



Endogenous inequality and fluctuations in a two-country model [☆]

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Abstract

We study a two-country version of Matsuyama's [K. Matsuyama, Financial market globalization, symmetry-breaking, and endogenous inequality of nations, *Econometrica* 72 (2004) 853–884] world economy model. As in Matsuyama's model, symmetry-breaking can be observed, and symmetry-breaking generates endogenously determined levels of inequality. In addition, we show that when the countries differ in population size, their interaction through credit markets may lead to persistent endogenous fluctuations. © 2009 Elsevier Inc. All rights reserved.

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1. Introduction

One of the primary aims of growth theory is to explain large and persistent cross-country differences in wealth. Exogenous factors mapped to long-run outcomes via a convex growth model appear insufficient to explain this variation [3]. An alternative approach is provided by poverty trap models, where identical economies can support multiple long-run outcomes (cf., e.g. [1,2,8]). A drawback common to many of these models is that they do not provide a satisfac-

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tory theory of the cross-country income distribution *per se*, given that outcomes for individual economies are determined in isolation.

A number of researchers have confronted this deficiency by developing models where cross-country income inequality arises endogenously, often as a result of capital market imperfections (e.g., [4,5,7,10]). For example, in Matsuyama [7] a world economy is populated by a continuum of small countries, each of which participates in an international market for credit. A wealth-dependent borrowing constraint implies that poorer economies are more restricted in their ability to raise credit, which, in turn, impacts on domestic wealth. The opposite is true in richer economies. As a result, small initial differences may be magnified, with the world economy polarizing into rich and poor.

Matsuyama analyzes this endogenous inequality within the framework of symmetry-breaking. Symmetry-breaking is a mechanism whereby diversity is endogenously generated through the economic interactions of entities such as households, firms or countries. Typically, the setting is one where the entities of interest have inherently identical characteristics. This symmetry across agents or other entities leads to the existence of a symmetric equilibrium. In some settings, the dynamics associated with economic interaction causes the symmetric equilibrium to lose stability, and small initial variations are amplified over time.¹

This symmetry-breaking phenomenon is particularly clear in Matsuyama's world economy model. The steady state for the world economy when all countries operate in autarky remains a (symmetric) steady state after credit markets are integrated. Under certain parameterizations, this steady state is unstable, and stable asymmetric steady states exist. Any small deviation from the symmetric steady state leads to a process whereby countries are endogenously divided into rich and poor.

While Matsuyama's continuum model is remarkably tractable, the infinite-dimensional state space does not lend itself to analysis of dynamics outside of the steady states. In this paper, we consider an alternative form of the model, where the world economy consists of two large countries. Although the spillover effects generated by their interactions in the credit market make the analysis less tractable, one can still observe symmetry-breaking phenomena similar to those in Matsuyama's model. In addition, we show that when population sizes differ, the interactions between the two economies may create persistent endogenous cycles.

The structure of the paper is as follows. Section 2 formulates the model and derives equilibrium conditions. Section 3 studies dynamics when population size is equal in the two countries, while Section 4 analyzes dynamics for the case of unequal population size. Remaining proofs can be found in Appendix A.

2. Set up

We now formulate and analyze a two-country version of the continuum world economy model in Matsuyama [7]. Aside from the individual countries having positive measure, the economic environment is the same. As a first step, we describe the environment faced by an individual country when the interest rate is given. We then connect the two countries via the interest rate, which is determined according to world supply and demand for credit.

In each country, successive generations have unit mass. Every agent lives for two periods, supplying one unit of labor in the first period and consuming in the second. At time t , produc-

¹ For a more complete definition of symmetry-breaking, see [9].

tion combines the current stock of capital k_t supplied by the old with the unit quantity of labor supplied by the young.² The resulting per-capita output is $f(k_t)$, where $f : \mathbb{R}_+ \rightarrow \mathbb{R}_+$ is C^2 and satisfies $f(0) = 0$, $f''(k) < 0 < f'(k)$, $\lim_{k \uparrow \infty} f'(k) = 0$, and $\lim_{k \downarrow 0} f'(k) = \infty$.

Factor markets are competitive, paying young agents the wage $W(k_t) := f(k_t) - k_t f'(k_t)$, and old agents a gross return on capital given by $f'(k_t)$. After production and the distribution of factor payments, the old consume and exit the model, while the young take their wage earnings and invest them.

2.1. Investment behavior

When investing, young agents can either lend in the credit market at (gross) interest rate r_{t+1} , or run a discrete indivisible project, which takes one unit of the consumption good and returns R units of the capital good.³ If all young agents start projects, then the capital stock at $t + 1$ is R . This leads to the resource constraint

$$0 \leq k_{t+1} \leq R. \tag{1}$$

The gross rate of return on the project, measured in units of the consumption good, is $Rf'(k_{t+1})$. Thus, investors are willing to start the project whenever

$$r_{t+1} \leq Rf'(k_{t+1}). \tag{2}$$

We refer to this inequality as the profitability constraint.

Let R^+ be the solution to $W(R^+) = 1$. Following Matsuyama, we assume that $W(R) < 1$. In other words, we consider only $R \in (0, R^+)$. Given the resource constraint (1), we then have $W(k_t) < 1$ for all t , and young agents who start projects must borrow $1 - W(k_t)$ at rate r_{t+1} . As a result, their obligation at $t + 1$ is given by $r_{t+1}(1 - W(k_t))$. Against this obligation, borrowers can only credibly pledge a fraction $\lambda \in [0, 1]$ of their expected earnings $Rf'(k_{t+1})$. The borrowing constraint is therefore

$$r_{t+1}(1 - W(k_t)) \leq \lambda Rf'(k_{t+1}). \tag{3}$$

The parameter λ can be interpreted as a measure of credit market imperfection, with higher values corresponding to lower imperfection.

2.2. Equilibrium

Let us now consider determination of the interest rate and next period capital stock, given the current stock k_t . As in Matsuyama [7], when international financial markets are absent and each economy operates in autarky, the interest rate adjusts so that domestic savings is equal to domestic investment, and hence domestic capital stock evolves according to

$$k_{t+1} = RW(k_t) \tag{4}$$

independent of the credit market imperfection.⁴

² “Capital” may be either human or physical. It depreciates fully between periods, so capital stock is equal to investment. Given the unit mass of agents, per-capita and aggregate values coincide. Later we consider other population sizes, and in this case k_t should be thought of as a per-capita quantity.

³ Factors of production are not internationally mobile, and projects can only be run domestically.

⁴ Assuming that $k_0 \leq R$, the assumption $W(R) < 1$ implies that the resource constraint (1) is never binding in autarky.

When international credit markets are present, the world interest rate is determined by international supply and demand for credit. Suppose first that the interest rate is fixed at r . In either of the two countries, equilibrium implies that the mass of agents who start projects (and hence k_{t+1}) increases until one of the constraints (1)–(3) binds. From this reasoning we obtain

$$k_{t+1} = \Psi(k_t, r) := \min \left\{ R, \Phi \left(\frac{r}{R} \right), \Phi \left(\frac{1 - W(k_t)}{\lambda} \cdot \frac{r}{R} \right) \right\} \tag{5}$$

where $\Phi(k) := (f')^{-1}(k)$. This leads to the two-country law of motion

$$k_{t+1}^1 = \Psi(k_t^1, r(k_t^1, k_t^2)) \quad \text{and} \quad k_{t+1}^2 = \Psi(k_t^2, r(k_t^1, k_t^2)) \tag{6}$$

where $r(k_t^1, k_t^2)$ is the equilibrium interest rate. Specifically, $r(k_t^1, k_t^2)$ is the r that solves

$$\Psi(k_t^1, r) + \Psi(k_t^2, r) = R(W(k_t^1) + W(k_t^2)). \tag{7}$$

The right-hand side of this expression is the world supply of physical capital in each period, and the left-hand side is demand for that capital.

Suppose that the resource constraint (1) is not binding (i.e., $k_{t+1} < R$). In this situation, the minimizer on the right-hand side of (5) is one of the last two terms. Let $K(\lambda)$ be the solution to $1 - W(K(\lambda)) = \lambda$. If $k_t > K(\lambda)$, then $1 - W(k_t) < \lambda$, and k_{t+1} is equal to $\Phi(r/R)$. If, on the other hand, $k_t \leq K(\lambda)$, then k_{t+1} is equal to the last term, and the borrowing constraint is binding. Evidently $\lambda \mapsto K(\lambda)$ is strictly decreasing on $(0, 1)$.

3. Dynamics: equal population size

In this section we investigate the dynamics of the autarkic and integrated world economies, as determined by (4) and (6) respectively. We show that in many respects the two-country economy studied here has dynamics similar to the continuum model of Matsuyama [7], with symmetry-breaking occurring over an identical range of parameter values. In other words, symmetry-breaking is robust with respect to the introduction of countries having positive mass.

3.1. Autarky

Consider first the joint dynamics of the pair $(k_t^1, k_t^2)_{t \geq 0}$ when financial markets are not integrated, and each economy operates in autarky. In this case k_t^1 and k_t^2 both individually follow the law of motion (4). The state space is taken to be $\mathbb{X} := (0, R] \times (0, R]$.⁵ As observed by Matsuyama [7, p. 863] and earlier authors, the OLG dynamics in (4) can exhibit multiple steady states even with neoclassical technology, which serves only to distract from analysis of symmetry-breaking. Hence we assume that $\lim_{k \downarrow 0} W'(k) = \infty$ and $W''(k) < 0$, thereby assuring the existence of a unique steady state $K^*(R)$ for each $R \in (0, R^+)$.⁶ The map $R \mapsto K^*(R)$ is strictly increasing and satisfies $K^*(R) < R$ for all R . Under these assumptions we have the following elementary result.

Proposition 3.1. *If the two countries operate in autarky, then for any value of $R \in (0, R^+)$, any $\lambda \in (0, 1)$, and any initial condition $(k_0^1, k_0^2) \in \mathbb{X}$, the bivariate process $(k_t^1, k_t^2)_{t \geq 0}$ converges to the unique steady state $(K^*(R), K^*(R))$.*

⁵ We exclude zero from the state space in order to rule out trivial steady states.

⁶ Specifically, $K^*(R)$ is the solution to $k = RW(k)$ for each R .

3.2. Integrated credit markets

Consider now the dynamics when financial markets are integrated. In this case k_t^1 and k_t^2 follow the law of motion (6). To analyze these dynamics we introduce some new notation. Let R_c be defined by $f(K^*(R_c)) = 1$, and R_λ by $K^*(R_\lambda) = K(\lambda)$. Let $\lambda_c \in (0, 1)$ be the solution to $f(K(\lambda_c)) = 1$. We will make use of the following result.

Lemma 3.1. *If $\lambda < \lambda_c$, then $R_c < R_\lambda$. If $\lambda > \lambda_c$, then $R_c > R_\lambda$.*

Proof. If $\lambda < \lambda_c$, then $f(K^*(R_c)) = 1 = f(K(\lambda_c)) < f(K(\lambda))$. Hence $K^*(R_c) < K(\lambda)$, and $R_c < R_\lambda$. The proof of the second claim is similar. \square

We can now state the main result of this section.⁷

Proposition 3.2. *Let $R \in (0, R^+)$ and let $\lambda \in (0, 1)$. When financial markets are integrated, there exists one and only one symmetric steady state, which is given by $(K^*(R), K^*(R))$. Moreover, this symmetric steady state is*

1. *locally stable whenever $0 < R < R_c$,*
2. *saddle path stable whenever $R_c < R < R_\lambda$, and*
3. *locally stable whenever $R_\lambda < R < R^+$.*

Combining Propositions 3.1 and 3.2, we see that the financially integrated two-country model possesses a unique symmetric steady state, which coincides with the long-run equilibrium when the two economies coexist in autarky. In the latter (i.e., autarkic) case, this steady state is always stable. In the financially integrated case, the same steady state $(K^*(R), K^*(R))$ may be stable or unstable, depending on parameters. Moreover, asymmetric steady states may exist.⁸

In view of Lemma 3.1, if $\lambda > \lambda_c$, then $(R_c, R_\lambda) = \emptyset$, and the symmetric steady state of the financially integrated two-country model is stable for all R . Thus, when the credit constraint is sufficiently weak, financial integration cannot be a source of symmetry-breaking. Fig. 1 illustrates this scenario when $f(k) = k^\alpha$. In both sub-figures, the dynamics of the integrated two-country model defined in (6) and (7) are represented by a phase diagram (curve intersections are steady states) superimposed on the vector field.⁹ In (a) the borrowing constraint is binding at the steady state. In (b) R is larger, and the constraint is not binding.¹⁰

Now consider the case where $\lambda < \lambda_c$. From Lemma 3.1 we have $R_c < R_\lambda$, and hence the set of $R \in (0, R^+)$ for which the saddle path stable equilibrium obtains is nonempty. An illustration of this case is given in Fig. 2. The parameters are $\alpha = 0.6$ and $\lambda = 0.3$, implying $R_c = 2.5$ and $R_\lambda = 3.62$. In (a) we have $R = 2.3 < R_c$, and the symmetric steady state is stable. In (b), on the

⁷ The result is analogous to Proposition 3 in Matsuyama [7], with instability (i.e., saddle path stability) of the symmetric steady state occurring for the same set of R in $(0, R^+)$.

⁸ It is not difficult to show that for the integrated two-country model, if (k, k') is a steady state, then so is (k', k) . In consequence, the number of *asymmetric* steady states is always even, and the total number of steady states is always odd.

⁹ In these and all other state space figures, k^1 is plotted on the x -axis, and k^2 is on the y -axis. Each plot is of the state space $\mathbb{X} = (0, R] \times (0, R]$.

¹⁰ The other parameters are $\alpha = 0.6$ and $\lambda = 0.65$. In the Cobb–Douglas case, $\lambda_c = \alpha$, so the inequality $\lambda > \lambda_c$ holds for these parameter values.

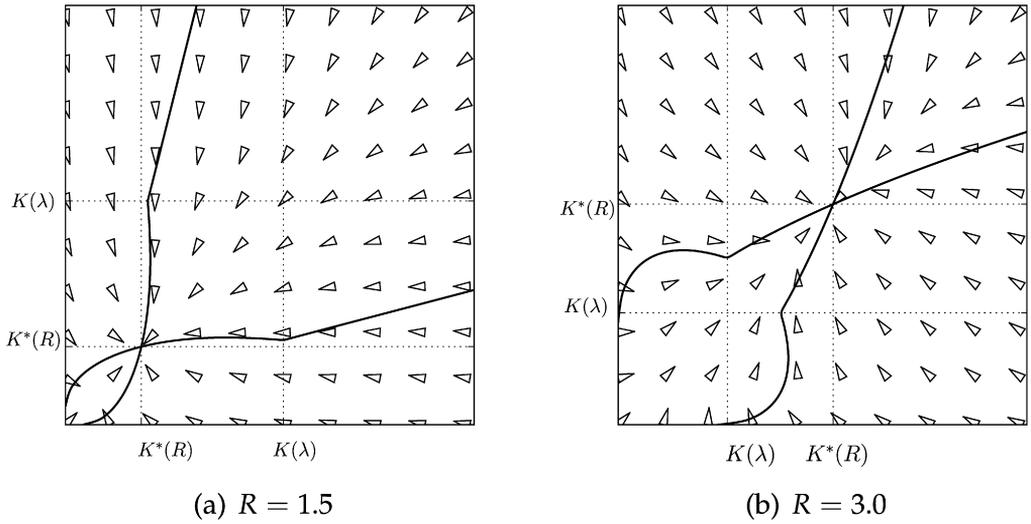


Fig. 1. Weak borrowing constraint ($\lambda > \lambda_c$).

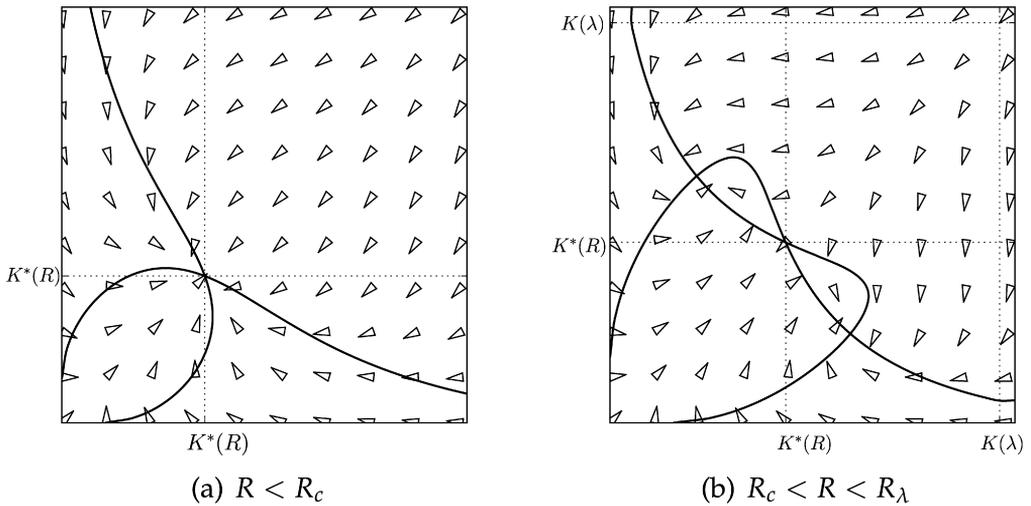


Fig. 2. Stability and instability at $(K^*(R), K^*(R))$.

other hand, $R = 2.65 \in (R_c, R_\lambda)$, and the symmetric steady state is saddle-path stable. Observe that instability of $(K^*(R), K^*(R))$ coincides with existence of stable *asymmetric* steady states.

3.3. Symmetry breaking

We now consider more closely how integration of financial markets affects the dynamics of a previously autarkic world economy, given a fixed parameterization of the model. We show that the symmetry-breaking observed in Matsuyama's [7] continuum world economy model is also a feature of the two-country world system.

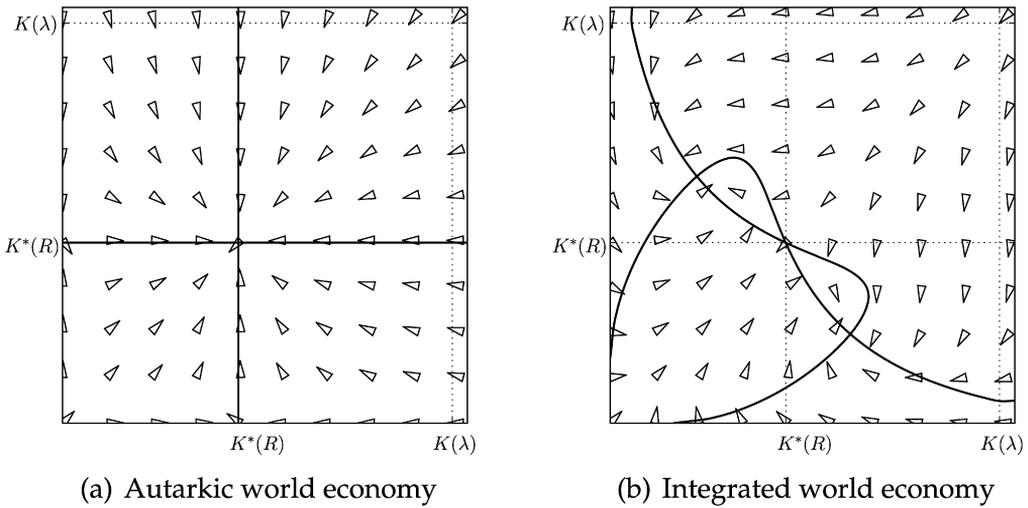


Fig. 3. Symmetry-breaking.

Consider Fig. 3. Sub-figure (a) shows the dynamics of the autarkic world economy, while (b) shows the dynamics under the same parameters when credit markets are integrated.¹¹ Introduction of a global credit market induces symmetry-breaking, with the symmetric steady state $(K^*(R), K^*(R))$ losing stability, combined with the emergence of stable asymmetric steady states. A slight perturbation of the current state (k_t^1, k_t^2) from the symmetric steady state leads to the endogenous formation of a polarized world economy, where one country has a higher steady state than $K^*(R)$ and the other country has a lower steady state.

It is worth noting that while the mechanism behind symmetry-breaking in this model is essentially the same as that of Matsuyama [7], some stable asymmetric steady states have properties that are not shared by those in his model. For example, the borrowing constraint can be binding at the asymmetric steady state in both countries (see Fig. 3(b)). These differences in dynamics can be attributed to spillover effects between the two countries that are not present in a model having a continuum of small open economies.

4. Dynamics: unequal population size

The previous section showed how the introduction of a global market for credit can lead to emergence of diversity in a world economy consisting of two identical countries. It is also interesting to consider how *exogenous* diversity can impact on outcomes in the two-country model. A natural form of heterogeneity to introduce is that of population size. In this section we investigate how variations in relative population size affect equilibrium dynamics.

To this end, we now consider a setting where the mass of agents per generation in country 1 is given by $L \in (0, 1)$, while in country 2 the mass is $1 - L$. The equilibrium conditions (1)–(3) are stated in terms of per capita values, and hence are unaffected. However, the interest rate condition (7) is stated in terms of aggregates, and now becomes

$$L\Psi(k_t^1, r) + (1 - L)\Psi(k_t^2, r) = R(LW(k_t^1) + (1 - L)W(k_t^2)). \tag{8}$$

¹¹ Here $R = 2.65 \in (R_c, R_\lambda)$, as in Fig. 2(b).

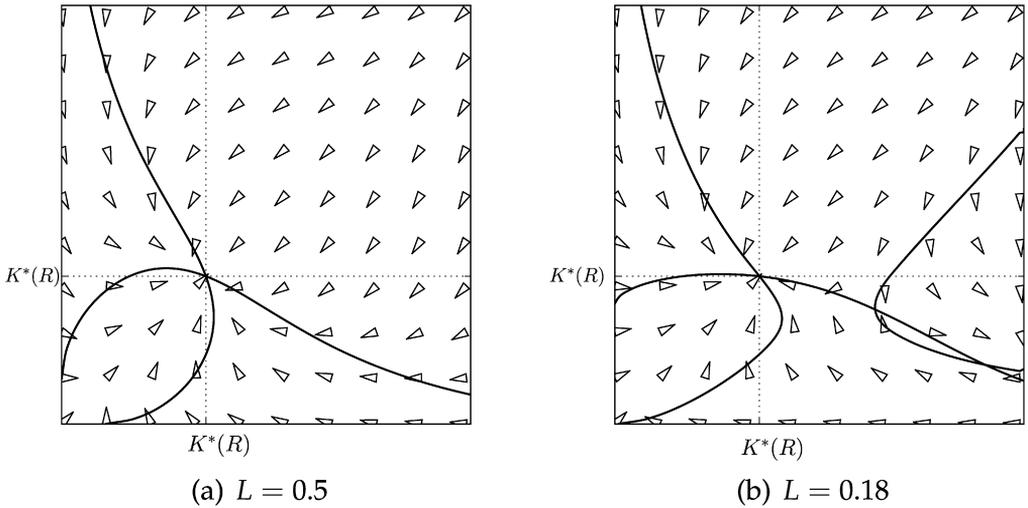


Fig. 4. Broken symmetric structure.

As before, this equation determines r as the interest rate that equates aggregates world demand for credit with aggregate world supply.

For some parameterizations, the impact of population size on long-run outcomes is substantial. For example, consider Fig. 4. The figure on the left is a replication of Fig. 2(a). (Here $R < R_c$, and, by Proposition 3.2, the symmetric steady state is stable. Fig. 4(a) also shows that no asymmetric steady states exist.) In Fig. 4(b), on the other hand, we set $L = 0.18$, so that the mass of agents in country 1 is substantially smaller than that of country 2.

A preliminary observation regarding the dynamics in Fig. 4(b) is that the symmetric steady state is unchanged, and it retains local stability. This follows from (8), because if $k_t^1 = k_t^2$ in that equation, then L vanishes, and the local laws of motion are homeomorphic to those for the case of equal population size. A second observation is that, in addition to the stable symmetric steady state, two asymmetric steady states exist. The asymmetric steady state closest to the symmetric steady state is unstable, and the other asymmetric steady state is stable. A further reduction of L leads to a bifurcation, whereby the stable asymmetric steady state becomes unstable, and cycles arise.

This outcome is illustrated as a bifurcation diagram in Fig. 5. In the figure, the x -axis represents different values of the parameter L , while the y -axis is the state space. The points plotted in the figure are long-run outcomes for each value of L , starting from initial condition $(k_0^1, k_0^2) = (2, 0.2)$.¹² Each value (k_t^1, k_t^2) is represented as two points in the figure. As discussed below, $k_t^1 > k_t^2$ generally holds, so that the higher cluster of values represents k^1 values, while the lower represents k^2 values.

When $L = 0.185$, orbits converge to the symmetric steady state $K^*(R) \simeq 0.81$. Around $L = 0.18$, the basin of the stable asymmetric steady state expands to include our initial condition (k_0^1, k_0^2) , and the orbit converges to this steady state.¹³ Country 1, which has the lower population

¹² Other parameters are fixed. As in Fig. 4, $\alpha = 0.6$, $\lambda = 0.3$ and $R = 2.3$.

¹³ This is the right-most steady state in Fig. 4(b).

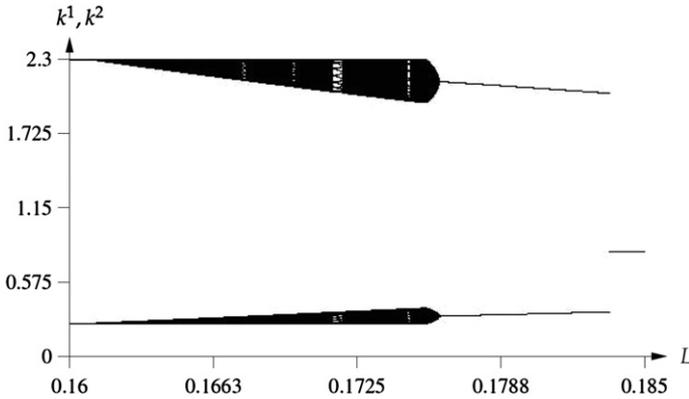


Fig. 5. Bifurcation diagram.

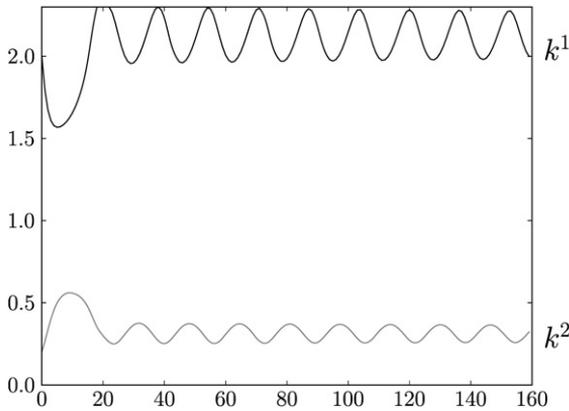


Fig. 6. Time series.

size (i.e., mass), has the higher capital stock per-capita in this equilibrium. At $L \simeq 0.176$, the asymmetric steady state undergoes a bifurcation that leads to endogenous cycles.¹⁴

The presence of endogenous cycles is further illustrated in Figs. 6 and 7. Here the value of L is taken to be 0.176, which is close to the bifurcation value, and the initial conditions are as in Fig. 5. Fig. 6 plots the evolution of capital stock in the two countries over time. Each time series converges to a stable cycle, with the low population economy fluctuating at a higher value than the high population economy. Fig. 7 shows these dynamics in (a subset of) the state space \mathbb{X} . The plot is of a single orbit $(k_t^1, k_t^2)_{t \geq 0}$, which converges to a closed invariant curve, and the two-country world economy exhibits persistent fluctuations over time.¹⁵

¹⁴ The bifurcation that occurs at this value of L is of the Neimark–Sacker type, as can be verified by examining the trace and determinant of the Jacobian at the asymmetric steady state. For details see Kikuchi [6]. If L goes below $L \simeq 0.161$, cycles disappear and the orbits converge to an asymmetric steady state where the resource constraint is binding in country 1 (i.e., $\lim_t k_t^1 = R$). Moreover, as $L \downarrow 0$, the limiting value of k_t^2 tends to $K^*(R)$, which is its value at the symmetric steady state.

¹⁵ The model of Boyd and Smith [4] also generates cycles, but the cycles are damped, and disappear asymptotically.

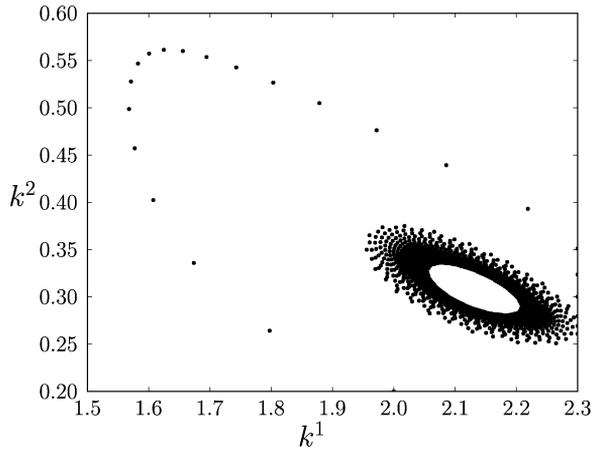


Fig. 7. Attractor plot.

Since $R < R_c$, the symmetric steady state is stable when we observe these fluctuations. The unequal population size generates a stable asymmetric steady state, which undergoes the bifurcation. Moreover, the borrowing constraint is binding in both countries along the endogenous cycles. Together, unequal population size and spillover effects drive the cycles.

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Appendix A

The proof of Proposition 3.1 is trivial, and hence omitted.

Proof of Proposition 3.2. First we verify the claim that there exists one and only one symmetric steady state, which is equal to $(K^*(R), K^*(R))$. To see this, observe that if (k, k) is a symmetric steady state, then $k = \Psi(k, r)$, where r is defined by (7). From (7) it then follows that $k = RW(k)$, or $k = K^*(R)$. Conversely, $k^* := K^*(R)$ is always a symmetric steady state, because if we set $k^1 = k^2 = k^*$ in (7) we obtain $\Psi(k^*, r) = RW(k^*)$, where r is the unique solution to this equation. But $RW(k^*) = k^*$ by definition, so $\Psi(k^*, r) = k^*$. In other words, $(k^*, k^*) := (K^*(R), K^*(R))$ is a fixed point of the system.

Regarding stability of the symmetric steady state $(K^*(R), K^*(R))$, let

$$J(x, y) = \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} \Psi_x(x, r(x, y)) & \Psi_y(x, r(x, y)) \\ \Psi_x(y, r(x, y)) & \Psi_y(y, r(x, y)) \end{bmatrix}$$

be the Jacobian associated with the dynamical system (6). Recall that $K^*(R) < R$ always holds, so Ψ is determined by one of the last two expressions in the minimum on the right-hand side of (5).

In order to assess stability, we wish to evaluate the eigenvalues of this matrix at $x = y = K^*(R)$. Observe that $a = d$ and $b = c$ when $x = y$. In this case, the characteristic polynomial is given by $p(\mu) = \mu^2 - 2a\mu + a^2 - b^2$, and the eigenvalues of the system are $\mu_1 = a + b$ and $\mu_2 = a - b$.

Generically, there are two cases to consider: Either $K^*(R) < K(\lambda)$ or $K^*(R) > K(\lambda)$.

Case 1. In the first case (i.e. $K^*(R) < K(\lambda)$), we consider the dynamical system for $x, y < K(\lambda)$, which is given by the third expression in the minimum on the right-hand side of (5). Some long but straightforward calculations show that

$$a = \frac{1}{2} \left[RW'(x) + \frac{xf'(x)}{1 - W(x)} \right], \quad b = \frac{1}{2} \left[RW'(x) - \frac{xf'(x)}{1 - W(x)} \right] \quad (9)$$

where we are using $x = y$ when the Jacobian is evaluated at the symmetric steady state. Thus

$$\mu_1(x) = \frac{xW'(x)}{W(x)} \quad \text{and} \quad \mu_2(x) = \frac{xf'(x)}{1 - W(x)}.$$

In view of our assumptions on W (in particular, $W'' < 0$) we have $0 < \mu_1(x) < 1$ for every x . Moreover $\mu_2(x) > 0$. Hence stability or instability depends on the sign of $\mu_2(x) - 1$ when $x = K^*(R)$.

Case 2. The other case to consider is $K^*(R) > K(\lambda)$. If $x, y \geq K(\lambda)$, then dynamics are given by the second expression in the minimum on the right-hand side of (5). In this case $a = b$, and $\mu_1(x)$ is as above, while $\mu_2(x) = 0$. Since both eigenvalues lie inside the unit disk, the symmetric steady state is always stable.

Now suppose that $0 < R < R_c$. If the second case prevails (i.e., $K^*(R) > K(\lambda)$), then stability obtains. Suppose instead that $K^*(R) < K(\lambda)$, so that the first case holds. Since $\mu_2(x)$ is strictly increasing in x and $f(K^*(R_c)) = 1$, evaluating at $x = K^*(R)$ and using $R < R_c$ yields $\mu_2(K^*(R)) < 1$. Hence stability obtains.

Next, suppose that $R_c < R < R_\lambda$. Then $K^*(R) < K(\lambda)$ by the definition of R_λ , and the first case holds. Since $R > R_c$ we have $\mu_2(K^*(R)) > 1$, and the symmetric steady state is saddle path stable.

Finally, suppose that $R_\lambda < R < R_+$. Then $K^*(R) > K(\lambda)$ by the definition of R_λ . Hence the second case holds, and the symmetric steady state is stable. \square

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