Financial Liberalization: Poverty Trap or Chaos*

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Abstract

We construct an overlapping generations model in which people are subject to limited pledgeability and uncertainty over entrepreneurial projects. We show that whether financial liberalization generates inequality, volatility, or both depends on the interaction of pledgeability and risk. Volatility requires a high level of both pledgeability and risk. Multiplicity of steady states requires a low level of both. For an intermediate level of both, we observe large heterogeneity in dynamics even between *inherently identical* economies: some may experience volatility and others stagnate though financial liberalization depending on their initial conditions.

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1 Introduction

Financial crises in East Asia, Southeast Asia and Latin America in the 1990s were preceded by financial liberalization, which facilitated borrowing in the international financial

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market. Capital inflows created investment booms. When crises hit, the forces reversed. Capital flowed out and investment collapsed. Critics attributed the link between financial liberalization and financial crises to insufficiently developed financial markets in the emerging economies. Gaytan and Ranciere (2004) and Daniel and Jones (2007) provide evidence that emerging markets are particularly vulnerable to financial crises.

Since Bernanke and Gertler (1989) and Kiyotaki and Moor (1997) studied the macroeconomic effects of financial frictions, the literature has extended the analysis to open economies. Gertler and Rogoff (1990), Boyd and Smith (1997), Matsuyama (2004), Aoki et al. (2009) show that contracting frictions can restrict an economy's ability to borrow from the international financial market thereby generating capital outflows even in a capital-scarce economy. When countervailing forces are present, financial liberalization may destabilize the economy and periods of capital inflows may be followed by periods of capital outflows generating endogenous fluctuations. Studies of financial frictions as a cause of endogenous fluctuations in open economies include Aghion et al. (2004), Caballe et al. (2006), Kikuchi (2008), Kikuchi and Stachurski (2009) and Martin and Taddei (2013). Our paper is closely related to the small open economy model of Aghion et al. (2004), Caballe et al. (2006), and Martin and Taddei (2013).[1] Their models are nonetheless different in that the countervailing force to the effect of financial frictions is a rising cost of the country-specific input factor in Aghion et al. (2004) and Caballe et al. (2006) and a declining cross-subsidization in the presence of adverse selection in Martin and Taddei (2013). In our model, entrepreneurs are willing to run risky entrepreneurial projects only for a higher expected return when the wage is higher. Therefore, the fraction of entrepreneurs in the economy—and thus total capital investment—may decline with capital stock.[2]

Taking limited pledgeability as a measure for financial development, Aghion et al. (2004) and Caballe et al. (2006) predict that an economy with either a very developed or a very undeveloped financial market is globally stable, while an emerging market economy with an intermediate level of financial development is unstable. This prediction is not consistent with the world financial crisis in 2008 when even the most developed financial markets in Europe and in the USA were not immune to typical forces of financial crises. Our model shows that the economy fluctuates endogenously even when entrepreneurs can borrow up to the net present value of their project.

We construct an overlapping generations model in which people are subject to limited

[1] Interaction between two economies through financial markets give rise to endogenous fluctuations in Kikuchi (2008) and Kikuchi and Stachurski (2009).

[2] The role of entrepreneurial risk for global imbalance is a focus in a two country model by Angeletos and Panousi (2011) who study the intertemporal costs and benefits of capital-account liberalization.

pledgeability and uncertainty over entrepreneurial projects. In the formulation of a borrowing constraint we follow Matsuyama (2004). In autarky the interest rate adjust so that savings equal investment. Hence, capital accumulation is not affected by the level of risk and pledgeability. The economy then converges monotonically to a steady state. We compare this result with the dynamics in the small open economy affected by interaction of the borrowing constraint and the entrepreneurial risk. Capital inflows relax the borrowing constraint and promote further capital accumulation. In the presence of diminishing return, however, the fraction of entrepreneurs in the economy, and thus capital investment, declines with capital stock as more saving is channeled to the international financial market for a safe return.[3]

We completely characterize the dynamics of the model.[4] Depending on the parameters, three steady states may emerge: a low, middle, and high steady state. The low steady state is always stable and the middle steady state is always unstable. Whether financial liberalization destabilizes the economy depends on the stability of the high steady state, which is unstable only if the risk associated with entrepreneurial projects is high. When the high steady state is unstable, we obtain four types of dynamics. First, the dynamics converges to the low steady state for low pledgeability. Second, the dynamics converges to a cycle of order two for high pledgeability. Hence, our model predicts that financial liberalization can be a source for instability even with no borrowing constraint.[5] Third, the economy moves back and forth between two regimes generating chaotic dynamics as in Caballe et al. (2006) for intermediate pledgeability. Fourth, when multiple steady states emerge for intermediate pledgeability, the economy is history dependent and converges either to the low steady state or to a chaotic dynamics around the high steady state. To the best of our knowledge we do not know any other models in the literature that generate the large heterogeneity in observed dynamics—volatility and poverty trap—between *inherently identical* economies.

Theoretical contributions have identified several channels how financial liberalization may magnify both inequality (e.g. Gertler and Rogoff 1990, Boyd and Smith 1997, Matsuyama 2004, and Aoki et al. 2009) and volatility (e.g. Aghion et al. 2004, Caballe et al. 2006, and Martin and Taddei 2013). The literature has nonetheless treated the two effects separately. Our model shows that whether financial liberalization generates in-

[3]Unlike in Angeletos and Panousi (2011), the overlapping generations structure in our model implies that the returns from investing overseas are consumed and the economy does not accumulate wealth.

[4]Our map for the evolution of capital stock is unimodal. Studies of dynamics arising from unimodal maps can be found in Matsuyama (1999), Mitra (2001), Mukherji (2005) and Gardini et al. (2008).

[5]Mendoza et al. (2007) show that the lack of insurance markets in developing economies fosters precautionary savings and the consequent capital outflows.

equality, volatility, or both depends on the interaction of pledgeability and risk of entrepreneurial projects. To demonstrate our results we provide a complete characterization of bifurcation curves in the space of the level of pledgeability and the riskiness of entrepreneurial projects.

The structure of the paper is as follows. Section 2 sets up the model. Section 3 derives equilibrium conditions. Section 4 characterizes the autarky economy as a benchmark. Section 5 characterizes the small open economy. Section 6 analyzes steady states and the global dynamics of the small open economy. Section 7 summarizes the main results. Section 8 concludes.

2 Set up

The economy consists of overlapping generations who live for two periods, supplying one unit of labor when young and consuming only when old. Successive generations have unit mass. Production combines the current stock of capital k_t supplied by the old with the unit quantity of labor supplied by the young.[6] The resulting per-capita output is $f(k_t) := k_t^\alpha$ where $\alpha \in (0,1)$ is the capital share in production. Factor markets are competitive: young agents receive the wage $w_t = W(k_t) := (1-\alpha)k_t^\alpha$ and old agents the return on capital $f'(k_t) := \alpha k_t^{\alpha-1}$. After production and the distribution of factor payments, the old consume and exit the model while the young make investment decisions.

When investing in period t, the young can either lend in the competitive financial market for a safe return r_{t+1} or run a risky project. The project takes one unit of the consumption good, and returns Z_{t+1} units of the capital good in period $t+1$.[7] The return Z_{t+1} is assumed to be a random variable which is independent and identically distributed across agents and over time, and takes a value $z > 0$ with probability $p \in (0,1]$ and 0 with probability $1-p$.

If all the young start projects, the capital stock in period $t+1$ is zp. Hence, the resource constraint is

$$0 \leq k_{t+1} \leq zp. \tag{1}$$

We assume that $W(zp) = 1$. Given the resource constraint (1), we then have $w_t < 1$ for all t, and the young must borrow $1 - w_t$ to start a project. We assume that the investment risk

[6] Capital depreciates fully between periods, so that capital stock in any period is equal to investment in the previous period.

[7] Factors of production are not internationally mobile and projects can only be run domestically.

is uninsurable. This may be justified on the ground that entrepreneurial investment such as venture capital investment is typically associated with high risk that is uninsurable.[8] We also assume that entrepreneurs meet the obligation when the project succeeds, and declare bankruptcy when the project fails.

Perfect competition in the credit market implies that entrepreneurs must repay $r_{t+1}(1 - w_t)/p$ in the period $t+1$; the lending rate is higher than the borrowing rate because of the default risk. Hence, the consumption of entrepreneurs in period $t+1$ is

$$c_{t+1} = \begin{cases} zf'(k_{t+1}) - \frac{r_{t+1}}{p}(1 - w_t) & \text{with } p \\ 0 & \text{with } 1 - p. \end{cases}$$

The young are willing to start an investment project—become an entrepreneur—whenever the expected utility from running a project is at least as high as lending in the financial market:

$$u(r_{t+1}w_t) \leq \mathbb{E}u(c_{t+1}). \tag{2}$$

We refer to this inequality as the participation constraint. Following Matsuyama (2004) we also assume that a borrower can only credibly pledge a fraction $\lambda \in (0, 1]$ of the revenue when the project succeeds. The borrowing constraint is therefore

$$\frac{r_{t+1}}{p}(1 - w_t) \leq \lambda z f'(k_{t+1}). \tag{3}$$

The parameter $\lambda \in (0, 1]$ measures the level of pledgeability, with a higher value corresponding to higher pledgeability (See Matsuyama 2004 for further interpretation issues). When $\lambda = 1$, then the borrowing constraint never binds.

3 Equilibrium

To become an entrepreneur the young must be both willing and able to borrow, i.e., both participation (2) and borrowing (3) constraints must be satisfied. Equilibrium implies that capital stock k_{t+1} adjust so that either (2) or (3) binds.

Let the utility function be given by $u(c) = c^\gamma$ where $\gamma \in (0, 1]$. When the participation constraint (2) binds, the young are indifferent between borrowing for a project and

[8] Macroeconomic literature on entrepreneurship commonly assumes substantial uninsurable investment risk (e.g. Evans and Jovanovic 1989, Quadrini 2000 and Cagetti and De Nardi 2006).

lending for a safe return. Then the safe return is given by

$$r_{t+1} = \frac{zp}{1+\xi w_t} f'(k_{t+1}) \tag{4}$$

where $\xi := p^{\frac{\gamma-1}{\gamma}} - 1 \geq 0$. The parameter measures the certainty equivalent of running a risky project because $u\left(r_{t+1}w_t\right) = \mathbb{E}u\left(c_{t+1}\right) = u(\frac{\mathbb{E}c_{t+1}}{1+\xi})$ when (2) binds. If $\xi = 0$, the certainty equivalent is the expected return of a risky project, which is possible only when either entrepreneurs are risk neutral ($\gamma = 1$), or when there is no uncertainty ($p = 1$).[9] Entrepreneurial projects are more attractive when projects are less risky or entrepreneurs are less risk-averse. Hence, the attractiveness of entrepreneurial projects declines with ξ. In other words, a project is perceived more risky if ξ is higher. In particular, entrepreneurs would choose to start investment projects only when the expected consumption exceeds $(1+\xi)$ times that of investors.

When the borrowing constraint (3) binds the safe return is given by

$$r_{t+1} = \frac{\lambda zp}{1-w_t} f'(k_{t+1}) \tag{5}$$

In this case, $\mathbb{E}u(c_{t+1}) > u(r_{t+1}w_t)$, and thus all the young would strictly prefer to take credit and run a project. However, the minimum investment requirement does not allow all to take credit and therefore a fraction of the young will be credit rationed; the rest will become entrepreneurs. Combining (4) and (5)

$$r_{t+1} = \begin{cases} \frac{\lambda\alpha zp}{1-w_t}\left(\frac{1-\alpha}{w_{t+1}}\right)^{\frac{1-\alpha}{\alpha}} & \text{if } w_t < w^c \\ \frac{\alpha zp}{1+\xi w_t}\left(\frac{1-\alpha}{w_{t+1}}\right)^{\frac{1-\alpha}{\alpha}} & \text{if } w_t \geq w^c \end{cases} \tag{6}$$

where $w^c := \frac{1-\lambda}{1+\lambda\xi} \in (0,1)$.

4 The autarky economy

The autarky economy serves as a benchmark to investigate the effects of financial liberalization. In the absence of the international financial market the interest rate adjusts so that all savings is used to finance the risky investment projects. Therefore, the capital stock evolves according to

$$k_{t+1} = zpw_t$$

[9] In this case, the model reduces to the one analyzed by Matsuyama (2004).

which is independent of individual risks. Given the assumption $W(zp) = 1$ we can rewrite the equation as

$$w_{t+1} = w_t^{\alpha}. \tag{7}$$

We obtain that $w_{t+1} \in (0,1]$ for any $w_t \in (0,1]$, and the economy converges monotonically to a unique steady state $w^* = 1$ for any initial wage rate $w_0 \in (0,1]$. The resource constraint (1) never binds and a dynamical system of the autarky is defined on the state space $(0,1]$. Substituting (7) into (6) we obtain the safe return in autarky

$$r_{t+1} = \begin{cases} \frac{\alpha}{1-\alpha}\frac{\lambda}{1-w_t}\left(\frac{1}{w_t}\right)^{1-\alpha} & \text{if } w_t < w^c \\ \frac{\alpha}{1-\alpha}\frac{1}{1+\xi w_t}\left(\frac{1}{w_t}\right)^{1-\alpha} & \text{if } w_t \geq w^c. \end{cases} \tag{8}$$

5 The small open economy

Everyone in the small open economy can trade intertemporally the final goods with the rest of the world. In other words, international lending and borrowing is allowed. The safe return is exogenously given in the international financial market and assumed to be invariant over time: $r_{t+1} = r > 0$. Simply by solving for w_{t+1} in (6) we obtain the evolution of the wage in the small open economy

$$w_{t+1} = \phi(w_t) := \min\{g(w_t), h(w_t), 1\} \tag{9}$$

where

$$g(w_t) := \left(\frac{1}{r}\frac{\alpha}{1-\alpha}\frac{\lambda}{1-w_t}\right)^{\frac{\alpha}{1-\alpha}} \quad \text{and} \quad h(w_t) := \left(\frac{1}{r}\frac{\alpha}{1-\alpha}\frac{1}{1+\xi w_t}\right)^{\frac{\alpha}{1-\alpha}}.$$

Map (9) defines a dynamical system $((0,1],\phi)$. The map g is increasing and convex and thus has at most two fixed points, which we denote $w_L \in (0, 1-\alpha)$ and $w_M > 1-\alpha$. The steady state w_L is always stable, while the steady state w_M is always unstable. In contrast, the map h is decreasing and convex and has a unique fixed point, which we denote $w_H \in (w^c, 1)$. We can also see that g is increasing in λ and h is decreasing in ξ. This implies that multiple steady states emerge for low pledgeability (low λ) and that the steady state becomes unstable when the project risk is high (high ξ).

The map h originates from the participation constraint (2) and the map g originates from the borrowing constraint (3). The two maps have opposing effects on capital accumulation. Under the participation constraint (2) the fraction of entrepreneurs in the population decreases with the capital stock—the wage—because the return to capital diminishes. With the fraction of entrepreneurs shrinking and capital flowing to the international market, capital investment decreases with the wage. On the other hand, under the

borrowing constraint (3), the fraction of entrepreneurs increases with the wage because entrepreneurs need to borrow less to fund their investment project. With the fraction of entrepreneurs expanding and capital flowing in the economy, capital investment increases with the wage.

The resource constraint (1) restricts the number of projects to unit mass—the population. In other words, foreigners cannot start an investment project: foreign direct investment is ruled out.[10] Combined with the restriction $W(zp) = 1$ on the investment technology the wage is then bounded above by 1. Capital inflows increase the fraction of entrepreneurs in the population by giving them access to the world saving to fund their projects and capital outflows decrease the fraction by allowing more people to invest in the international market for a safe return. In other words, we focus on how risk sharing in the international financial market changes the allocation of resources in a small open economy.

Before analyzing the dynamics in detail, we first impose some parameter restrictions to focus on the dynamics unique to our model. If $\alpha < 0.5$, the steady sate w_H is stable (shown later in Proposition 3) and the dynamics of our model is qualitatively similar to the Matsuyama model: there is either a unique steady state (either w_L or w_H) or multiple steady states (w_L, w_M, w_H). In either case, the volatility of the small open economy diminishes in the long run as it converges to a steady state. Hence, in the what follows, we assume $\alpha > 0.5$. This assumption is not unreasonable when physical capital is interpreted as human capital. Moreover, we can easily specify a utility function such that w_H is unstable even for $\alpha < 0.5$. One such example is given in the appendix.

Let

$$\xi_1^c := \tfrac{\alpha}{1-\alpha}\tfrac{1}{r} - 1, \quad \lambda_1^c := \tfrac{1-\alpha}{\alpha} r, \quad \text{and} \quad \Lambda_1(\xi) := \tfrac{1}{1+\xi_1^c}\left(1 - \tfrac{\xi_1^c}{\xi}\right).$$

If $r > \frac{\alpha}{1-\alpha}$, then $\lambda_1^c > 1$ and $\xi_1^c < 0$, and the resource constraint (1) never binds: the resulting map (9) is $\min\{g(w_t), h(w_t)\}$. The remainder of the paper focuses on the case when $r < \frac{\alpha}{1-\alpha}$. Let $\Omega := \{(\xi, \lambda) | (\xi, \lambda) \in (0, \infty) \times (0, 1]\}$. The next proposition gives us all the possible configurations that the map (9) can take.[11]

[10] This restriction is reasonable if physical capital and the investment project are interpreted as human capital and education.

[11] Note $g(0) > 1 \Leftrightarrow \lambda > \lambda_1^c$, $h(1) > 1 \Leftrightarrow \xi < \xi_1^c$, and $g(w^c) = h(w^c) < 1 \Leftrightarrow \lambda < \Lambda_1(\xi)$.

Proposition 1. *Let $r < \frac{\alpha}{1-\alpha}$. For any pair $(\xi, \lambda) \in \Omega$ we have*

$$\phi(w) = \begin{cases} \min\{g(w), 1\} & \text{if } (\xi,\lambda) \in \Omega_0 := \{(\xi,\lambda) \in (0,\xi_1^c] \times (0,\lambda_1^c)\} \\ 1 & \text{if } (\xi,\lambda) \in \Omega_1 := \{(\xi,\lambda) \in (0,\xi_1^c] \times [\lambda_1^c,1]\} \\ \min\{g(w), h(w)\} & \text{if } (\xi,\lambda) \in \Omega_2 := \{(\xi,\lambda) \in (\xi_1^c,\infty) \times (0,\Lambda_1(\xi)]\} \\ \min\{g(w), 1, h(w)\} & \text{if } (\xi,\lambda) \in \Omega_3 := \{(\xi,\lambda) \in (\xi_1^c,\infty) \times (\Lambda_1(\xi),\lambda_1^c)\} \\ \min\{1, h(w)\} & \text{if } (\xi,\lambda) \in \Omega_4 := \{(\xi,\lambda) \in (\xi_1^c,\infty) \times [\lambda_1^c,1]\} \end{cases}$$

The proposition follows directly from the definitions of ξ_1^c, λ_1^c, and $\Lambda_1(\xi)$. For analyzing how the risk of investment projects (ξ) and the pledgeability (λ) affect the small open economy, the following numerical examples take $r = 1.35$ and $\alpha = 2/3$, which satisfy $r < \frac{\alpha}{1-\alpha}$.[12] Figure 1 illustrates the proposition by dividing the parameter space Ω into the five disjoint regions.

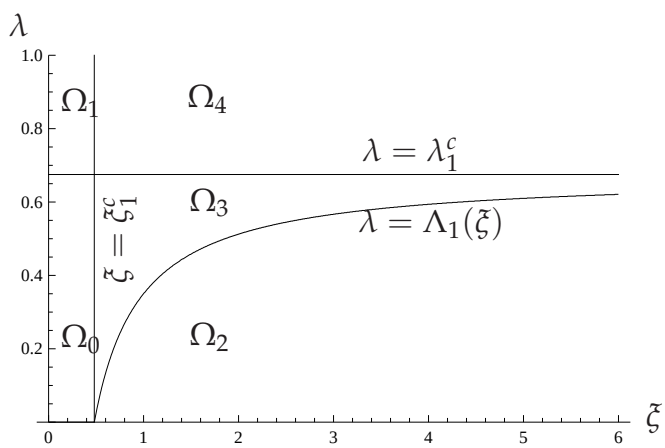
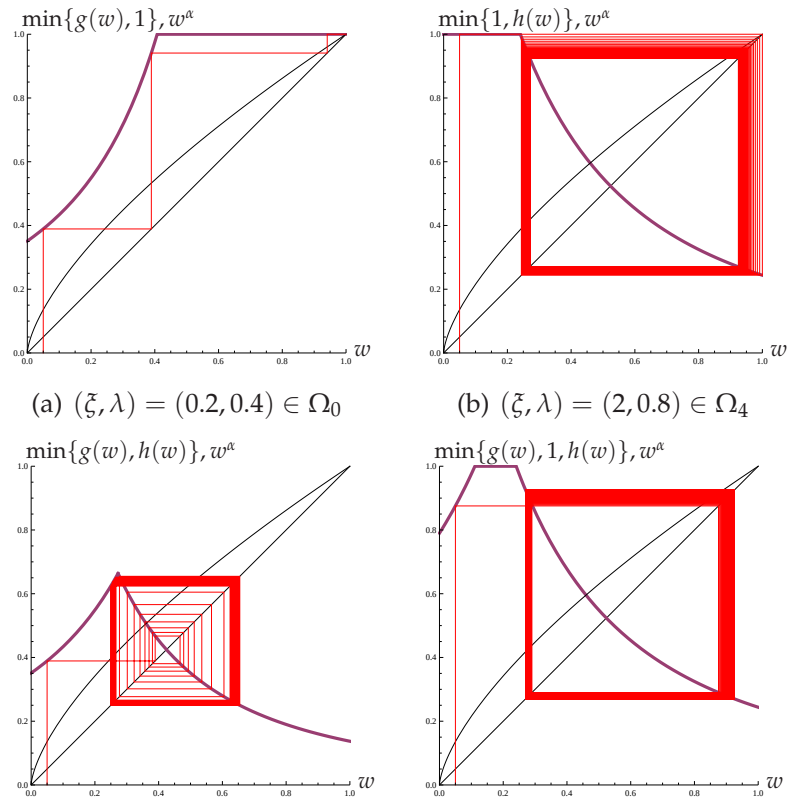


Figure 1: Different configurations of ϕ: $\alpha = 2/3$ and $r = 1.35$.

When $(\xi, \lambda) \in \Omega_1$ neither the borrowing constraint nor the risk plays any role; financial liberalization leads to inflows of the world saving and the economy converges immediately to the maximum steady state 1. To illustrate the map $\phi(w)$ for the remaining parameter space, Figure 2 shows cobweb plots for the map $\phi(w)$—the trajectory of the economy for a given initial condition— together with w^α. The current account balance is the difference between saving and investment, $w_t - \frac{k_{t+1}}{zp}$. Hence, the small open economy runs a current account deficit whenever $\phi(w_t) > w_t^\alpha$.

[12]The average risk-free rate in US is taken as the world interest rate, which was about 1.19% over the period 1892-1987 (See Cecchetti (1993) for more details). When we take one period to be 25 years, $r = (1 + 0.0119)^{25} \approx 1.35$.



(a) $(\xi, \lambda) = (0.2, 0.4) \in \Omega_0$ (b) $(\xi, \lambda) = (2, 0.8) \in \Omega_4$

(c) $(\xi, \lambda) = (3, 0.4) \in \Omega_2$ (d) $(\xi, \lambda) = (2, 0.6) \in \Omega_3$

Figure 2: Evolution of the wage: $\alpha = 2/3$ and $r = 1.35$

Panel (a) shows a case when $(\xi, \lambda) \in \Omega_0$ with intermediate pledgeability such that $\phi(w) \geq w^\alpha$ and thus inflows of the world saving help the economy to converge to 1. The map is similar to the one in Matsuyama (2004). We see in the next section that multiple steady states (a poverty trap) emerge for a severe borrowing constraint.

Panels (b), (c), and (d) show that the map $h(w)$ appears for a high project risk, generating the steady state w_H where the economy runs a current account surplus at the steady state $w_H < (w_H)^\alpha$. The figures show that when the steady state is unstable the economy converges to a cycle in which a period of current account surplus alternates with a period of current account deficit.

Panel (b) shows a case when $(\xi, \lambda) \in \Omega_4$ and that a 2-cycle emerges only if w_H is unstable. Panels (c) and (d) show a case when $(\xi, \lambda) \in \Omega_2 \cup \Omega_3$. We see in the next section that high order fixed points are stable—the steady state w_H is unstable—when the entrepreneurial project is risky. When stable high order fixed points co-exist with multiple steady states, financial liberalization generates a poverty trap for the poor and volatility of income for the rich. We will turn to the study of these higher order fixed points in Section 6.2.

6 Financial liberalization, inequality, and instability

6.1 Existence and stability of steady states

We start with dividing the parameter space $(\xi, \lambda) \in \Omega$ according to whether there exists either a unique steady state (w_L or w_H) or multiples steady states (w_L, w_M, w_H) and whether the steady state w_H is stable or not.

Let $\lambda = \Lambda_2(\xi)$ denote a unique solution to $h(w^c) = w^c$. If $\lambda > \Lambda_2(\xi)$, then $h(w^c) > w^c$ and thus $w_H \in (w^c, 1)$ always exists. Let $\lambda = \lambda_2^c := (1-\alpha)^{\frac{1}{\alpha}} r$ denote a unique solution to $g(1-\alpha) = \alpha$. If $\lambda < \lambda_2^c$, the map g admits two fixed points. It is straightforward to verify that $\Lambda_2(\xi)$ and λ_2^c are tangent to each other at $\xi_2^c := \frac{1}{1-\alpha}\left[\frac{\alpha}{(1-\alpha)^{1/\alpha} r} - 1\right]$.

Proposition 2. *Let $r < \frac{\alpha}{1-\alpha}$. For any pair $(\xi, \lambda) \in \Omega$ there exist*

- *a unique steady state* 1 *if and only if $\xi \leq \xi_1^c$ and $\lambda > \lambda_2^c$*
- *a unique steady state w_H if and only if $\xi > \xi_1^c$ and $\lambda > \lambda_2^c$*
- *a unique steady state w_L if and only if either $\xi \in (\xi_1^c, \xi_2^c)$ and $\lambda < \Lambda_2(\xi)$ or $\xi > \xi_2^c$ and $\lambda < \lambda_2^c$*
- *three steady states $\{w_L, w_M, 1\}$ if and only if $\xi \leq \xi_1^c$ and $\lambda < \lambda_2^c$*
- *three steady states $\{w_L, w_M, w_H\}$ if and only if $\xi \in (\xi_1^c, \xi_2^c)$ and $\lambda \in (\Lambda_2(\xi), \lambda_2^c)$.*

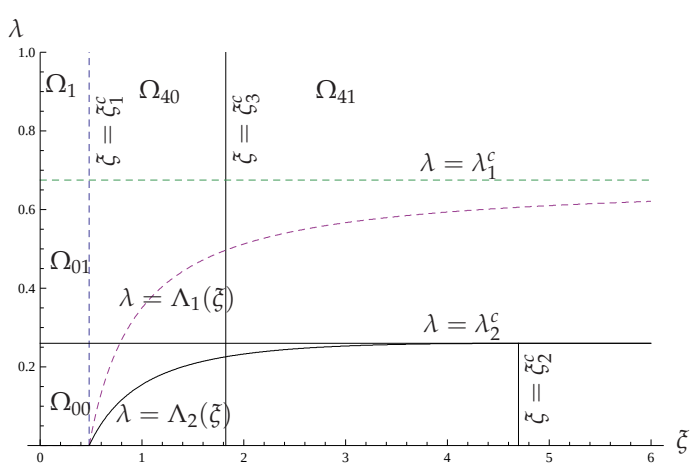


Figure 3: Uniqueness and Local Stability of Steady States: $\alpha = 2/3$, and $r = 1.35$.

Figure 3 shows the parameter regions defined in Proposition 2.[13] When $(\xi, \lambda) \in \Omega_{01} \cup \Omega_1$ the economy converges to 1 as in Figure 2 (a). When $(\xi, \lambda) \in \Omega_{00}$ a poverty trap emerges. In other words, when the project risk is low, a poverty trap arises only for low pledgeability. When $(\xi, \lambda) \in \Omega_{40}$, the economy converges to a unique steady state w_H. When $(\xi, \lambda) \in \Omega_{41}$ the economy converges to a unique cycle of order two as in Figure 2 (b). In other words, the high steady state loses stability when the project risk is high.

Proposition 3. *Suppose the steady state w_H exists. Then w_H is locally unstable if and only if*

$$\alpha \in (0.5, 1) \quad \textit{and} \quad \xi \geq \xi_3^c := \left(\frac{1-\alpha}{2\alpha-1}\right)^{\frac{1}{1-\alpha}} r^{\frac{\alpha}{1-\alpha}}.$$

Recalling from Propositions 2 that the high steady state w_H exists[14] if and only if $\xi > \xi_1^c$ and $\lambda > \Lambda_2(\xi)$, Proposition 3 implies that financial liberalization generates endogenous cycles if and only if the capital share in production is higher than 0.5, the project risk is perceived sufficiently high and the borrowing constraint is not too severe. In contrast to Aghion et al. (2004) and Caballe et al. (2006) it is worthwhile to point out that endogenous cycles emerge even with no borrowing constraint ($\lambda = 1$).

6.2 Global dynamics

We now focus on the remaining case $(\xi, \lambda) \in \Omega_2 \cup \Omega_3$ when the project risk is high and the pledgeability is low. The high steady state w_H is stable if $\xi < \xi_3^c$ (c.f. Proposition 3). When the high steady state is stable, the economy fails to converge to it globally only if a poverty trap emerges under low pledgeability, $\lambda \in (\Lambda_2(\xi), \lambda_2^c)$. For $\lambda < \Lambda_2(\xi)$, the low steady state w_L is a unique globally stable steady state. Rich dynamics arises when w_H is unstable; bifurcations lead to emergence of endogenous cycles—both periodic and chaotic. We use the technique developed in Gradini et al. (2008) to obtain formal expressions of all bifurcations curves.

[13] If $r < \frac{\alpha}{1-\alpha}$, then $\xi_1^c < \xi_2^c$.

[14] Let $r^* := \frac{\alpha}{1-\alpha}\frac{1}{1+\xi}$ denote the autarky steady state interest rate. We obtain $r > r^* \Leftrightarrow \xi > \xi_1^c$. This implies that the world interest rate has to be higher than the autarky steady state interest rate for the steady state w_H to exist.

6.2.1 Period doubling

Let us first consider the case when $(\xi, \lambda) \in \Omega_3$. From Proposition 1 we obtain the map

$$\phi(w) = \begin{cases} g(w) & \text{if } w < w_1^c \\ 1 & \text{if } w \in [w_1^c, w_2^c) \\ h(w) & \text{if } w \geq w_2^c \end{cases}$$

where

$$w_1^c := 1 - \tfrac{\lambda}{r}\tfrac{\alpha}{1-\alpha} = 1 - \tfrac{\lambda}{\lambda_1^c} \quad \text{and} \quad w_2^c := \tfrac{1}{\xi}\left(\tfrac{1}{r}\tfrac{\alpha}{1-\alpha} - 1\right) = \tfrac{\xi_1^c}{\xi}.$$

Figure 4 shows the bifurcation curves we define below. When $(\xi, \lambda) \in \Omega_{30}$ we have

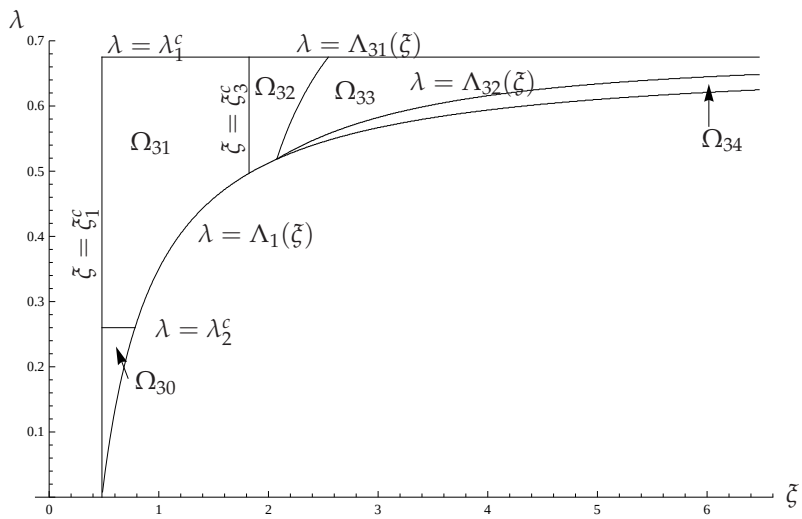


Figure 4: Bifurcation scenarios when $\alpha = 2/3$, $r = 1.35$, and $(\xi, \lambda) \in \Omega_3$.

three steady states $(w_L, w_M, 1)$. When $(\xi, \lambda) \in \Omega_{31}$ we have one stable steady state w_H. It looses stability through a flip bifurcation at $\xi = \xi_3^c$ and a stable 2-cycle $(w_R, h(w_R))$ emerges—w_R is the solution to $w = h^2(w) := h[h(w)]$ and both points $(w_R, h(w_R))$ belong to domain of definition of h. When $(\xi, \lambda) \in \Omega_{32}$ we have a unique and globally stable 2-cycle $(w_R, h(w_R))$ as in Figure 2 (d). The periodic point w_R merges with the critical point w_2^c and a border-collision bifurcation occurs when

$$h^2(w^c) = w^c \quad \text{or equivalently} \quad \lambda = \Lambda_{31}(\xi).$$

When $(\xi, \lambda) \in \Omega_{33}$ we have a globally stable 2-cycle $(1, h(1))$. The 2-cycle $(1, h(1))$ looses stability when

$$h(1) = w_1^c \quad \text{or equivalently} \quad \lambda = \Lambda_{32}(\xi) := \lambda_1^c \left(1 - \left(\tfrac{1+\xi_1^c}{1+\xi}\right)^{\frac{\alpha}{1-\alpha}}\right).$$

When $(\xi, \lambda) \in \Omega_{34}$ we have a globally stable 2-cycle $(w_F, g(w_F))$—$w_F \in (0, w_1^c)$ is a unique solution to $h(g(w)) = w$ and w_F belongs to domain of definition of g while $g(w_F) \in (w_2^c, 1)$ belongs to the domain of definition of h.

6.2.2 Route to chaos and multiple steady states

We now turn to the case when $(\xi, \lambda) \in \Omega_2$. From Proposition 1 we obtain the map

$$\phi(w) = \begin{cases} g(w) & \text{if } w < w^c \\ h(w) & \text{if } w \geq w^c. \end{cases}$$

Figure 5 shows the bifurcation curves we define below.

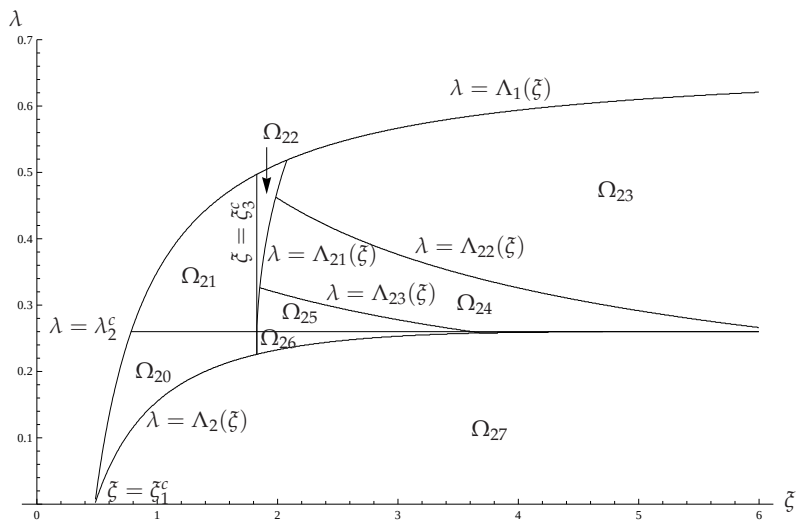


Figure 5: Bifurcation scenarios when $\alpha = 2/3$, $r = 1.35$, and $(\xi, \lambda) \in \Omega_2$.

When $(\xi, \lambda) \in \Omega_{27}$ we have one stable steady state w_L. When $(\xi, \lambda) \in \Omega_{20}$ we have multiple steady states (w_L, w_M, w_H) where both w_L and w_H are asymptotically stable. When $(\xi, \lambda) \in \Omega_{21}$ we have one globally stable steady state w_H. The steady state w_H looses stability through a flip bifurcation at $\xi = \xi_3^c$ and a stable 2-cycle $(w_R, h(w_R))$ emerges. When $(\xi, \lambda) \in \Omega_{22}$ we have a unique and globally stable 2-cycle $(w_R, h(w_R))$. The periodic point w_R merges with the critical point w^c and a border-collision bifurcation occurs when

$$h^2(w^c) = w^c \quad \text{or equivalently} \quad \lambda = \Lambda_{21}(\xi).$$

When $(\xi, \lambda) \in \Omega_{23}$ we have a globally stable 2-cycle $(w_F, g(w_F))$. The 2-cycle $(w_F, g(w_F))$ looses its stability when the points

$$\{w^c, g(w^c), h(g(w^c)), g(h(g(w^c)))\}$$

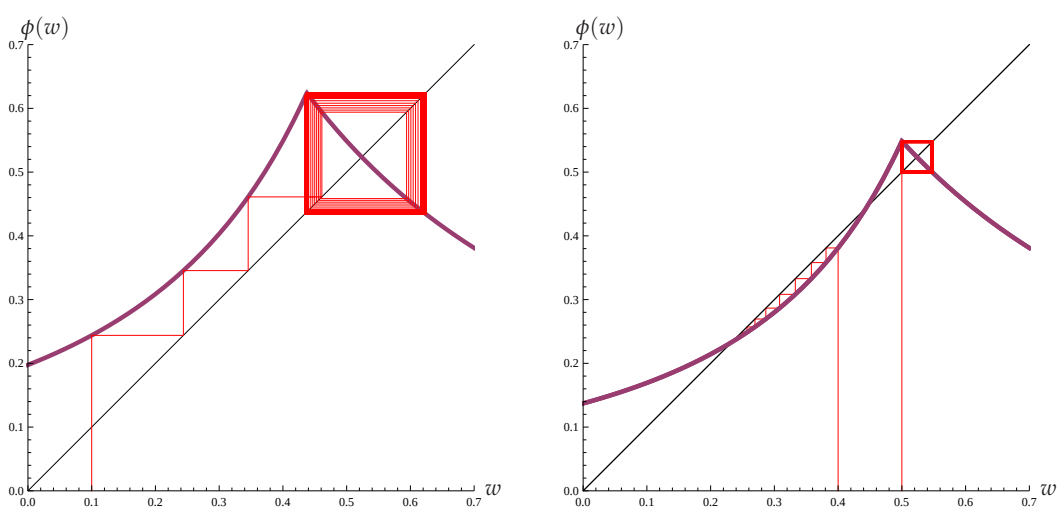
form a 4-cycle and the points of the segments $[h(g(w^c)), w^c]$, $[g(h(g(w^c))), g(w^c)]$ are all fixed points for the map $\phi^4(w)$—corresponding to all 4-cycles for ϕ and only one 2-cycle with periodic points approximately in the center of the two intervals—when

$$\phi^4(w^c) \equiv h[g[h^2(w^c)]] = w^c \quad \text{or equivalently} \quad \lambda = \Lambda_{22}(\xi)$$

(cf. Proposition 1 in Gardini et al. 2008). We cannot obtain a stable 4-cycle after this bifurcation because ϕ^4 always crosses the identity line with a slope grater than one in absolute value, i.e., $|\frac{d}{dw}\phi^4(w)| \geq 1$. In fact, we cannot obtain any stable cycle of any period because ϕ^4 is expanding in the absorbing interval $[h(g(w^c)), g(w^c)]$. The critical bifurcation of the 2-cycle also corresponds to the bifurcation curve at which the 4-cyclical chaotic intervals undergo a border-collision bifurcation. When $(\xi, \lambda) \in \Omega_{24}$ we have globally stable 4-cyclical chaotic intervals as in Figure 2 (c). The 4-cyclical chaotic intervals merge in pairs into 2-cyclical chaotic intervals and a homoclinic bifurcation of the repelling 2-cycle occurs when the fifth iterate of the point of maximum merges into the periodic point w_F:

$$\phi^6(w^c) = (\phi^2(w^c))^3 = w_F \quad \text{or equivalently} \quad \lambda = \Lambda_{23}(\xi).$$

When $(\xi, \lambda) \in \Omega_{25}$ we have globally stable 2-cyclical chaotic intervals. Two additional steady states w_L and w_M emerge and a tangent bifurcation occurs at $\lambda = \lambda_2^c$. When $(\xi, \lambda) \in \Omega_{26}$ we simultaneously have a stable steady state w_L and stable 2-cyclical chaotic intervals. Figure 6 displays a cobweb plot before and after the tangent bifurcation. We



(a) Chaos—before the tangent bifurcation, $\lambda = 0.3$

(b) Poverty trap and chaos—after the tangent bifurcation, $\lambda = 0.25$

Figure 6: Before and after the tangent bifurcation at $\lambda = \lambda_2^c$, $\xi = 2$

can see that the chaotic intervals are globally attractive before the tangent bifurcation

and after the tangent bifurcation they co-exist with a stable steady state w_L. Trajectories with $w_0 \in (0, w_M)$ converge to w_L while trajectories with $w_0 \in (w_M, 1)$ converge to the chaotic intervals. Financial liberalization simultaneously creates a poverty trap and a magnification of volatility.

7 Summary

Matsuyama (2004) shows how a borrowing constraint can cause multiplicity of steady states in a small open economy. It is well-known that a borrowing constraint can generate a diverging force under financial liberalization. The same mechanism is also present in our model. We incorporate risk in running entrepreneurial projects and analyze how it interacts with the borrowing constraint to generate inequality, volatility, or both.

It is intuitive that for a low risk, the forces of the borrowing constraint dominates the dynamics of the economy (generating a poverty trap for low pledgeability). When the risk is high, however, the steady state w_H—if it exists—is unstable. The instability causes volatility in the small open economy though in-and-outflows of capital. This holds even for high pledgeability. When the pledgeability is low, the steady state w_H ceases to exist and the economy converges to the low steady state w_L. When the pledgeability is at an intermediate level, multiple steady states emerge while the high steady state is unstable. In this case, financial liberalization magnifies either inequality or volatility—the small open economy falls into a poverty trap or fluctuates chaotically—depending on initial conditions. This mechanism explains why inherently identical countries may experience volatility or stagnate though financial liberalization.

The dynamics of the model is summarized in the following list and figure. The economy converges to

- 1 for $(\xi, \lambda) \in \Omega_{01} \cup \Omega_1$, and w_H for $(\xi, \lambda) \in \Omega_{21} \cup \Omega_{31} \cup \Omega_{40}$
- w_L for $(\xi, \lambda) \in \Omega_{27}$
- either w_L or w_H for $(\xi, \lambda) \in \Omega_{00} \cup \Omega_{20} \cup \Omega_{30}$
- a 2-cycle for $(\xi, \lambda) \in \Omega_{22} \cup \Omega_{23} \cup \Omega_{32} \cup \Omega_{33} \cup \Omega_{41}$
- 2-and-4-cyclical chaotic intervals for $(\xi, \lambda) \in \Omega_{25} \cup \Omega_{24}$
- either w_L or 2-cyclical chaotic intervals for $(\xi, \lambda) \in \Omega_{26}$

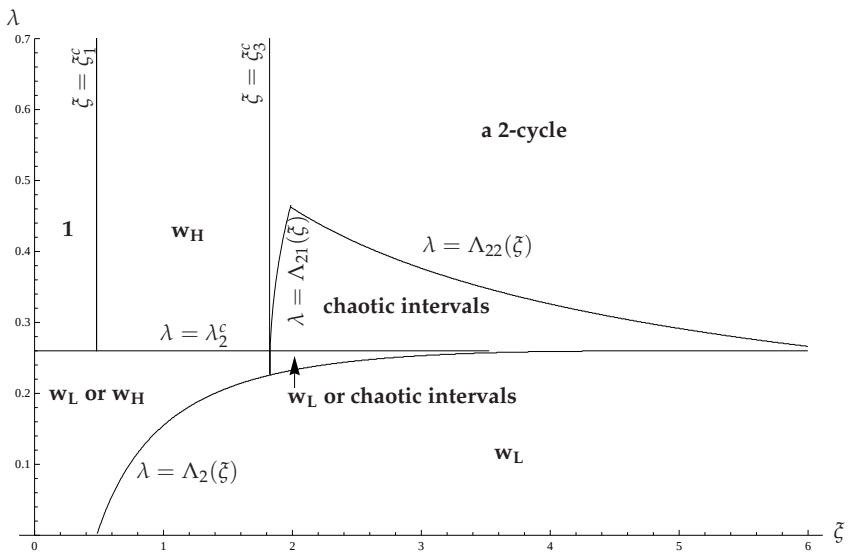


Figure 7: Long-run dynamics when $\alpha = 2/3$, and $r = 1.35$.

The figure displays how the pledgeability and the risk interact. Volatility requires a high level of both pledgeability and risk. Multiplicity of steady states requires a low level of both. For an intermediate level of both *inherently identical* economies face the most dramatic consequence of financial liberalization: They may fall into a poverty trap or fluctuate chaotically depending on their initial conditions. For low pledgeability and high risk the economy converges to the low steady state. For high pledgeability and low risk the economy converges to a high steady state. The next table provides a comprehensive overview of the results.

Table 1: Interaction of pledgeability and risk

	low pledgeability	intermediate pledgeability	high pledgebility
low risk	poverty trap	poverty trap	high steady state
intermediate risk	low steady state	poverty trap & volatility	high steady state
high risk	low steady state	volatility	volatility

8 Concluding remarks

We construct an overlapping generations model in which people are subject to limited pledgeability and uncertainty over entrepreneurial projects and provide a complete characterization of dynamics (bifurcation curves) in the space of the level of pledgeability and risk of entrepreneurial projects.

We compare two extreme cases: an autarky economy and a small open economy fully integrated in the international financial market. The model is set up such that the long run volatility of income is zero in autarky because the economy converges monotonically to a steady state. In the small open economy, in contrast, the volatility of income can either diminish asymptotically to zero or persist forever. We show that financial liberalization can be inherently destabilizing even in the absence of exogenous sources of volatility such as technology and preference shocks. Destabilizing market forces lead to periods of capital inflows are followed by a sudden reversal of international capital flows. On the other hand, financial market liberalization can also generate a poverty trap in the presence of a borrowing constraint.

Our main contributions are twofold. First, we show that whether financial liberalization generates inequality, volatility, or both depends on the interaction of risk and pledgeability of entrepreneurial projects. Second, we show that financial liberalization can even simultaneously generate a poverty trap and a chaotic fluctuation. Hence, we observe large heterogeneity in dynamics even between inherently identical economies: some may experience volatility and others stagnate though financial liberalization depending on their initial conditions.

A Appendix

A.1 Instability of the steady state w_H for $\alpha < 0.5$

We give an example of a utility function for which w_H can be unstable for $\alpha < 0.5$. Let us consider a general utility function, u, which is strictly increasing, strictly concave and satisfies $u(0) = 0$. The participation constraint holds if and only if

$$pu\left(zf'(k_{t+1}) - \frac{r}{p}(1 - w_t)\right) = u\left(rw_t\right).$$

The corresponding h function is

$$h(w_t) = \left(\frac{\alpha}{1-\alpha}\frac{1}{r + \Xi(rw_t)}\right)^{\frac{\alpha}{1-\alpha}} \tag{A.1}$$

where $\Xi(x) := pu^{-1}(u\left(x\right)/p) - x$. Let the utility function be given by $u(c) = (1+c)^{\gamma} - 1$. Then,

$$\Xi(x) = p\left\{\left[\frac{(1+x)^{\gamma}-(1-p)}{p}\right]^{\frac{1}{\gamma}} - 1\right\} - x \quad \text{and} \quad \Xi'(x) = (1+x)^{\gamma-1}\left[\frac{(1+x)^{\gamma}-(1-p)}{p}\right]^{\frac{1-\gamma}{\gamma}} - 1.$$

We can easily very that Ξ is strictly increasing and that (A.1) has a unique fixed point, which we denote w_H. The fixed point is unstable if and only if

$$\frac{rw_H\Xi'(rw_H)}{\Xi(rw_H)} \geq \frac{1-\alpha}{\alpha}\frac{r+\Xi(rw_H)}{\Xi(rw_H)}. \tag{A.2}$$

If $u(c) = c^\gamma$, then $\Xi(x) = \xi x$ and thus $\frac{x\Xi'(x)}{\Xi(x)} = 1$. This with (A.2) implies that w_H is unstable only when $\alpha > 0.5$. This is no longer the case when $u(c) = (1+c)^\gamma - 1$ because $\frac{x\Xi'(x)}{\Xi(x)}$ can exceed one for a sufficiently small value of $p \in (0,1)$ and $\gamma \in (0,1)$. Figure 8 gives a numerical example when (A.2) is satisfied for $\alpha < 0.5$.

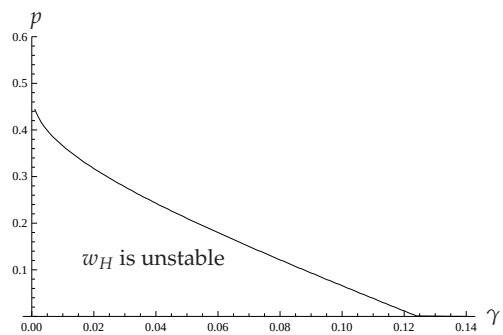


Figure 8: Parameter region where w_H is unstable; $r = 1.35$ and $\alpha = 1/3$.

A.2 Remaining Proofs

Proof of Proposition 2. If $\xi \leq \xi_1^c$ and $\lambda > \lambda_2^c$, then $\phi(w) = 1$ and thus $w = 1$ is a unique steady state. The map h admits a unique fixed point on $(w^c, 1)$ if $\lambda > \Lambda_2(\xi)$ and no fixed point if $\lambda < \Lambda_2(\xi)$. The map g is tangent to the identity line at $w = 1-\alpha$ if and only if $\lambda = \lambda_2^c$. If $\lambda < \lambda_2^c$, then g has two fixed points $w_L \in (0, 1-\alpha)$ and $w_M \in (1-\alpha, 1)$. In order for w_M to belong to the interval $(0, w^c)$—remember $(0, w^c)$ is the domain of definition of g— it must hold that $\lambda > \Lambda_2(\xi)$. □

Proof of Proposition 3. Suppose w_H exists. Since

$$\frac{wh'(w)}{h(w)} = -\frac{\alpha}{1-\alpha}\frac{\xi w}{1+\xi w}, \quad \text{it follows that} \quad h'(w_H) < -1 \quad \Leftrightarrow \quad \xi w_H(2\alpha - 1) > 1-\alpha.$$

This is possible only if $\alpha \in (0.5, 1)$ and $\xi \geq \xi_3^c$. □

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