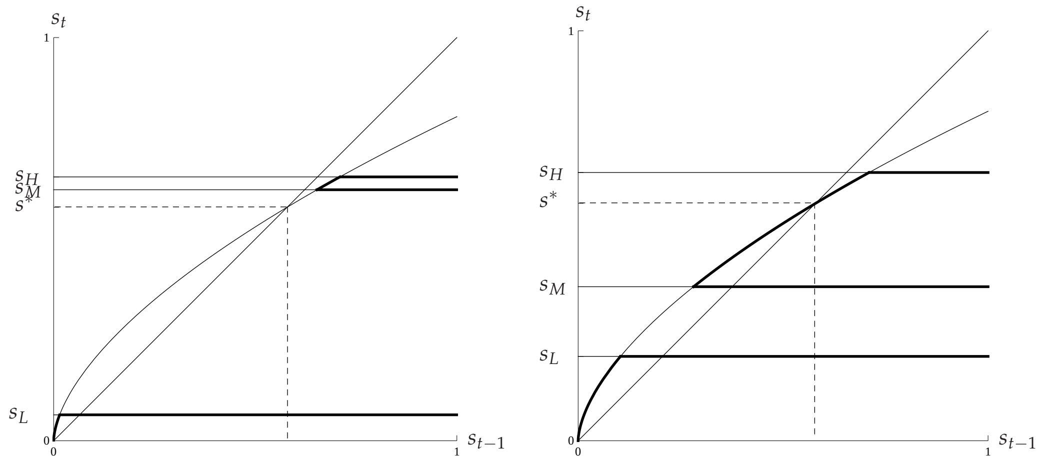
[10] We exclude zero from the state space to rule out trivial steady states.

[11] This can be confirmed by changing λ for a fixed $R \in (0, R^+)$ in Figure 3.

[12] If $s^* > 1 - \lambda$, then $\lambda > \psi_2(R)$ ensures that the borrowing constraint is not binding in s^*.

[13] Any stochastic process ensuring a positive probability to each equilibrium at all times t would define a proper IFS. The i.i.d assumption is made only to derive the stationary probability distribution over all the possible states in Section 3.2.

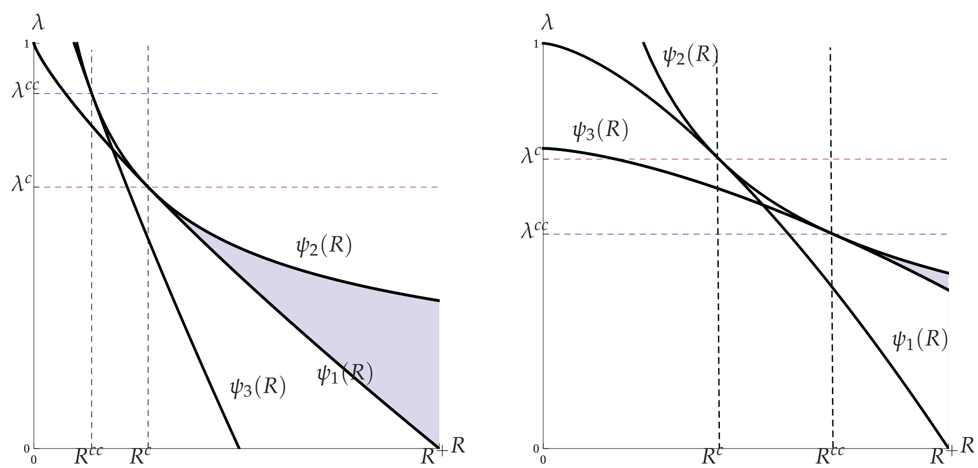
In the set $\mathcal{E}$ we obtain two cases as shown in Figure 5. The figure shows that multiple steady exist if and only if $(\beta, \lambda, R) \in E$ and $s^* > s_M$.



(a) Unique steady state: $s^* < s_M$ (b) Multiple steady states: $s^* > s_M$

Figure 5: Multiple equilibria: $(\lambda, R) \in \mathcal{E}$

To examine the parameter set for which $s^* > s_M$, we assume a Cobb-Douglas production function $f(k) = k^\alpha$ where $\alpha \in (0,1)$. Let $\psi_3(R)$ be the value of λ such that $H(s^*, \lambda, R) = 1/\beta$ and R^{cc} be the value of R such that $\phi(R) = s^*$. We can show that $s^* > s_M$ is equivalent to $\lambda > \psi_3(R)$ and $R > R^{cc}$. The shaded area in Figure 6 shows when the conditions are satisfied in the set $\mathcal{E}$. Note that there exists a unique pair



(a) $R^c > R^{cc}$ (b) $R^c < R^{cc}$

Figure 6: Multiple steady states: $s^* > s_M$

(λ^{cc}, R^{cc}) such that $\lambda^{cc} = \psi_2(R^{cc}) = \psi_3(R^{cc})$. We obtain that

$$R^{cc} \in \begin{cases} (0, R^c] & \text{iff } \beta \in (0, \underline{\beta}(\alpha)] \\ (R^c, R^+) & \text{iff } \beta \in (\underline{\beta}(\alpha), \overline{\beta}(\alpha)) \\ [R^+, \infty) & \text{iff } \beta \in [\overline{\beta}(\alpha), \infty) \end{cases} \tag{9}$$

where $\underline{\beta}(\alpha) := \frac{1-\alpha}{\alpha}$ and $\overline{\beta}(\alpha) := \frac{1-\alpha}{\alpha}(\frac{2-\alpha}{1-\alpha})^{1-\alpha}$. The equivalence in (9) means that the conditions $R^c > R^{cc}$ and $R^c < R^{cc}$ restrict the parameters (β, α). These restrictions are shown in Figure 7. In the region (a) multiple steady states exist if multiple equilibria

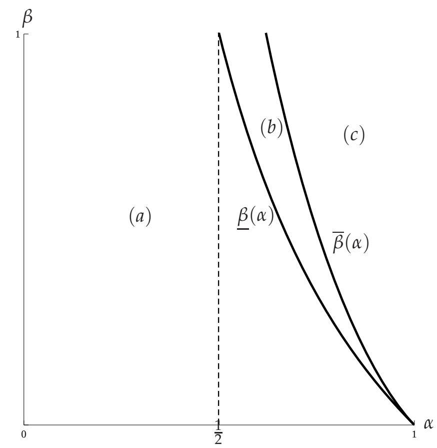


Figure 7: Multiple equilibria and multiple steady states

exist.[14] In the region (b) multiple steady states exist only if $\lambda \in (\psi_3(R), \psi_2(R))$ and $R \in (R^{cc}, R^+)$. In the region (c) multiple steady states do not exist.

Let

$$\begin{aligned} \mathcal{E}_1 &= \left\{ (\beta, \lambda, R) \mid (\beta, \lambda, R) \in (0, \underline{\beta}(\alpha)) \times (\psi_1(R), \psi_2(R)) \times (R^c, R^+) \right\} \\ \mathcal{E}_2 &= \left\{ (\beta, \lambda, R) \mid (\beta, \lambda, R) \in (\underline{\beta}(\alpha), \overline{\beta}(\alpha)) \times (\psi_3(R), \psi_2(R)) \times (R^{cc}, R^+) \right\}. \end{aligned}$$

The next proposition summarizes the results of this section.

Proposition 3.1. *There exist*

(a) *a unique steady state* $\min\{s_L, s^*\}$ *if and only if* $(\beta, \lambda, R) \in \mathcal{E} \setminus (\mathcal{E}_1 \cup \mathcal{E}_2)$.

(b) *three steady states* $\{s_L, s_M, s_H\}$ *if and only if* $(\beta, \lambda, R) \in \mathcal{E}_1$.

(c) *three steady states* $\{s_L, s_M, s^*\}$ *if and only if* $(\beta, \lambda, R) \in \mathcal{E}_2$.

[14]Notice that $\alpha < 1/2$ is sufficient to ensure (a).

Multiple steady states arise when the economy is endowed with a highly productive investment technology ($R > \max\{R^c, R^{cc}\}$) but is subject to an intermediate level of financial market imperfection.

3.2 Poverty trap and endogenous fluctuations

We now analyze the dynamics when multiple steady states exist, i.e., when $(\beta, \lambda, R) \in \mathcal{E}_1 \cup \mathcal{E}_2$. If multiple steady state exist, we have $s_L < \phi(R) < s_M < s^*$. This implies $\Psi(s_L) < \Psi(s_M)$ and $s_M < \Psi(s_M)$. Hence, we need to consider two cases: $\Psi(s_L) < s_M < \Psi(s_M)$ and $s_M < \Psi(s_L) < \Psi(s_M)$. The following lemma defines a threshold of λ such that $\Psi(s_L) = s_M$.

Lemma 3.1. *For any $\alpha \in (0,1)$, $\beta \in (0, \overline{\beta}(\alpha))$, and $R \in (\max\{R^c, R^{cc}\}, R^+)$ there exists a threshold $\bar{\lambda} = \{\lambda \in (\psi_1(R), \psi_2(R)) \cup (\psi_3(R), \psi_2(R)) \mid \Psi(s_L) = s_M\}$.*

Since s_M is decreasing in λ the case $\Psi(s_L) < s_M < \Psi(s_M)$ holds for a high level of financial market imperfection. In this case there exist multiple equilibria only in the neighborhood of s_M and $\min\{s_H, s^*\}$ as illustrated in Figure 8.

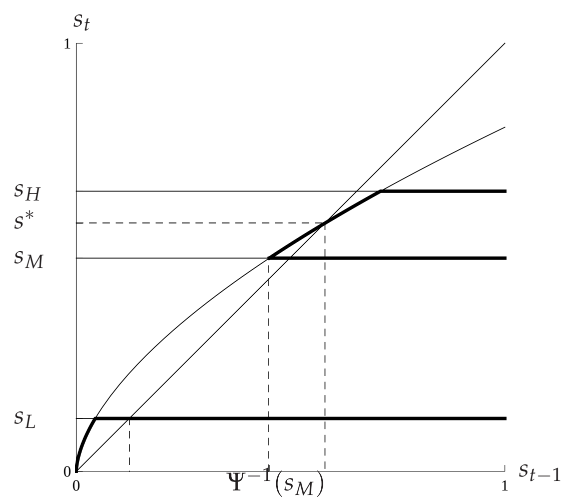


Figure 8: Poverty trap: $\Psi(s_L) < s_M < \Psi(s_M)$

The figure shows that for any $s_0 \in (0,1)$, the economy will eventually converge to s_L due to the positive probability p_L attached to s_L. Whenever agents coordinate on a low-saving equilibrium the economy falls into self-reinforcing "vicious circles" of a low-wealth/low-investment state where the borrowing constraint is binding. When the wage $\Psi(s_{t-1})$ is higher than s_M agents find it rational to save at a high level with virtuous consequences (a high wage for the next generation for example) if everyone else does so. The critical nature of the poverty trap in our model is, however, that once the economy falls into the trap, the low wage no longer generates incentives for agents to save at a high level.

The case $s_M < \Psi(s_L) < \Psi(s_M)$ holds for a low level of financial market imperfection. Figure 9 illustrates this case where multiple equilibria exist in the neighborhood of all steady states. The existence of multiple equilibria in a neighborhood of s_L causes

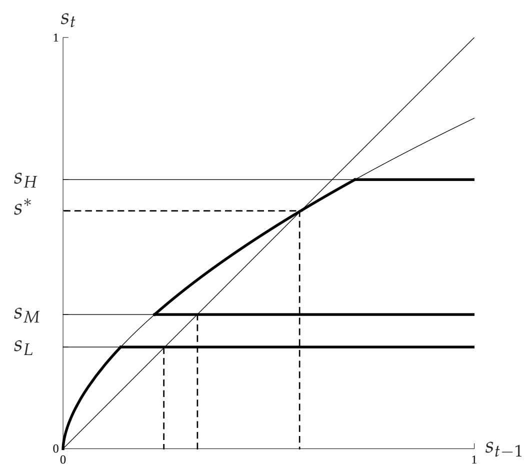


Figure 9: Endogenous fluctuation: $s_M < \Psi(s_L) < \Psi(s_M)$

the economy to move out of s_L with a positive probability. If all the young share the belief that others will save at a middle or high level (i.e, s_M or $\min\{\Psi(s_{t-1}), s_H\}$) the economy escapes from the poverty trap s_L. Unlike in the case above, the economy, after finite periods, is always in a state in which agents find it rational to save at a different level, with a vicious or virtuous consequence, if everyone else does so. The underlying mechanism of escaping the poverty trap in our model is reminiscent to the "big push" in Murphy et al. (1989) and Matsuyama (1991).[15] In our model, however, the economy also moves, with a positive probability, out of the middle steady state s_M and the high steady state $\min\{s^*, s_H\}$. Hence, for any $s_0 \in (0,1)$, the economy fluctuates endogenously if beliefs switch as in a sunspot-type equilibrium. The next proposition summarizes the findings.

Proposition 3.2. *Let $\alpha \in (0,1)$, $(p_L, p_M, p_H) \in \triangle^3$ and $(\beta, \lambda, R) \in E_1 \cup E_2$. For any initial condition $s_0 \in (0,1)$, self-fulfilling beliefs cause the economy to*

[15]In Murphy et al. (1989) it is sufficient for firms to expect that there will be demand for their products to decide to invest in the modern production technology, making their expectation a self-fulling prophecy.

(a) *fall into a poverty trap and converge to s_L if $\lambda < \bar{\lambda}$.*

(b) *fluctuate indefinitely if $\lambda > \bar{\lambda}$.*

The i.i.d. assumption on how the young coordinate their saving decisions allows us to derive the stationary probability distribution over all the possible states of the economy.

Let $m = \min_{t\in\mathbb{N}} \left\{t|\Psi^t(s_L) = s_H\right\}$ and $n = \min_{t\in\mathbb{N}} \left\{t|\Psi^t(s_M) = s_H\right\}$.

Proposition 3.3. *Consider the case (b) in Proposition 3.2. The possible states and their probability distribution are given by*

$$\{s_L,\Psi(s_L),...,\Psi^{m-1}(s_L),s_M,\Psi(s_M),...,\Psi^{n-1}(s_M),s_H\} \tag{10}$$

$$\{p_L,p_Lp_H,...,p_Lp_H^{m-1},p_M,p_Mp_H,...,p_Mp_H^{n-1},\frac{p_Lp_H^m + p_Mp_H^n}{p_L + p_M}\} \tag{11}$$

for $(\beta,\lambda,R) \in E_1$ and by

$$\{s_L,\Psi(s_L),\Psi^2(s_L),\Psi^3(s_L),...,s_M,\Psi(s_M),\Psi^2(s_M),\Psi^3(s_M),...\} \tag{12}$$

$$\{p_L,p_Lp_H,p_Lp_H^2,p_Lp_H^3,...,p_M,p_Mp_H,p_Mp_H^2,p_Mp_H^3,...\} \tag{13}$$

for $(\beta,\lambda,R) \in E_2$.

The IFS defined in (7) contains two constant maps s_L and s_M and one non-constant map $\min\{\Psi(s_{t-1}),s_H\}$, which converges either to s_H (c.f. Proposition 3.1 b) in a finite number of iterations or monotonically approaches s^* (c.f. Proposition 3.1 c). This implies that the attractor of the IFS can neither be a fractal set nor can the system exhibit chaotic dynamics in the sense of Gardini et al. (2009), Mitra, Montrucchio and Privileggi (2004), Mitra and Privileggi (2004, 2009). The stationary behavior features a mix of steady states, cycles and finite monotone sequences across which the system randomly wanders.

3.3 Income inequality

The borrowing constraint in our model keeps the return on lending below the marginal product of capital and therefore creates a wedge of income between lenders and borrowers as in Aghion and Bolton (1997), Banerjee and Newman (1993) and Galor and Zeira (1993). In both equilibria s_L and s_M where the borrowing constraint is binding, the return from lending is lower than the return from running investment projects, i.e,

$r(s_t) = 1/\beta < Rf'(Rs_t)$ for $s_t = s_L, s_M$. The wedge measures income inequality between entrepreneurs and investors, which is greater in s_L than in s_M. In equilibrium s_H where the borrowing constraint is not binding, $r(s_H) = \frac{1}{\beta} = Rf'(Rs_H)$ and thus no inequality exists. Combing these observations with the long-run behavior of the economy, our model then predicts for a given investment technology R that the economy converges to a low (high) saving steady state with (without) income inequality if financial market imperfection is high (low) (c.f. Proposition 2.2 a and c and Proposition 3.1 a). If we take the level of financial market imperfection as a proxy for financial development, our model predicts financial development reduces inequality. When the economy fluctuates due to self-fulfilling beliefs, income inequality can also rise and fall.[16]

4 Discussion

The dynamics of the aggregate saving drives the dynamics of all the other economic variables such the wage, the return on capital, etc. For the following discussion we focus on the aggregate saving rate, which in period t is defined as $\frac{s_t}{f(s_{t-1}R)}$. The steady state aggregate saving rate is increasing in the aggregate saving and is equal to $1-\alpha$ in s^* and less than $1-\alpha$ in all other steady states.[17] When multiple equilibria exist in period t, the aggregate saving rate can jump up or down as the saving can be s_L, s_M or $\min\{\Psi(s_{t-1}), s_H\}$ for a given $f(s_{t_1}R)$. This implies self-fulfilling beliefs can cause excessive volatility and sudden changes in the aggregate saving rate.

4.1 Volatility of the saving rate

The model predicts that countries which have a productive investment technology ($R > \max\{R^c, R^{cc}\}$) but a moderately developed financial market (an intermediate level of λ) are vulnerable to self-fulfilling beliefs and exhibit potentially high volatility (c.f. Proposition 3.2 b).[18] Although testing these predictions is beyond the scope of this paper, we briefly review a panel of OECD countries.[19] To this end, Figure 10 plots the

[16] We note that s_L is increasing in λ while s_M is decreasing in λ. This implies that higher financial market imperfection leads to higher volatility of the saving rates as well as greater inequality in s_L but smaller inequality in s_M.

[17] The steady state aggregate saving rate $\frac{s}{f(sR)}$ is increasing in s because f is concave.

[18] Existence of multiple steady states also requires that $\beta \in (0, \overline{\beta}(\alpha))$ for any $\alpha \in (0,1)$.

[19] See Benhabib and Farmer (1999) for a survey on econometrics of multiple equilibria in macroeconomics.

sample standard deviation of the gross saving rate against the sample average level of the deposit to lending rate ratio using data from the World Bank Development Indicators.[20] The gross saving rate is measured as a percentage of GDP, and the deposit to lending rate ratio reflects the monitoring cost of banks, which is taken as a proxy for the level of financial market imperfection. The points in the figure correspond to 32 countries.[21] For each country, time series observations are obtained for the period 1960-2011.

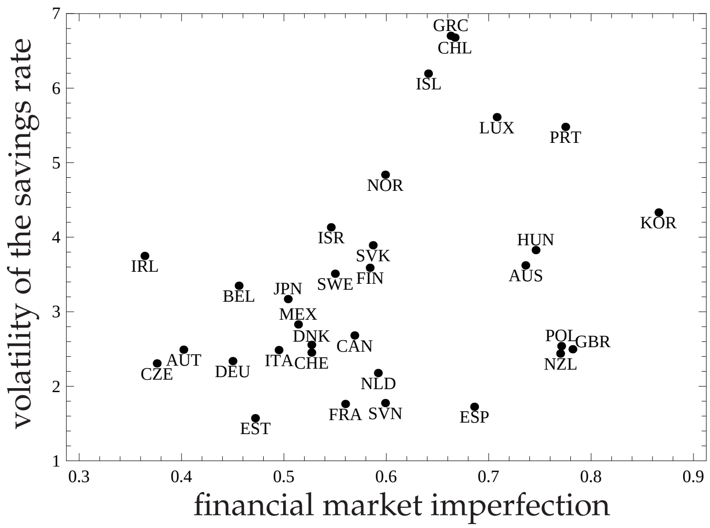


Figure 10: Volatility of the saving rate against financial market imperfection

The figure shows that countries with a relatively high volatility in the saving rate have an intermediate level of financial market imperfection. On the other hand, countries with either a low or a high level of the imperfection have a low volatility. These findings are consistent with the predictions of our model.[22]

[20]Detailed descriptions of the measurement of the data is given in the appendix.

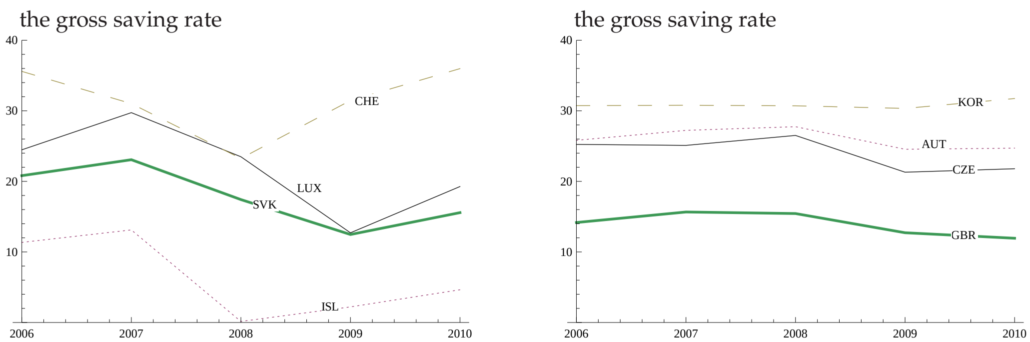
[21]Country codes presented in Figure 10 are: AUS - Australia, AUT - Austria, BEL - Belgium, CAN - Canada, CHL - Chile, CZE - Czech Republic, DNK - Denmark, EST - Estonia, FIN - Finland, FRA - France, DEU - Germany, GRC - Greece, HUN - Hungary, ISL - Iceland, IRL - Ireland, ISR - Israel, ITA - Italy, JPN - Japan, KOR - Korea, Republic, LUX - Luxembourg, MEX - Mexico, NLD - Netherlands, NZL - New Zealand, NOR - Norway, POL - Poland, PRT - Portugal, SVK - Slovak Republic, SVN - Slovenia, ESP - Spain, SWE - Sweden, CHE - Switzerland, GBR - United Kingdom.

[22]Note that our theoretical model does not predict that all countries with an intermediate level of the financial market imperfection necessarily have a high volatility as the dynamics still depends on the probability attached to each state.

4.2 Sudden changes in the saving rate

Figure 10 shows that Chile has the highest volatility together with Greece. However, unlike Greece, which has a clear downward trend for the entire sample period, Chile's volatility in the saving rate comes mostly from its deep dip during 1982-1984.[23] Our model suggests that the debt and banking crisis may have played a role of a sunspot, which coordinated self-fulfilling beliefs causing the sudden decline and the subsequent increase of the saving rate in Chile.

The financial crisis in 2008 may also provide an example of the possible role of a sunspot. Figure 11 shows time series of the gross saving rate during 2006-2010 for two groups of selected countries.[24] Four countries with the highest volatility are depicted



(a) sharp decline and recovery

(b) relatively moderate changes

Figure 11: The saving rate before and after the financial crisis in 2008

in the panel (a) and they all have an intermediate level of financial market imperfection according to Figure 10.[25] The finding is consistent with our model's prediction

[23]The gross saving rate in Chile during 1975-80 was about 15.58%, then it felt to 1.48% in 1982 and by 1988 it stabilized to 22.39%. Some argue that the debt and banking crisis during 1982-83 was largely responsible for the initial rapid decline and the privatization of the pension system implemented in 1980-81 for the subsequent increase (e.g. Moranfeé 1998). In contrast, Schmidt-Hebbel (2001) estimates that most of the saving rate movement is unaccounted and only up to 12.2% of the increase in the national saving rate can be attributed to the pension reform. Samwick (2000) provides empirical evidence that the saving rate did not increase in other OECD countries such as Switzerland and Australia which also reformed their pensioned systems. Even more, Samwick (2000) concludes based on cross-country regressions of 94 reforming countries that pension reforms have significant negative effects on the saving rate. These studies show that pension reforms in different countries have mixed effects.

[24]We have dropped Ireland for this example as the saving rate in Ireland shows a clear downward trend during this period.

[25]We have not used the sample average of financial market imperfection during the period 2006 and 2010 as there are too many countries with missing data. This may not be too much of concern as we do not look at the variation within a country but across countries. No clear time trend in financial market

that countries that experience rapid changes in the saving rate have an intermediate level of financial market imperfection. Four countries with the lowest and the highest level of the imperfection are depicted in the panel (b). In contrast to those in the panel (a) countries in the panel (b) have relatively moderate fluctuations of the saving rate during the period. The finding is also consistent with our model's prediction that countries with either a high or a low financial market imperfection are not subject to rapid changes in the saving rate.

5 Conclusion

In Matsuyama's (2004) two period overlapping generations model, the resource in the economy at any point in time is not sufficient for all the young to invest in capital due to investment indivisibility. When the borrowing constraint is binding the young strictly prefer borrowing to lending. He assumes that the young do not consume and therefore simply save the wage. The autarky economy converges to a unique steady state independently of the interest rate. Allowing for intertemporal saving decisions in his model raises new modeling issues. In this class of models, in which there exists credit rationing in equilibrium, it is not obvious how ex-ante identical agents should make their saving decisions in a competitive market. In this paper we considered a symmetric Nash equilibrium, in which the credit is allocated randomly and therefore everyone makes the same saving decision. We may find an alternative equilibrium in which ex-ante identical agents make ex-post different saving decisions. Some may be willing to save more than others and pay a higher interest rate to obtain credit. We leave the investigation to future research.

In the symmetric Nash equilibrium we show that a strategic complementarity in saving decisions arises due to a borrowing constraint and investment indivisibility. The complementarity generates multiple equilibria and steady states and thus the possibility of self-fulfilling beliefs. Self-fulfilling beliefs determine the long-run dynamics when the financial market is not developed enough to match the productivity of investment technology. Under this circumstance, phenomena such as a poverty trap, a big push and a sunspot equilibrium occur accompanied by a sudden change in the saving rate and income inequality without any shocks to fundamentals.

imperfection was observed for the selected countries.

6 Appendix

6.1 Remaining proofs

Proof of Lemma 2.1. Taking log and differentiating $H(s,\lambda,R)$ with respect to s we obtain that $H_s(s,\lambda,R) = 0 \Leftrightarrow s/(1-s) = \varepsilon(Rs)$. Assumption 2.1 implies that there exists a unique solution $s^c = \phi(R) \in (0,1)$ to $s/(1-s) = \varepsilon(Rs)$. Since $H(0,\lambda,R) = \infty$ and $H(1,\lambda,R) = \infty$, H obtains its minimum at the solution. The implicit function theorem implies the differentiability of ϕ and

$$\frac{\phi(R)}{1-\phi(R)} \equiv \varepsilon(R\phi(R)). \tag{14}$$

Differentiating both sides of (14) we obtain

$$\phi'(R) = \frac{\varepsilon'(R\phi(R))\phi(R)}{(1+\varepsilon(R\phi(R)))^2 - \varepsilon'(R\phi(R))R} \leq 0, \tag{15}$$

where the weak inequality follows from $\varepsilon' \leq 0$ in Assumption 2.1. □

The two functions $\psi_1 : (0,R^+] \to \mathbb{R}_{++}$ and $\psi_2 : (0,R^+] \to \mathbb{R}_{++}$ are given by

$$\psi_1(R) := 1 - (f')^{-1}\left(\tfrac{1}{\beta R}\right)\tfrac{1}{R} \quad \text{and} \quad \psi_2(R) := \tfrac{1-\phi(R)}{\beta R f'(R\phi(R))}.$$

Lemma 6.1. *If Assumption 2.1 is satisfied, then*

(a) ψ_1 is strictly decreasing with $\lim_{R\downarrow 0}\psi_1(R) = 1$ and $\psi_1(R^+) = 0$.

(b) ψ_2 is strictly decreasing.

(c) There exists a unique $R^c \in (0,R^+)$ which solves $\psi_1(R) = \psi_2(R)$. Moreover, $\psi_1(R) < \psi_2(R)$ for any $R \neq R^c$ and $\psi_1(R)$ and $\psi_2(R)$ are tangent to each other at $R = R^c$.

Proof of Lemma 6.1. (a) From the definition of ψ_1 we have

$$\beta R f'(R(1-\psi_1(R))) \equiv 1. \tag{16}$$

Taking log of both sides of (16) and then differentiating it, we obtain

$$\psi_1'(R) \quad = \quad \tfrac{1-\psi_1(R)}{R}\left(1 - \tfrac{1}{\varepsilon(R(1-\psi_1(R)))}\right) < 0 \tag{17}$$

where the last inequality follows from Assumption 2.1 (a) and $\psi_1(R) < 1$ (for any $R > 0$).

From (16),

$$\beta\rho(R)f'(R(1-\psi_1(R))) \equiv f'(R). \tag{18}$$

By definition $\beta\rho(R_1^+) = 1$. This with (18) and the monotonicity property of f' implies that $\psi_1(R_1^+) = 0$.

By multiplying both sides of (16) by $1-\psi_1(R)$ and taking the limits of both sides as R approaches zero, we obtain $0 = \lim_{R\downarrow 0}\rho(R(1-\psi_1(R))) = 1 - \lim_{R\downarrow 0}\psi_1(R) = 1-m > 0$. This with continuity and monotonicity of ψ_1 implies that $\lim_{R\downarrow 0}\psi_1(R) = 1$.

(b) Taking log of both sides of $\psi_2(R) := (1-\phi(R))/(\beta R f'(R\phi(R)))$ and differentiating it, we obtain

$$\begin{aligned} \frac{R\psi_2'(R)}{\psi_2(R)} &= -\frac{R\phi'(R)}{\phi(R)}\frac{\phi(R)}{1-\phi(R)} - 1 + \varepsilon(R\phi(R))\left(1+\frac{R\phi'(R)}{\phi(R)}\right) \\ &= -1+\varepsilon(R\phi(R)) < 0, \end{aligned} \tag{19}$$

where the second line follows from $\varepsilon(R\phi(R)) \equiv \phi(R)/(1-\phi(R))$ and the inequality follows from Assumption 2.1 (a).

(c) We observe that

$$H(1-\psi_1(R), \psi_1(R), R) \equiv H(\phi(R), \psi_2(R), R) < H(1-\psi_1(R), \psi_2(R), R),$$

if and only if $1-\psi_1(R) \neq \phi(R)$. Because $H(s, \cdot, R)$ is increasing for any given (s, R), this implies that if $1-\psi_1(R) \neq \phi(R)$, then $\psi_2(R) > \psi_1(R)$ and if $1-\psi_1(R) = \phi(R)$, then $\psi_2(R) = \psi_1(R)$. Thus solving $\psi_2(R) = \psi_1(R)$ is equivalent to solving $1-\psi_1(R) = \phi(R)$ or

$$\beta\rho\left(R\phi(R)\right) - \phi(R) = 0. \tag{20}$$

First, we observe that $R \mapsto R\phi(R)$ is strictly increasing since, using (15),

$$R\phi'(R) + \phi(R) = \frac{\phi(R)(1+\varepsilon(R\phi(R)))^2}{(1+\varepsilon(R\phi(R)))^2 - \varepsilon'(R\phi(R))R} > 0$$

where the last inequality follows from $\varepsilon' \leq 0$ in Assumption 2.1. Second, we observe that $\lim_{R\downarrow 0}\phi(R) = \frac{\underline{\varepsilon}}{1+\underline{\varepsilon}}$ where $\underline{\varepsilon} = \lim_{k\downarrow 0}\varepsilon(k)$. Together with Lemma 2.1 stating that $\phi(R)$ is non-increasing, this implies that the left hand side of (20) is strictly increasing in R and satisfies the boundary conditions

$$\lim_{R\downarrow 0}\beta\rho\left(R\phi(R)\right) - \phi(R) = -\frac{\underline{\varepsilon}}{1+\underline{\varepsilon}} < 0$$

and, as $\phi(R^+) < 1$ and f' is decreasing, by definition of ρ we get

$$\beta\rho\left(R^+\phi(R^+)\right) - \phi(R^+) > \phi(R^+)\left(\beta\rho(R^+) - 1\right) > 0.$$

The intermediate value theorem ensures a unique solution $R^c \in (0, R^+)$ to (20). Next, we verify that $\psi_1(R)$ and $\psi_2(R)$ are tangent to each other at $R = R^c$. Multiplying both sides of (19) by $R/\psi_1(R)$, we obtain

$$\frac{R\psi_1'(R)}{\psi_1(R)} = -\frac{1-\psi_1(R)}{\psi_1(R)}\frac{1-\varepsilon(R(1-\psi_1(R)))}{\varepsilon(R(1-\psi_1(R)))}. \tag{21}$$

Since $\psi_1(R) = \psi_2(R) = 1 - \phi(R)$, it follows from (19) and (21) that $\psi_1'(R) = \psi_2'(R)$. □

Proof of Proposition 2.1. The proposition follows directly from Lemma 6.1. □

Proof of Proposition 2.2. The proposition follows directly from Proposition 2.1. □

For the Cobb-Douglas production function $f(k) = k^\alpha$ we obtain

$$\phi(R) = \tfrac{1-\alpha}{2-\alpha}, \quad R^c = (\tfrac{1}{\alpha\beta})^{\frac{1}{\alpha}}(\tfrac{1-\alpha}{2-\alpha})^{\frac{1-\alpha}{\alpha}}, \quad R^{cc} = \frac{1}{(1-\alpha)(2-\alpha)^{\frac{1-\alpha}{\alpha}}}, \quad R^+ = (\tfrac{1}{\alpha\beta})^{\frac{1}{\alpha}}$$

$$\psi_1(R) = 1 - (\alpha\beta R^\alpha)^{\frac{1}{1-\alpha}}, \quad \psi_2(R) = \tfrac{1}{\alpha\beta}\tfrac{(1-\alpha)^{1-\alpha}}{(2-\alpha)^{2-\alpha}R^\alpha}, \quad \psi_3(R) = \tfrac{(1-\alpha)[1-((1-\alpha)R^\alpha)^{\frac{1}{1-\alpha}}]}{\alpha\beta}.$$

Lemma 6.2.

(a) *ψ_3 is strictly decreasing.*

(b) *$\psi_2(R)$ and $\psi_3(R)$ are tangent to each other at $R = R^{cc}$ and $\psi_3(R) < \psi_2(R)$ for any $R \neq R^{cc}$.*

(c) *$(\psi_1(R), \psi_2(R)) \subset (\psi_3(R), \psi_2(R))$ for $R \in (R^c, R^+)$*
$(\psi_3(R), \psi_2(R)) \subset (\psi_1(R), \psi_2(R))$ for $R \in (R^{cc}, R^+)$

Proof of Proposition 3.1. It can be easily verified that if $(\beta, \lambda, R) \in \mathcal{E}$ and either $R < R^{cc}$ or $\lambda < \psi_3(R)$, there exists a unique steady state $\min\{s_L, s^*\}$. The necessary and sufficient condition for existence of multiple steady states is $s_M < s^*$. Note that $\beta < \underline{\beta}(\alpha) \Leftrightarrow s_H < s^*$. Lemma 6.2 establishes that $(\beta, \lambda, R) \in \mathcal{E}_1$ and $(\beta, \lambda, R) \in \mathcal{E}_2$ are necessary and sufficient conditions for $s_M < s^*$. □

Proof of Lemma 3.1. If $(\beta, \lambda, R) \in E_1 \cup E_2$, then $s_L < \phi(R) < s_M < s^*$. If λ is sufficiently close to $\psi_2(R)$, then $s_L \approx s_M$. This implies that $s_L \approx \phi(R) \approx s_M < \Psi(s_L)$ since $s_L < s^*$. Hence, $s_M < \Psi(s_L)$. Continuity of both s_L and s_M in parameters ensures the claim. □

Proof of Proposition 3.3. First, we consider the parameter set $\mathcal{E}_1$ corresponding to Proposition 3.1 (b) where $s_H < s^* \Leftrightarrow \beta < \underline{\beta}(\alpha)$. As all possible equilibria have positive probability, for any initial condition $s_0 \in (0,1)$, the trajectory will end up in a few iterations in one of the two constants s_L or s_M and from that point on the possible states are only those of the finite set given by (10). The assumption of i.i.d for the stochastic process easily guarantees existence and uniqueness of the stationary distribution. Since the probability of returning to states s_L, s_M and s_H is p_L, p_M and p_H respectively, it follows that the stationary probability distribution over all possible states is given by (11). In this case, the interval $[s_L, s_H]$ is a forward invariant set.

Second, we consider the parameter set $\mathcal{E}_2$ corresponding to Proposition 3.1 (c) where $s_H \geq s^* \Leftrightarrow \beta \geq \underline{\beta}(\alpha)$. For any initial condition $s_0 \in (0,1)$, the trajectory will end up in a few iterations in one of two constants s_L or s_M and from that point on the possible states are only those of the countable set given by (12). It is worthwhile to note that s^* does not belong to the above set but belongs to its closure. Since the probability of returning to state s_L and s_M is p_L and p_M respectively and both p_L and p_M are independent of the current state, it follows that the stationary probability distribution over all possible states is given by (13). In this cases, the interval $[s_L, s^*)$ is a forward invariant set. $\square$

6.2 Data

The data is obtained from the World Bank Development Indicators for the period 1960-2011. First, we calculate the sample average standard deviation of the time series of the saving rate for all 34 OECD countries. Second, we calculate the sample average of the deposit to lending rate ratio for 32 countries. The deposit rate data is missing for the United States and the lending rate data is missing for Turkey. The deposit rate is measured as the rate at which commercial banks pay on demand, time or savings deposits while lending rate is measured as the rate which commercial banks charge on loans to prime customers.

# of observations	1-5	6-10	11-15	16-20	21-25	26-30	31-35	36-40	41-45	46-51
# of countries	0	0	0	6	0	1	5	12	7	3

Table 1: Annual data for the saving rate

# of observations	1-5	6-10	11-15	16-20	21-25	26-30	31-35	36-40	41-45	46-51
# of countries	1	0	1	7	7	4	6	4	1	1

Table 2: The lending rate to deposit rate ratio

The two time series data might not perfectly match for some countries. For example, France has the saving rate from 1975 to 2011 (37 observations) and the lending rate to deposit rate ratio from 1966 to 2004 (39 observations). In this case 1975-2004 is the overlapping time period (30 observations) and we define the overlap coefficient as 30/36=81%. Among 32 countries, which are included in the sample, the overlap coefficient is 100% for Japan. Table 3 shows that the overlap coefficient is at least 50% for 29 countries which is 91% of all counties included in the sample.

overlap coeff.	5-49%	50-59%	60-69%	70-79%	80-89%	90-100%
# of countries	3	3	7	5	8	6
% of total	9%	9%	22%	16%	25%	19%

Table 3: Overlap coefficient

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