A Mechanism for Booms and Busts in Housing Prices*

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Abstract

The paper studies the dynamics of housing prices in a pure exchange overlapping generations framework a la Samuelson (1958) and Gale (1973), which is extended to include housing as a utility-yielding durable good and a credit sector. We completely characterize the equilibrium dynamics, which alternates between an expansive regime where leveraged borrowing increases housing prices, and a contractive regime where these variables decrease. Regime switches occur due to small but persistent income changes giving rise to boom-bust cycles in housing prices. Price deviations from fundamentals are caused by leveraged borrowing, and turn out to be fully welfare-neutral.

Keywords: Pure exchange OLG, housing prices, credit volume, boom-bust cyles, regime switching

JEL Classification: C62, E32, G21

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1 Introduction

In the absence of financial markets, housing prices are naturally bounded by consumer incomes which represent the fundamentals of the economy. As mortgage loans are common means to finance housing purchases, a theoretical framework to study housing prices should also incorporate a financial sector which supplies loans to finance housing. Suppose a consumer takes a loan to buy a house and uses the revenue from selling the house to repay his loan in the future. If the housing price increases over time, he may be able to afford a house whose value exceeds his income. At the aggregate level, housing prices and mortgage loans may have an off-setting effect such that each of these variables alone becomes arbitrarily large while the net-payment flow remains bounded by consumer incomes. In this paper, we develop a model to demonstrate that this simple idea constitutes a mechanism which generates boom-bust cycles in housing prices, and why mortgage loans may cause housing prices to permanently exceed their fundamental value.

The present model builds on the pure exchange overlapping generations economy with fiat money a la Samuelson (1958) and Gale (1973). We focus on Gale's classical case where the young take loans mediated by a 'central clearing house' (p.29), which we interpret as a banking sector. Thus, the implied direction of intergenerational transfers is from old to young. In the absence of housing, loans in Gale's framework are bounded by consumer incomes. For our purpose we extend his framework by including housing as a utility-yielding durable good, and analyze the equilibrium when consumers take mortgage loans to finance housing. As in Gale (1973) the loan volume supplied by banks evolves according to a simple rollover condition, and therefore expands when the interest rate is positive. However, unlike the case in Gale (1973) we show that the presence of housing permits the loan volume to expand without bound as trading of housing can offset the transfers from old to young in the form of loans.

Suppose that consumer incomes take two values only. Consumption smoothing then implies that when the current income is relatively low compared to the future income, consumers are willing to pay a high interest rate which increases the credit volume over time. Moreover, the abundance of credit translates into a higher demand for housing, and can increase housing prices without bound. This state is called the *expansive regime*. Reversing the argument, the economy enters a *contractive regime* when the current income is relatively high. In this case the credit volume shrinks and the housing price decreases to a lower bound, which we define as the fundamental price. A switch between these two regimes occurs

under a quite moderate income change. If the change is persistent, the system stays in one regime for a number of periods generating boom-bust cycles in the housing price and credit volume. We also show that a large component of the housing price is a pure bubble defined as a deviation from the fundamental value. The deviation is caused by leveraged borrowing, and found to have no bearing on consumption decisions which are exclusively based on the fundamental housing price. Thus, a remarkable consequence is that housing bubbles in our model are fully welfare-neutral.

There is a recent outburst of literature on housing market dynamics after the financial crises unfolded in the US in 2007. Studies that extend the pure exchange overlapping generations framework by including a durable good to study housing market dynamics include Ortalo-Magné & Rady (2006) and Arce & López-Salido (2011). They share a common feature with our model in that housing generates utility. The main difference, however, is that borrowing is limited to meet collateral requirements or down payments in their models while our model does not assume any kind of financial frictions. Other recent studies on housing dynamics using different frameworks include Bajari et al. (2010), Chen & Winter (2011), He et al. (2011) and Burnside et al. (2011) who incorporate various kinds of financial friction and heterogeneity, each with a different focus on the ingredient for the driver of housing dynamics. These contributions are helpful in obtaining insights into different aspects of housing markets and evaluating the quantitative impacts of the interaction between financial markets, housing price and consumption.

In this paper, our objective is different. Rather than developing a stylized model to match data, our aim is to show that the pure exchange overlapping generations framework can be extended in a straightforward way to generate booms and busts in housing prices accompanied by the expanding and contracting credit volume. This co-movement in the time of booms and busts is well-documented (e.g. Chen & Winter (2011) and He et al. (2011)). However, it is not well understood how it is related to fundamentals in the economy. Our analysis uncovers a simple mechanism through which small but persistent income changes generate large movements in housing value and aggregate credit volume. With rational expectations, no heterogeneity and no financial frictions our model can serve as an analytically tractable benchmark for further studies.

The paper is organized as follows. Section 2 introduces the model. Section 3 derives the forward-recursive structure of equilibria while Section 4 studies the equilibrium dynamics under constant incomes. Section 5 generalizes the deterministic structure to the case with time-varying incomes, and analyzes the scope for boom-bust

scenarios to emerge due to persistent income changes. The theoretical findings are illustrated in Section 6 with the help of numerical simulations. Section 7 concludes. Proofs for all results can be found in the mathematical appendix.

2 The Model

We extend the pure exchange rate overlapping generation framework a la Samuelson (1958) and Gale (1973) by including a durable commodity that we refer to as 'housing'. The non-durable good is called 'the consumption good' and is chosen as the numeraire.

Consumption sector

The consumption sector consists of overlapping generations of homogeneous, two-period lived consumers who receive an exogenous random income stream of the non-durable commodity. While the non-durable commodity is consumed in both periods, housing consumption is confined to the second period of life. We denote by $c_t := (c_t^y, c_{t+1}^o)$ the non-durable consumption and $e_t := (e_t^y, e_{t+1}^o) \gg 0$ the lifetime income of the generation born in $t \geq 0$. The latter takes values in the compact set $\mathcal{E} \subset [e_{\min}^y, e_{\max}^y] \times [e_{\min}^o, e_{\max}^o] \subset \mathbb{R}_{++}^2$ for all times $t \geq 0$.

Housing

Houses are retradable and in constant supply normalized to unity. The young purchase houses from the old at the end of period t at the price $p_t > 0$, for which they incur a cost $\kappa > 0$ per unit to be paid in the following period $t + 1$. This parameter can be interpreted as a proportional cost associated with holding houses such as maintenance and remodeling costs or insurance payments, and will turn out to be a crucial ingredient to our model. Housing investment transfers incomes intertemporally and yields utility in the following period.

Financial sector

The financial sector consists of a large number of banks which offer loans at a riskless interest factor $R_t > 0$. Let $b_t \geq 0$ denote the aggregate credit volume corresponding to the resource available to the financial sector at time t. This resource corresponds to the loan repayment of the old and is provided as loans to

the young. Thus, the credit volume evolves according to the rollover condition[1]

$$b_t = R_{t-1} b_{t-1}, \qquad t \geq 1. \tag{1}$$

The initial value $b_0 \geq 0$ is historically given. The contracts supplied by banks allow consumers to transfer wealth from the second to the first period of life. Following Gale (1973), this corresponds to inside money in the *classical case* in contrast to the *Samuelson case* where the intertemporal wealth transfer is from the first to the second period of life.[2] Inside money in the classical case corresponds to an I.O.U. allowing consumers to take loans when young. The role of the financial sector is to enforce the I.O.U. permitting an intergenerational transfer, which otherwise would never take place simply because the old are no longer alive when the young make their loan repayment. Without such intergenerational transfer, the housing price would be bounded by consumer incomes.

Consumer demand

The young choose lifetime consumption of non-durables and housing (c^y, c^o, h) to maximize their lifetime utility function U, which is additively separable over time:

$$U(c^y, c^o, h) = u(c^y) + v(c^o, h).$$

The function u is taken to be of the isoelastic form

$$u(c) = \frac{c^{1-\alpha}}{1-\alpha}, \quad \alpha > 0$$

with the usual interpretation that $u(c) = \log c$ if $\alpha = 1$. Second-period utility v is the composition of u and an aggregator function $g : \mathbb{R}^2_{++} \to \mathbb{R}_+$, which aggregates durable and non-durable consumption in the second period to a composite commodity. Following Lustig & Nieuwerburgh (2005) and Bajari et al. (2010), we use a CES aggregator

$$g(c, h) = [\beta c^\rho + (1-\beta) h^\rho]^{\frac{1}{\rho}}, \ 0 < \beta < 1, \ \rho < 1.$$

The young discount second-period utility by $\gamma > 0$ and thus v takes the form

$$v(c, h) = \gamma\, u(g(c, h)) = \gamma \frac{[\beta c^\rho + (1-\beta) h^\rho]^{\frac{1-\alpha}{\rho}}}{1-\alpha}. \tag{2}$$

[1]Equation (1) may be seen as a reduced-form condition of a financial sector with $L \geq 1$ banks which grant loans to consumers at a safe return $R_t > 0$ and exchange resources in some interbank market at return $R_t^\star > 0$. Let b_t^l denote the initial net resources of bank $l \in \{1, \ldots, L\}$ which chooses its demand d on the interbank market to maximize expected profit $R_t(b_t^l + d) - R_t^\star d$. This and interbank market clearing requires $R_t^\star = R_t$ and $\sum_{l=1}^L d_t^l = 0$. Setting $b_t := \sum_{l=1}^L b_t^l$ then gives (1) at the aggregate level.

[2]For convenience, we define the credit volume to be a positive number $b_t \geq 0$ which reverses the sign convention employed in Gale (1973).

If $\rho = 0$, second period utility is Cobb-Douglas while it is additively separable in housing and consumption if $\rho = 1 - \alpha$. Given incomes $e_t = (e_t^y, e_{t+1}^o)$, the credit return $R_t > 0$, and housing prices $(p_t, p_{t+1}) \gg 0$, the budget constraints are

$$c^y = e_t^y + b - p_t\, h \quad \text{and} \quad c^o = e_{t+1}^o - R_t\, b + (p_{t+1} - \kappa)\, h. \tag{3}$$

Here b and h are the loan demand and housing investment respectively. Note that we deviate from the traditional sign convention by denoting a positive loan demand by $b \geq 0$. Using (3), the young consumers' objective function at time t is

$$V_t(b, h) := U(e_t^y + b - p_t\, h, e_{t+1}^o - R_t\, b + (p_{t+1} - \kappa_{t+1})\, h, h).$$

The consumers' decision problem reads

$$\max_{b,h} \Big\{ V_t(b, h) \,|\, h \geq 0,\ p_t\, h \leq e_t^y + b,\ e_{t+1}^o - b\, R_t + h(p_{t+1} - \kappa) \geq 0 \Big\}. \tag{4}$$

Note that no sign restriction on b is imposed at the individual level.

Equilibrium

The 'fundamentals' of the economy are given by the exogenous income sequence $\{e_t\}_{t\geq 0}$. The following definition of equilibrium reconciles market clearing and individual optimality under rational expectations.

Definition 1

Given the fundamentals $\{e_t\}_{t\geq 0}$ and an initial credit volume $b_0 \geq 0$, an equilibrium is a sequence $\{b_t, h_t, R_t, p_t\}_{t\geq 0}$, which satisfies $p_t > 0$, $R_t > 0$ and the following conditions for each $t \geq 0$:

(i) The decision (b_t, h_t) solves (4) given housing prices and lifetime income.

(ii) Markets clear, i.e., $h_t = 1$ and b_t evolves according to (1).

Note that Walras' law implies consumption good market clearing, i.e, $c_t^y + c_t^o = e_t^y + e_t^o - \kappa$ for all $t \geq 0$.

3 Recursive Equilibrium Structure

Recursive equilibrium

As a first step, we unveil the forward-recursive structure of equilibrium and the state dynamics of the model. Essentially, we will show that the dynamics is driven by the evolution of the variable

$$q_t := p_t - b_t, \quad t \geq 0. \tag{5}$$

Below, we will argue that q_t can be interpreted as the *fundamental housing price*. We focus on equilibria where $q_t \geq 0$ for all $t \geq 0$. Under this restriction, loan repayments must fully be backed by future housing values in equilibrium, i.e., $R_t b_t \leq p_{t+1}$. From (1), the supply of loans is predetermined by the repayments of the old to the financial sector. In equilibrium, the interest rate and housing prices are determined such that the young are willing to demand the loans supplied by the financial sector, and purchase the constant stock of houses from the old. Since no sign-restriction is imposed on loan demand b at the individual level, the first order conditions of the young consumers' decision problem (4) must be satisfied in equilibrium. Hence, the following Euler equations have to hold for each period $t \geq 0$:

$$u'(e_t^y - q_t) = R_t\, v_c(e_{t+1}^o - \kappa + q_{t+1}, 1) \tag{6a}$$

$$p_t\, u'(e_t^y - q_t) = (p_{t+1} - \kappa)\, v_c(e_{t+1}^o - \kappa + q_{t+1}, 1) + v_h(e_{t+1}^o - \kappa + q_{t+1}, 1). \tag{6b}$$

Combining (6a) and (6b) using (1) gives the following equilibrium condition

$$F(q_{t+1}, q_t; e_t) := q_t\, u'(e_t^y - q_t) - v_c(e_{t+1}^o - \kappa + q_{t+1}, 1)\,(q_{t+1} - \kappa) - v_h(e_{t+1}^o - \kappa + q_{t+1}, 1) = 0 \tag{7}$$

which has to hold for each $t \geq 0$. Condition (7) determines q_{t+1} implicitly as a function of q_t and e_t. The following result states necessary and sufficient conditions under which a unique solution to (7) can be determined. This provides the basis for deriving a dynamical system.

Lemma 1

Suppose $\rho \geq 0$ and $\alpha < 1$. Then, for each $e = (e^y, e^o) \gg 0$ and $q < e^y$ there exists a unique value $q_1 > \kappa - e^o$, which satisfies $F(q_1, q; e) = 0$ with F defined in (7).

Lemma 1 permits to define a map $f(\cdot; e) : (-\infty, e^y) \to (\kappa - e^o, \infty)$ which determines the unique zero of $F(\cdot, q; e) = 0$ for each $q < e^y$.[3] Thus, whenever $q_t < e_t^y$, the unique solution to (7) can be written as

$$q_{t+1} = f(q_t; e_t). \tag{8}$$

Preview of the equilibrium dynamics

Equation (8) relates the fundamental housing price to its previous value and the

[3] The restrictions $\rho \geq 0$ and $\alpha < 1$ are necessary and sufficient for $\lim_{c \to \infty} v_c(c, 1)\, c = \infty$ which is crucial for existence of a solution to (7) for arbitrary q_t and e_t. Although the restriction $\alpha < 1$ excludes a logarithmic function u used in Bajari et al. (2010), this case can be approximated as the limiting case $\alpha \to 1$ in our setup.

exogenous income sequence which will constitute the driver of our equilibrium dynamics below. At this stage, however, (8) does not yet define a dynamical system because we have not provided conditions for a well-defined state space. The latter is given by a subset $\mathbb{Q} \subset \mathbb{R}_+$ in which the equilibrium sequence $\{q_t\}_{t\geq 0}$ remains under arbitrary values of fundamentals $\{e_t\}_{t\geq 0}$. Formally, recalling that $e_t \in \mathcal{E}$ we require that $\mathbb{Q}$ be self-supporting for the family of mappings $(f(\cdot; e))_{e\in\mathcal{E}}$ in the sense that $q \in \mathbb{Q}$ implies $q' = f(q; e) \in \mathbb{Q}$ for all $e \in \mathcal{E}$. The following Sections 4 and 5 establish existence of such a state space formally, which requires additional restrictions on incomes and the cost parameter κ.

For the remainder of this section, however, let us simply assume that a state space $\mathbb{Q} \subset \mathbb{R}_+$ exists. Then, the forward-recursive structure in (8) is generated by mixing the family of mappings $(f\cdot; e)_{e\in\mathcal{E}}$. That is, given q_t, the income $e_t \in \mathcal{E}$ of generation t 'selects' a particular map $f(\cdot; e_t)$ which determines the next value $q_{t+1} = f(q_t; e_t)$. In this way, the evolution of the sequence $\{q_t\}_{t\geq 0}$ is directly related to the sequence of fundamentals $\{e_t\}_{t\geq 0}$. To obtain the evolution of the other equilibrium variables, use (8) in (6a) to observe that the equilibrium interest factor is given as

$$R_t = \mathcal{R}(q_t; e_t) := \frac{u'(e_t^y - q_t)}{v_c(e_{t+1}^o - \kappa + f(q_t; e_t^y), 1)}. \tag{9}$$

Equation (9) equates the equilibrium interest factor to the intertemporal marginal rate of substitution in consumption. Using (9) in (1) and combining (8) with (1) and (5), we obtain the following three-dimensional dynamical system:

$$q_{t+1} = f(q_t; e_t) \tag{10a}$$

$$b_{t+1} = \mathcal{R}(q_t; e_t) b_t \tag{10b}$$

$$p_{t+1} = f(q_t; e_t) + \mathcal{R}(q_t; e_t) b_t. \tag{10c}$$

Given initial values $b_0 \geq 0$ and $p_0 > 0$ such that $q_0 = p_0 - b_0 \in \mathbb{Q}$, equations (10a–c) completely describe the equilibrium dynamics. The formal properties of the system will be studied extensively in the following two sections under different assumptions on consumer incomes. Even at this stage, however, several qualitative observations can already be made. First, we observe that (10a) constitutes a sub-system of the equilibrium dynamics that evolves independently of the other equilibrium variables and is bounded by consumer incomes. Second, it is obvious from (10b) that the credit volume expands in periods where $R_t > 1$ and contracts if $R_t < 1$. Below we refer to these cases as the *expansive regime* and the *contractive regime*, respectively. Moreover, by (10c) these changes in the credit volume translate directly into a corresponding change in housing prices. Thus, if income changes cause the system

to alternate between the expansive and contractive regime, large movements in housing prices occur. The large movements can almost exclusively be attributed to movements in the credit volume while the fundamental price is uniformly bounded by incomes. This linkage constitutes the key mechanism of the model studied in this paper.

The role of the cost parameter κ

The dynamic analysis presented in the following sections will reveal that the housing cost parameter κ is a crucial ingredient to our model. Even at this point, some intuition why this holds can be developed. Use (6a,b) to write (10c) as

$$p_{t+1} = \mathcal{R}(q_t, e_t)p_t + \kappa - \frac{v_h(e^o_{t+1} - \kappa + f(q_t; e_t), 1)}{v_c(e^o_{t+1} - \kappa + f(q_t; e_t), 1)}. \tag{11}$$

From (11) we infer that the equilibrium housing return $(p_{t+1}-\kappa)/p_t$ must be smaller than R_t. This is because the old derive utility from housing—which may be seen as a housing dividend—and therefore accept a return on housing investment which is lower than on loans. If $\kappa = 0$, then $p_{t+1}/p_t < b_{t+1}/b_t = R_t$, i.e., the credit volume (10b) grows faster than the housing price (10c). As a consequence, loan repayments will asymptotically no longer backed by housing values which violates our restriction $q_t > 0$. In fact, equilibria will fail to exist in cases where the return R_t is asymptotically smaller than unity. To avoid these problems, a sufficiently large housing cost is necessary. A smaller cost raises the demand for housing, which must be counteracted by a smaller ratio p_{t+1}/p_t to meet the constant supply of housing. Subsequently, loans grow faster than the housing price.

4 Housing Price Dynamics

Constant incomes

In this section we characterize the equilibrium dynamics (10a–c) for the case with constant incomes $e_t \equiv e = (e^y, e^o) \in \mathcal{E}$ for all $t \geq 0$. For notational convenience, the dependence of variables and functions on e will be suppressed. In the previous section we argued that the sign of the interest rate determines whether the economy is in an expansive or contractive regime. In this section we explore the linkage that relates the equilibrium interest rate to the fundamentals of the economy. Theorem 1 provides a complete characterization of equilibria under constant incomes. The insights obtained will serve as a building block to study the case with time-varying first-period incomes in the next section.

Steady state analysis

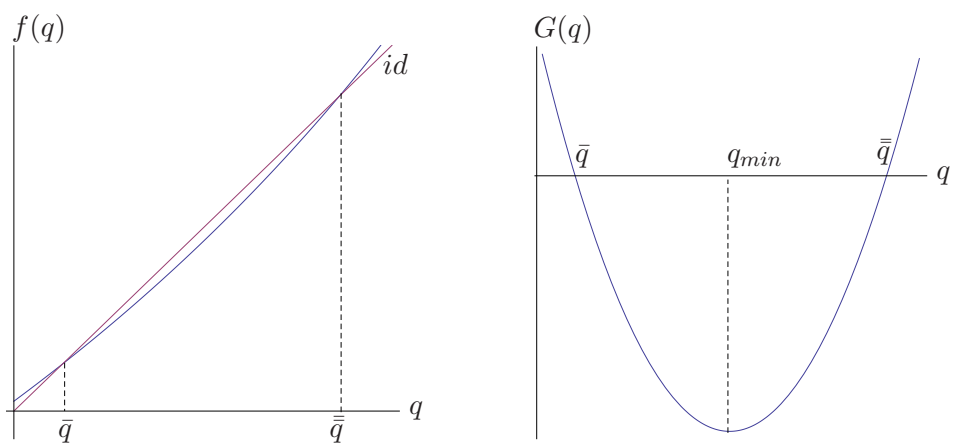
By the insights from previous section, the existence of equilibrium is essentially equivalent to determining a state space $\mathbb{Q}$ on which the dynamics (8) can live. To establish existence, we seek to determine an interval $\mathbb{Q} \subset [0, e^y)$ which is self-supporting for the map $f = f(\cdot; e)$, i.e., $f(q) \in \mathbb{Q}$ for all $q \in \mathbb{Q}$. By the properties of f established in Lemma 5, fixed points of f are natural boundary points of such intervals. Thus, as a first step, we study steady states of (10a) corresponding to fixed points of f and their properties.

Since f maps $(-\infty, e^y)$ into $(\kappa - e^o, \infty)$, it is clear that any steady state must lie in the open interval $(\kappa - e^o, e^y)$. Therefore, a necessary precondition for fixed points to exist is $e^y + e^o > \kappa$. This condition simply says that the resources available in each period must be large enough to cover housing costs. In what follows, we impose a stronger restriction that $e^o > \kappa$ which says that second period income alone is large enough to cover the housing cost at equilibrium.

It follows from (7) that fixed-points of f are zeros of the map $G : (\kappa - e^o, e^y) \to \mathbb{R}$

$$G(q) := F(q, q; e). \tag{12}$$

Formal properties of f and G are stated as Lemmata 5 and 6 in Appendix B. The qualitative result is illustrated in the following figures which depict the map f and the fixed point map G. Note that the zeros of G in Figure 1(b) correspond to intersections of (the graph of) f with the principal diagonal in Figure 1(a). By Lemma



(a) The time-one map f (b) The fixed point map G

Figure 1: Time-one map of state dynamics

6, G is a strictly convex function and there exists a unique value $q_{\min} \in (\kappa - e^o, e^y)$ at which $G'(q_{\min}) = 0$ and G attains its global minimum. Combined with the boundary behavior of G and excluding the case $G(q_{\min}) = 0$, the requirement

$G(q_{\min}) < 0$ is necessary and sufficient for fixed points to exist. In this case, f has precisely two fixed points $\bar{q} < \bar{\bar{q}}$. Requiring, in addition, that $G'(0) < 0 < G(0)$ ensures that $\bar{q} > 0$, a fact that directly be verified from Figure 1(b). In this case, both fixed points are necessarily positive. The numerical simulation of Section 6 shows that all three conditions are satisfied for a broad range of economically reasonable parameterizations.

The previous insights are stated formally in the following lemma which lists conditions for positive fixed points to exist and characterizes their stability properties.[4]

Lemma 2

Suppose $\rho \geq 0$ and $\alpha < 1$ and let incomes $e = (e^y, e^0) \in \mathcal{E}$ satisfy $e^o > \kappa$. In addition, suppose the minimum of G defined in (12) satisfies $G(q_{\min}) < 0$. Then,

(i) The time-one map f from (8) has precisely two fixed points $\bar{q} \in (\kappa - e^o, q_{\min})$ and $\bar{\bar{q}} \in (q_{\min}, e^y)$. If, in addition, $G'(0) < 0 < G(0)$, then $\bar{q} > 0$.

(ii) The fixed point $\bar{\bar{q}}$ is locally unstable while $\bar{q}$ is asymptotically stable. Moreover, $f(q) > q$ for all $q \in (-\infty, \bar{q}\,) \cup (\,\bar{\bar{q}}, e^y)$ and $f(q) < q$ for all $q \in (\bar{q}, \bar{\bar{q}})$.

Determining a state space

Using Lemma 2 it is now straightforward to determine the desired state space $\mathbb{Q}$ for the dynamics (10a). To ensure that $q_t \geq 0$ for all t, we assume the additional condition from Lemma 2(i) such that the fixed points of f satisfy $0 < \bar{q} < \bar{\bar{q}}$. Then, the interval $\mathbb{Q} := [0, \bar{\bar{q}})$ is self-supporting for the dynamics (8), an insight which is readily confirmed by Figure 1(a).[5] Thus, restricting f to this subset defines a discrete dynamical system whose long-run behavior is characterized in the next lemma.

[4] Qualitatively, the dynamics bear some resemblance to the equilibrium dynamics of real money balances in Gale (1973) in that there are two steady states, one of which – the larger one – is unstable and the other one asymptotically stable. The latter corresponds to the non-monetary steady state in which no intergenerational transfers take place. Precisely this scenario would be recovered in our setting in the absence of housing. In the presence of housing, however, the analog of real money balances in Gale's model is played by the fundamental price of housing which also has an unstable and a –smaller – stable steady state. Unlike the case in Gale (1973), however, the latter is bounded away from zero. More importantly, as argued above, even though the q_t dynamics is bounded, p_t and b_t can grow without bound in our model.

[5] We will exclude $\bar{\bar{q}}$ from the state space in order to rule out degenerate equilibria. In the case with time-varying incomes to be studied in the next section, this imposes no additional restriction regarding the long run behavior of the system.

Lemma 3
Suppose $\rho \geq 0$ and $\alpha < 1$ and let incomes $e = (e^y, e^0) \in \mathcal{E}$ satisfy $e^o > \kappa$. In addition, let G defined in (12) satisfy $G(q_{\min}) < 0$ and $G'(0) < 0 < G(0)$. Then,

(i) The restricted time-one map $f : \mathbb{Q} \to \mathbb{Q}$ has $\bar{q}$ as its unique fixed point.

(ii) This fixed point is globally stable and for each $q_0 \in \mathbb{Q}$ the sequence $\{q_t\}_{t\geq 0}$ defined recursively as $q_{t+1} = f(q_t)$, $t \geq 0$ converges monotonically to $\bar{q}$.

Equilibrium under constant incomes
We are now in a position to characterize the equilibrium dynamics (10a–c) formally under fixed incomes $e_t \equiv e = (e^y, e^o) \in \mathcal{E}$. Fix an initial value (p_0, b_0) which satisfies $b_0 \geq 0$, $p_0 > 0$ and $q_0 = p_0 - b_0 \in \mathbb{Q}$. Recall that the dynamics (10a) of q_t is decoupled from the other two variables and converge monotonically to a unique steady state $\bar{q}$ by Lemma 3. It is evident from (10b) and (10c) that the long-run dynamic behavior of the credit volume b_t and housing prices p_t depend on the steady state interest factor $\mathcal{R}(\bar{q}; e)$. If $\mathcal{R}(\bar{q}; e) < 1$, the credit volume asymptotically converges to zero while by (5) prices converge to $\bar{p} = \bar{q}$. Conversely, if $\mathcal{R}(\bar{q}; e) > 1$ and $b_0 > 0$, both credit volume and housing prices grow without bound and converge to infinity. Notice, however, that the equilibrium dynamics is well-defined in either case. The following theorem summarizes these insights and provides a complete characterization of equilibria under constant incomes.

Theorem 1
Let the hypotheses of Lemma 3 be satisfied. Then,

(i) Each $p_0 > 0$ and $b_0 \geq 0$ for which $p_0 - b_0 \in \mathbb{Q}$ defines an equilibrium where the evolution of the equilibrium variables follows (10a–c) and $\lim_{t\to\infty} q_t = \bar{q}$.

(ii) If $b_0 = 0$ or $\mathcal{R}(\bar{q}; e) < 1$, then $\lim_{t\to\infty} p_t = \bar{q}$ while $\lim_{t\to\infty} b_t = 0$.

(iii) If $b_0 > 0$ and $\mathcal{R}(\bar{q}; e) > 1$, then $\lim_{t\to\infty} p_t = \lim_{t\to\infty} b_t = \infty$.

Recall that if $b_0 = 0$, i.e., in the absence of a financial sector, the housing price coincides with the sequence $\{q_t\}_{t\geq 0}$ which is uniformly bounded. Thus, any potential unboundedness of housing prices is exclusively due to the financial sector. If $b_0 > 0$, the equilibrium properties are determined by the steady state return $\mathcal{R}(\bar{q}; e)$ and may either be *contractive* or *expansive*, the case (ii) or (iii) in the above theorem. In case (ii), the credit volume converges to zero and the housing price coincides with its fundamental value, at least asymptotically. In case (iii),

however, we show below that the housing price p_t has a persistent bubble equal to b_t which makes it permanently exceed its fundamental value q_t.

To build some intuition how the emergence of the regimes in Theorem 1 is related to consumer incomes e, suppose first that the income e^y of the young is low relative to second-period income e^o. Then, young consumers have a strong desire to smooth their consumption and borrow against future income which results in a high interest rate. By (1), this results in an expansion of loans over time. As the fundamental housing price is uniformly bounded and converges to a constant value, the increased credit volume translates directly into an increase in housing prices. In other words, the increased credit volume is fully used for increased expenditures for housing. Reversing these arguments, a sufficiently high first-period income e^y relative to e^o results in a low interest rate which causes the loan volume to contract and the housing price to converge to its fundamental value. From this reasoning we propose that suitable income changes provide a simple way of generating large movements in housing prices. This idea is formally explored in Section 5.

Bubbles and fundamental housing prices

If houses were absent—or could not be traded between generations—our model would essentially reduce to a version of Gale (1973) where the credit sector plays a role equivalent to negative money. In that case loans are not backed by any physical resources and correspond to an intrinsically valueless asset that is traded at a positive price. Thus, the credit volume is a pure *bubble*.

The bubbleless equilibrium in our economy corresponds to the initial choice $b_0 = 0$ which implies $b_t = 0$ and $p_t = q_t$ for all t. We call q_t the *fundamental housing price* that would prevail in the absence of mortgage loans. The same state is reached asymptotically in case (ii) of Theorem 1 where loans vanish asymptotically and the equilibrium is asymptotically bubbleless.

We call the equilibrium scenario from Theorem 1 (iii) where $\mathcal{R}(\bar{q}; e) > 1$ and the credit volume is asymptotically non-vanishing a *bubbly equilibrium*. We now demonstrate that this is indeed justified and the housing price p_t decomposes into a stationary component (equal to q_t) that reflects the fundamental value of housing and a bubbly component (equal to b_t) that captures the deviation from the fundamental value. Define the variable

$$d_t = \mathcal{D}(q_t; e) := \frac{v_h(e^0 - \kappa + q_t, 1)}{v_c(e^0 - \kappa + q_t, 1)} > 0 \tag{13}$$

which may be interpreted as the *housing dividend* corresponding to the marginal utility earned from consuming an additional unit of housing. Specifically, d_t is zero

if housing yields no utility, i.e., $v_h = 0$. Using (13) in the Euler equations (6b,b) permits to write $p_t = \frac{p_{t+1}+d_{t+1}-\kappa}{R_t}$ which may be solved forward to obtain

$$p_t = \sum_{n=0}^{\infty}(d_{t+1+n} - \kappa) \prod_{m=0}^{n} R_{t+m}^{-1} + \lim_{n\to\infty} p_{t+1+n} \prod_{m=0}^{n} R_{t+m}^{-1}. \tag{14}$$

Using $p_{t+1+n} = q_{t+1+n} + b_{t+1+n}$ by (5) together with $\lim_{n\to\infty} q_{t+1+n} = \bar{q} > 0$, $\lim_{n\to\infty} R_{t+n} = \mathcal{R}(\bar{q}; e) > 1$ by Lemma 3 and continuity of $\mathcal{R}$, and $b_{t+1+n} \prod_{m=0}^{n} R_{t+m}^{-1} = b_t$ for all $n \geq 0$ by (1) we can write (14) as[6]

$$q_t = p_t - b_t = \sum_{n=0}^{\infty}(d_{t+1+n} - \kappa) \prod_{m=0}^{n} R_{t+m}^{-1}. \tag{15}$$

Thus, q_t is exclusively determined by the discounted sum of future net dividends justifying our interpretation as a fundamental price of housing.

Welfare

The fundamental housing price is also a measure for the net intergenerational transfer of incomes where p_t measures the commodity transfer from young to old through the housing market whereas b_t measures that from old to young through the financial sector. Hence, q_t implies a net commodity transfer from young to old. Using (3) the induced consumption allocation can be written as

$$c_t^y = e_t^y - q_t \quad \text{and} \quad c_t^o = e_t^0 - \kappa + q_t. \tag{16}$$

This shows that the resulting consumption allocation is exclusively determined by the dynamics of fundamentals (10a). Thus, for a given value q_0, any injection of credit $b_0 > 0$ merely increases the bubbly component of the housing price. As a consequence, any price deviation from the fundamental price is fully neutral with respect to consumer welfare. These observations also extend to the case with time varying incomes studied in the next section.

To obtain additional insights into the welfare properties of equilibrium allocations, let $a = (c^y, c^o, h)$ be any stationary allocation which satisfies the feasibility constraints $c^y \geq 0, c^o \geq 0$, $c^y + c^o + \kappa h \leq e^y + e^o$ and $0 \leq h \leq 1$. The induced variables $R(a) := \frac{u'(c^y)}{v_c(c^o,h)}$ and $D(a) := \frac{v_h(c^o,h)}{v_c(c^o,h)}$ may be interpreted as the supporting return

[6]Here we use that $R_{t+n} = \mathcal{R}(q_{t+n}, e)$ and $\lim_{n\to\infty} R_{t+n} = \mathcal{R}(\bar{q}; e) > 1$ implies that there exists $1 < \Delta < \mathcal{R}(\bar{q}; e)$ and $n_0 \in \mathbb{N}$ such that $R_{t+n} \geq \Delta > 1$ for all $n > n_0$. Therefore,

$$0 \leq \lim_{n\to\infty} q_{t+1+n} \prod_{m=0}^{n} R_{t+m}^{-1} = \bar{q} \prod_{m=0}^{\infty} R_{t+m}^{-1} < \bar{q}\Delta^{n_0} \prod_{m=0}^{n_0} R_{t+m}^{-1} \prod_{m=0}^{\infty} \Delta^{-1} = 0.$$

and housing dividend. Standard arguments imply that a unique stationary allocation $a_\star = (c^y_\star, c^o_\star, h_\star)$ exists which maximizes consumer utility and satisfies the classical golden rule condition $R_\star := R(a_\star) = 1$ and $D_\star := D(a_\star) \geq \kappa$. The latter may be interpreted as an efficiency condition with respect housing consumption. Specifically, $D_\star = \kappa$ if $h^\star = 1$.

For a stationary equilibrium allocation $a_E = (e^y - q, e^o - \kappa + q, 1)$ where q is a steady state of (10a), the supporting prices satisfy $R^E := R(a^E) = \mathcal{R}(q; e)$ and $D^E := D(a^E) = \mathcal{D}(q; e)$ with $\mathcal{R}$ and $\mathcal{D}$ defined by (9) and (13). It follows directly from Theorem 1 that, in general, equilibrium allocations will not be optimal in the above sense, a typical feature of OLG models. Moreover, it is straightforward to show that $R^E \gtreqless 1$ iff $D^E \gtreqless \kappa$.[7] Therefore, the characterization in Theorem 1 satisfies $D^E < \kappa$ in case (ii) while $D^E > \kappa$ in case (iii).

Equation (16) resembles the consumption allocation in Gale (1973) along a monetary equilibrium with real monetary transfers $q_t > 0$. In his model, such equilibria exist whenever the intertemporal return R_0 supporting the autarky allocation satisfies $R_0 < 1$. It turns out that a similar characterization of the above existence conditions is also possible in our framework.[8] To this end, consider the autarky allocation $a_0 := (c^y_0, c^o_0, h_0) = (e^y, e^o, 1)$ where generations do not trade. Then, the condition $G(0) > 0$ from Lemma 3 holds if and only if a_0 is inefficient with respect to housing consumption, i.e., $D_0 := D(a_0) < \kappa$. Note that this condition can only be satisfied if $\kappa > 0$ reconfirming that the cost parameter $\kappa > 0$ is a key ingredient of our model. Similarly, the condition $G'(0) < 0$ from Lemma 3 holds, if and only if $R_0 := R(a_0) = \frac{u'(e^y)}{v_c(e^o-\kappa,1)}$ satisfies

$$R_0 < 1 + \frac{v_{ch}(e^o - \kappa, 1) + \kappa |v_{cc}(e^o - \kappa, 1)|}{v_c(e^o - \kappa, 1)}. \tag{17}$$

If $v_{ch} \geq 0$, i.e., if consumption and housing are complements—equivalent to $\rho \leq 1 - \alpha$—a sufficient condition for (17) to hold is $R_0 < 1$.

5 Boom-Bust Cycles in Housing Prices

Time-varying incomes

We now analyze the case where incomes vary over time. For ease of exposition, we will confine our attention to changes in first-period incomes while second-period

[7] As any equilibrium satisfies $R_t = \frac{p_{t+1}+d_{t+1}-\kappa}{p_t}$ for all $t \geq 0$, $R_t = \frac{q_{t+1}+d_{t+1}-\kappa}{q_t}$ by (1) and (5). For stationary allocations where $q_t \equiv q > 0$, $R_t \equiv R$ by (9) and $d_t \equiv D$ by (13), $R = \frac{q+D-\kappa}{q}$.

[8] We are grateful to an anonymous referee for this observation.

incomes are constant. Thus, assume as in the previous section that $e_t^o \equiv e^o > \kappa$ while e_t^y takes values in the set $\mathcal{E}^y := [e^y_{\min}, e^y_{\max}] \subset \mathbb{R}_{++}$. In the sequel, we will therefore drop the argument e^o writing e.g. $f(q; e^y)$ instead of $f(q; e^y, e^o)$.

Determining a state space

The existence of equilibrium requires to determine a state space for the equilibrium dynamics (10a). Formally, we seek to determine an interval $\mathbb{Q}$ which is self supporting under the family $(f\cdot; e^y)_{e^y \in \mathcal{E}^y}$ in the sense that $q \in \mathbb{Q}$ implies $f(q; e^y) \in \mathbb{Q}$ for all $e^y \in \mathcal{E}^y$. While the underlying construction principle is the same as in the previous section, the present case must incorporate that the map f and its fixed points vary with consumer incomes. As any equilibrium sequence $\{q_t\}_{t\geq 0}$ satisfies $0 < q_t < e_t^y$ for all $t \geq 0$, it is also clear that $\mathbb{Q}$ must be a subset of $[0, e^y_{\min}]$.

Let us assume that the hypotheses of Lemma 3 are satisfied for all $e^y \in \mathcal{E}^y$.[9] Then, each map $f(\cdot; e^y)$ has precisely two fixed points in $(0, e^y)$ which we denote by $\bar{q}(e^y)$ and $\bar{\bar{q}}(e^y)$. Lemma 7 in Appendix B establishes formally that both fixed points vary smoothly with incomes and that an increase in e^y increases $\bar{\bar{q}}(e^y)$ and decreases $\bar{q}(e^y)$. Thus, the values

$$\bar{q}_{\min} := \min_{e^y \in \mathcal{E}^y} \left\{ \bar{q}(e^y) \right\} = \bar{q}(e^y_{\max}) \tag{18a}$$

$$\bar{q}_{\max} := \max_{e^y \in \mathcal{E}^y} \left\{ \bar{q}(e^y) \right\} = \bar{q}(e^y_{\min}) \tag{18b}$$

$$\bar{\bar{q}}_{\min} := \min_{e^y \in \mathcal{E}^y} \left\{ \bar{\bar{q}}(e^y) \right\} = \bar{\bar{q}}(e^y_{\min}) \tag{18c}$$

denote the smallest and largest fixed point $\bar{q}$ and the smallest fixed point $\bar{\bar{q}}$ of the mappings f as incomes vary across the interval $\mathcal{E}^y = [e^y_{\min}, e^y_{\max}]$. Figure 2 illustrates this and the following results for two-valued incomes $e^y \in \{e^y_{\min}, e^y_{\max}\}$.

The values in (18a–c) are natural bounds for intervals which are self-supporting for the family of dynamic mappings $(f\cdot; e^y)_{e^y \in \mathcal{E}^y}$. Defining the intervals $\bar{\mathbb{Q}} := [\bar{q}_{\min}, \bar{q}_{\max}]$ and $\mathbb{Q} := [0, \bar{\bar{q}}_{\min})$[10], this conjecture is confirmed by the next lemma which essentially extends Lemma 3 to the more general stochastic case.

Lemma 4

Let the hypotheses of Lemma 3 be satisfied for each $e^y \in \mathcal{E}^y$. Then,

(i) Both intervals $\bar{\mathbb{Q}}$ and $\mathbb{Q}$ are self-supporting for the family $(f\cdot : e^y)_{e^y \in \mathcal{E}^y}$.

(ii) For each $q_0 \in \mathbb{Q}$ and any income sequence $\{e_t^y\}_{t\geq 0}$ where $e_t^y \in \mathcal{E}^y$, the sequence $\{q_t\}_{t\geq 0}$ generated by (10a) converges to the set $\bar{\mathbb{Q}}$.

[9] In fact, it would suffice if they are satisfied at the extreme points $e^y = e^y_{\min}$ and $e^y = e^y_{\max}$.

[10] Note that $0 < \bar{q}_{\min} < \bar{q}_{\max} < \bar{\bar{q}}_{\min}$ which implies the inclusions $\emptyset \neq \bar{\mathbb{Q}} \subsetneq \mathbb{Q}$.

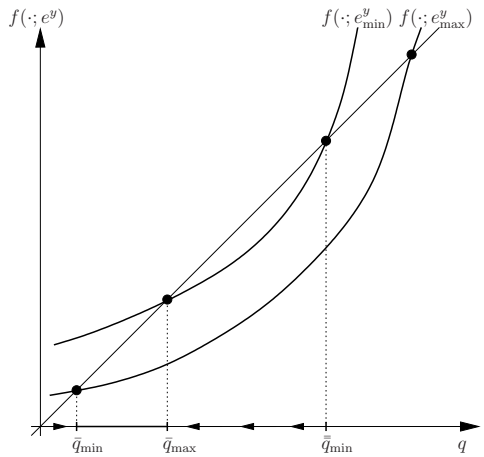


Figure 2: Time-one maps generating the dynamics under two incomes

Equilibrium under time-varying incomes
Based on the previous result, the next theorem generalizes the existence result from Theorem 1(i) to the case with time-varying first-period incomes. Note that Theorem 1(i) obtains as a special case where $e^y_{\min} = e^y_{\max} = e^y$.

Theorem 2
Let the hypotheses of Lemma 3 be satisfied for each $e^y \in \mathcal{E}^y$. Then, given initial values $p_0 > 0$ and $b_0 \geq 0$ satisfying $q_0 := p_0 - b_0 \in \mathbb{Q}$, any income sequence $\{e^y_t\}_{t\geq 0}$ where $e^y_t \in \mathcal{E}^y$ defines an equilibrium generated by (10a–c).

It follows from Lemma 4 that asymptotically the equilibrium sequence $\{q_t\}_{t\geq 0}$ will take values in the set $\bar{\mathbb{Q}}$. Since we are interested in the long-run properties of equilibrium, we can confine our attention to the set $\bar{\mathbb{Q}}$. The sign of the interest rate is again crucial for the long-run behavior of equilibrium housing prices and the credit volume. Lemma 8 in Appendix B establishes that the interest rate determined by the mapping $\mathcal{R}$ from (9) is a smooth function that is increasing in q and decreasing in e^y. Thus, the values

$$\begin{aligned} R_{\min} &:= \min\{\mathcal{R}(q; e^y) \mid q \in \bar{\mathbb{Q}}, e^y \in \mathcal{E}^y\} = \mathcal{R}(\bar{q}_{\min}; e^y_{\max}) \\ R_{\max} &:= \max\{\mathcal{R}(q; e^y) \mid q \in \bar{\mathbb{Q}}, e^y \in \mathcal{E}^y\} = \mathcal{R}(\bar{q}_{\max}; e^y_{\min}) \end{aligned}$$

define the maximum and minimum return observed as incomes and fundamental prices range across the state space $\bar{\mathbb{Q}} \times \mathcal{E}^y$. The following theorem generalizes the characterization of equilibria from Theorem 1(ii) and (iii) to the present case.

Theorem 3
Let the hypotheses of Lemma 3 be satisfied for each $e^y \in \mathcal{E}^y$. Then, for any $p_0 > 0$

and $b_0 \geq 0$ satisfying $q_0 := p_0 - b_0 \in \mathbb{Q}$ and any income sequence $\{e_t^y\}_{t\geq 0}$ where $e_t^y \in \mathcal{E}^y$, the equilibrium generated by (10a–c) satisfies the following:

(i) If $b_0 = 0$ or $R_{\max} < 1$, then $\lim_{t\to\infty} b_t = 0$ while $\lim_{t\to\infty} |p_t - q_t| = 0$.

(ii) If $b_0 > 0$ and $R_{\min} > 1$, then $\lim_{t\to\infty} p_t = \lim_{t\to\infty} b_t = \infty$.

The mechanism for boom-bust cycles

Excluding the non-generic cases $R_{\min} = 1$ and $R_{\max} = 1$, Theorem 3 shows that the equilibrium is uniformly contractive if $R_{\max} < 1$ and expansive if $R_{\min} > 1$. Qualitatively, the resulting dynamics are identical to the case with constant incomes studied in the previous section. In particular, the hypotheses of Theorem 3(i) and (ii) exclude boom-bust cycles in housing prices. Thus, we obtain $R_{\min} < 1 < R_{\max}$ as a necessary condition for such cycles to emerge.

Let this condition be satisfied and suppose, in addition, that $b_0 > 0$. To illustrate the mechanism that generates boom-bust cycles in housing prices, consider the simplest case where e^y takes two values $e^y_{\min}$ and $e^y_{\max}$. Suppose that incomes initially take the lower value $e_t^y = e^y_{\min}$. Then, the dynamics generated by the map $f(\cdot; e^y_{\min})$ start converging to the associated steady state $\bar{q}(e^y_{\min}) = \bar{q}_{\max}$ and we have $R_t > 1$ for t sufficiently large as $\mathcal{R}(\bar{q}_{\max}, e^y_{\min}) = R_{\max} > 1$. By (1), the credit volume starts to expand and so do housing prices while their difference q_t is uniformly bounded.

Now, suppose that at some time $\tilde{t} > 0$, incomes switch to the higher value $e^y_{\max}$. The corresponding dynamics is now generated by the map $f(\cdot; e^y_{\max})$ which has $\bar{q}_{\min}$ as its unique steady state to which the variable q_t starts converging. For sufficiently large $t > \tilde{t}$, we will have $R_t < 1$ implying that both credit volume and housing price will contract.

Combining these observations, it is clear that under time-varying incomes, the system will alternate between an expansionary regime and a contractive regime. These changes are most profound if $\mathcal{R}(q; e^y_{\min}) > 1$ and $\mathcal{R}(q; e^y_{\max}) < 1$ for all $q \in \bar{\mathbb{Q}}$. The first requirement is equivalent to $\mathcal{R}(\bar{q}_{\min}, e^y_{\min}) > 1$, and implies that the credit volume starts expanding immediately when $e_t = e^y_{\min}$. The second condition is equivalent to $\mathcal{R}(\bar{q}_{\max}, e^y_{\max}) < 1$, and implies that the credit volume starts contracting immediately when $e_t = e^y_{\max}$. Now if the income sequence is persistent, then long periods of credit expansion will follow long periods of credit contraction. This mechanism offers a potential to generate large movements in housing prices due to persistent income changes.[11]

[11] The mechanism straightforwardly generalizes to the case where incomes take values in the

At first sight it may seem puzzling that the collapse of housing prices is consistent with perfect foresight. The intuition is that the drop in the housing price is accompanied by a fall in the interest rate which reduces refinancing cost, and therefore consumers are willing to accept the capital loss. In other words, the economy experiences boom-bust cycles, along which consumers adjust their behavior perfectly without causing any bankruptcy problem.

6 A Numerical Example

We employ numerical simulations to demonstrate that the previous boom-bust scenario is compatible with parameter choices used in recent quantitative studies, and that the switch between the two regimes is triggered by relatively small income changes.

Parameters

We choose α close to unity to approximate a logarithmic function u used in Bajari et al. (2010). For simplicity, we follow Li & Yao (2007) by confining ourselves to the case of unit elasticity setting $\rho = 0$, which yields a Cobb-Douglas function for second-period utility.[12] For this choice, the parameter $1 - \beta$ can be interpreted as the share of housing expenditure in consumer income, and we choose $\beta = .67$.[13] As in Hurd (1989), consumers' annual time discount is taken to be 1/1.011 implying a discount factor $\gamma = 0.70 \approx (1/1.011)^{35}$. We normalize incomes setting $e^o = 1$. The sequence $\{e_t^y\}_{t\geq 0}$ is selected such that incomes exhibit a high degree of persistence and tend to stay in a given state for several periods. For this purpose, we generate $\{e_t^y\}_{t\geq 0}$ as a random draw from a symmetric two-state Markov process with values in $\mathcal{E}^y = \{e^y_{\min} = 1.425, e^y_{\max} = 1.5\}$ and a time-invariant transition probability $\pi = 0.2$.[14] Finally, our choice for $\kappa = 1/3$ implies that housing costs make up about 10% of consumers' lifetime income. The initial values are set to $p_0 = b_0 = 1$.

interval $[e^y_{\min}, e^y_{\max}]$ as long as the dynamics alternates between the expansive regime $\{(q, e) \in \bar{\mathbb{Q}} \times \mathcal{E}^y \,|\, \mathcal{R}(q; e) > 1\}$ and the contractive regime $\{(q, e) \in \bar{\mathbb{Q}} \times \mathcal{E}^y \,|\, \mathcal{R}(q; e) < 1\}$.

[12] Bajari et al. (2010) devise an elasticity of substitution between housing and second-period consumption slightly larger than unity (about 1.3) corresponding to $\rho = 0.24$.

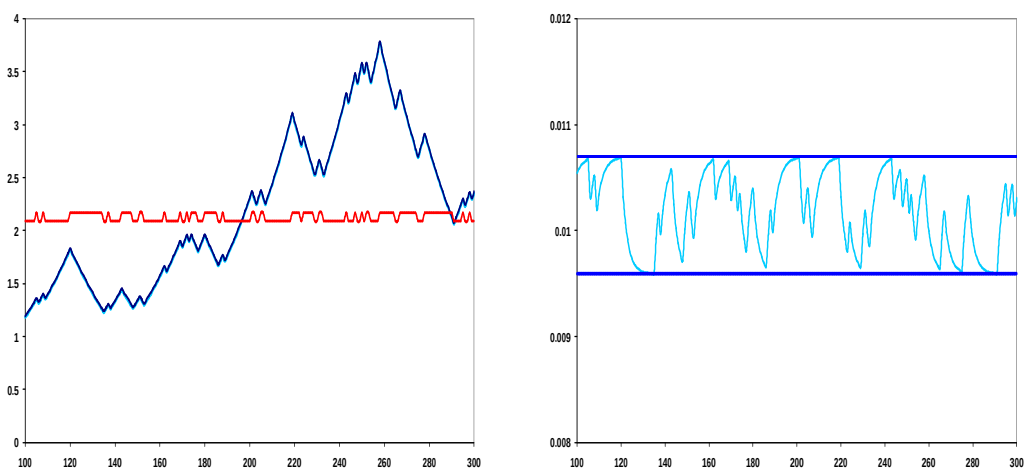
[13] Bajari et al. (2010) choose a value of $\beta \approx .77$. As housing is confined to the second period in our model, we choose a smaller value.

[14] The fact that incomes are higher in the first than in the second period seems broadly consistent with empirical evidence, cf. Table 3 in Bajari et al. (2010). This is also consistent if we were to replace the pure exchange setting by a production economy where the young earn labor income and the old capital income. Empirical evidence then suggests that the former is about twice as large as the latter.

Under this parametrization, the hypotheses of Lemma 2 hold for all $e^y \in \mathcal{E}^y$. Thus, for each fixed income stream $e_t^y \equiv e^y \in \mathcal{E}^y$, the dynamics (10a) converges to a unique steady state $\bar{q}(e^y) > 0$. In particular, the above parametrization implies that $R_{\min} = \mathcal{R}(\bar{q}(e^y_{\max}), e^y_{\max}) < 1 < \mathcal{R}(\bar{q}(e^y_{\min}), e^y_{\min}) = R_{\max}$ such that the necessary conditions for booms and busts of housing prices to occur are satisfied.

Simulation results

Figure 3 displays the time series of the model for $t = 300$ periods starting in $t = 100$ to capture the long run characteristics of the model. The left panel shows a time window of the housing price p_t and the credit volume b_t. To relate movements in these variables to the 'fundamentals' of the economy we also depict the aggregate net income $e_t^y + e^o - \kappa$ which represents the total resources available in period t net of housing costs. The right panel depicts $q_t = p_t - b_t$ together with the bounds $\bar{q}_{\min}$ and $\bar{q}_{\max}$ of the ergodic set $\bar{\mathbb{Q}}$ defined in (18a,b).



(a) p_t, b_t, and $e_t^y + e^o - \kappa$ (b) q_t, $\bar{q}_{\min}$, $\bar{q}_{\max}$

Figure 3: A boom-bust scenario

The figures confirm that small but persistent income changes generate large and persistent movements in the housing price and the credit volume. The co-movement of these two variables is apparent, and in fact the two variables are almost indistinguishable in Figure 3(a). By contrast, we know from our previous results that their difference $q_t = p_t - b_t$ is uniformly bounded and remains in a compact set $\bar{\mathbb{Q}} = [\bar{q}_{\min}, \bar{q}_{\max}]$. These properties can readily be inferred from Figure 3(b). Figure 3(a) confirms that both housing investment and credit volume increase (decrease) when the aggregate income is high (low), and they may exceed the aggregate income by an order of magnitude. For example, consider the situation in period $t_0 = 251$ when the housing market is in a boom phase. In this period, the young

are in the low-income state receiving $e^y_{t_0} = 1.425$ such that aggregate net income is $e^y_{t_0} + e^o - \kappa = 2.09$. They take a loan $b_{t_0} = 3.48$ at the interest rate of 2.6% to finance their housing purchase at the price $p_{t_0} = 3.49$. This corresponds to a leverage ratio (loans over discounted lifetime income $e^y_{t_0} + e^o R_t^{-1}$) of 145%. Moreover, the loan repayment $R_{t_0} b_{t_0} = 3.57$ is more than three times larger than second-period non-housing income e^o. Nevertheless, according to the mechanism of our model, the next period's housing price $p_{t_0+1} = 3.58$ allows consumers to repay their loan from the revenues of selling their houses at the end of period $t_0 + 1$. In fact, the net flow from the young to old consumers, which is equal to the fundamental price of housing, is only $q_{t_0} = p_{t_0} - b_{t_0} = 0.01$, and remains bounded by the income of the young. Conversely, when the housing market is depressed, the leveraged borrowing declines dramatically while the fundamental price moves within the tiny range as seen in Figure 3(b).

7 Conclusions

In the absence of a financial sector the only intergenerational transfer of commodities in our model is from the young to the old through the housing market. Consequently, housing values are bounded by young consumer incomes. Introducing a financial sector adds an additional channel of intergenerational trade in the form of a credit market, which mediates a commodity transfer from the old to the young. The combination of these channels permits each flow of intergenerational transfers to become arbitrarily large as long as the net flow remains bounded by consumer incomes. This structure amplifies small but persistent income changes into large movements of housing prices and credit volumes which can both grow unboundedly large while a linear combination of them remains stationary, i.e., uniformly bounded. The presence of such a cointegration relationship is a testable implication of the model.

In our model, the boom in housing prices occurs accompanied by expanding loan volumes when the interest rate is positive. Hence, the stationarity of endowments implies that bubbles emerge when the interest rate is greater than the growth rate of the economy, a feature of our model shared by Caballero et al. (2006), Arce & López-Salido (2011), Martin & Ventura (2012), and Ventura (2012). The boom comes to a halt when a higher income of the young causes the interest rate to become negative. This positive correlation between the interest rate and the credit volume occurs naturally in a model with inside money but may be at odds

with the empirical observation that the cost of refinancing is relatively low in many bubble episodes. Note, however, that the required changes in the interest rate for the economy to switch regimes can be arbitrarily small: the real interest rate only needs to change its sign. The key prediction of our model is the positive correlation between the credit volume and the housing price in boom-bust cycles.

The paper opens many avenues for future research. The model presented above was a baseline model in all dimensions, with rational expectations, no heterogeneity, no explicit financial frictions, no uncertainty, and no government or central bank, and these assumptions can potentially be weakened.

Introducing outside money in our model will permit us to investigate how monetary policies interact with the financial sector and the housing market. The assumption of perfect foresight ruled out problems of bankruptcy and default in our model (no interest rate spread or risk premium). The effects of uncertainty and a stochastic income process can potentially be investigated under a more general rational expectations concept. These extensions will affect the relationship between the interest rate and the credit volume.

Whether bubbles in our framework remain welfare neutral in the presence of capital accumulation is also an interesting question to be explored.

A Proofs

A.1 Proof of Lemma 1

Let $e = (e^y, e^o) \in \mathcal{E}$ and $q < e^y$ be arbitrary but fixed. For brevity, set $\underline{q} := \kappa - e^o$ and

$$H(q_1) := v_c(q_1 - \underline{q}, 1)\,(q_1 - \kappa) + v_h(q_1 - \underline{q}, 1), \quad q_1 > \underline{q}. \tag{A.1}$$

Since v in (2) is homogeneous of degree $1-\alpha$, Euler's theorem for homogeneous functions implies $v_c(c,1)\,c + v_h(c,1) = (1-\alpha)v(c,1)$ for all $c > 0$ permitting us to write

$$H(q_1) = (1-\alpha)\,v(q_1 - \underline{q}, 1) - v_c(q_1 - \underline{q}, 1)\,e^o, \quad q_1 > \underline{q}. \tag{A.2}$$

Since $\rho \geq 0$, the function v satisfies the Inada condition $\lim_{c\to 0} v_c(c,1) = \infty$. Thus,

$$\lim_{q_1 \to \underline{q}} H(q_1) = (1-\alpha)\,v(0,1) - e^o \lim_{q_1 \to \underline{q}} v_c(q_1 - \underline{q}, 1) = -\infty. \tag{A.3}$$

Furthermore, the restrictions $\rho \geq 0$ and $\alpha < 1$ together imply $\lim_{c\to\infty} c\, v_c(c,1) = \infty$. Using this in (A.1) yields the right limit as

$$\lim_{q_1\to\infty} H(q_1) \geq \lim_{q_1\to\infty} v_c(q_1 - \underline{q}, 1)\,(q_1 - \kappa) = \infty. \tag{A.4}$$

Existence of the desired solution thus follows from (A.3), (A.4), and continuity of H. Uniqueness is a consequence of (A.2) and the concavity of v which give the derivative

$$H'(q_1) = (1-\alpha)v_c(q_1 - \underline{q}, 1) - v_{cc}(q_1 - \underline{q}, 1)\, e^o > 0. \tag{A.5}$$

■

A.2 Proof of Lemma 2

(i) Using (12) in conjunction with (A.2), a routine calculation shows that $\lim_{q\to e^y} G(q) = \lim_{q\to\kappa-e^o} G(q) = \infty$. Thus, $G(q_{\min}) < 0$ implies that G has a fixed point in each of the intervals $(\kappa - e^o, q_{\min})$ and $(q_{\min}, e^y)$. By strict convexity and the boundary behavior of the first derivative (cf. Lemma 6), the map G is strictly decreasing on $(\kappa - e^o, q_{\min})$ and strictly increasing on $(q_{\min}, e^y)$. Thus, there can be at most one fixed point in each of the two intervals.

(ii) It is obvious from (i) that $G'(\bar{q}) < 0 < G'(\bar{\bar{q}})$. Utilizing the result from Lemma 5 and the definitions of D and H given in the proof of Lemma 2, this implies that $G'(\bar{q}) = D'(\bar{q}) - H'(\bar{q}) < 0$ and $G'(\bar{\bar{q}}) = D'(\bar{\bar{q}}) - H'(\bar{\bar{q}}) > 0$. Therefore,

$$0 < f'(\bar{q}) = \frac{D'(\bar{q})}{H'(\bar{q})} < 1 < \frac{D'(\bar{\bar{q}})}{H'(\bar{\bar{q}})} = f'(\bar{\bar{q}}) \tag{A.6}$$

which implies the local stability properties asserted. The remaining inequalities follow from this and the uniqueness of fixed points on the respective intervals. Finally, by the properties of G derived in Lemma 6, $G'(0) < 0$ implies $q_{\min} > 0$ while $G(q_{\min}) < 0 < G(0)$ ensure that $\bar{q}$ lies in the interval $(0, q_{\min})$. ■

A.3 Proof of Lemma 3

Assertion (i) follows immediately from Lemma 2(i). The result in (ii) is a consequence of local stability of $\bar{q}$ and Lemma 2(ii). Monotonicity of the sequence $\{q_t\}_{t\geq 0}$ follows from this and Lemma 5. ■

A.4 Proof of Theorem 1

(i) Lemma 3 and $q_0 \in \mathbb{Q}$ imply that $q_t \in \mathbb{Q}$ for all t and $\lim_{t\to\infty} q_t = \bar{q}$. By (1) and (9), $b_0 \geq 0$ implies $b_t \geq 0$ and, by (5) $p_t > 0$ for all t proving (i).
(ii) If $b_0 = 0$, then $b_t = 0$ and $q_t = p_t$ for all t and the claim is obvious. If $b_0 > 0$ and $\mathcal{R}(\bar{q}; e) < 1$, there exists $t_0 \geq 0$ such that $\mathcal{R}(q_t; e) < 1$ for all $t \geq t_0$ by stability of $\bar{q}$ and continuity of $\mathcal{R}$. In fact, since $q \mapsto \mathcal{R}(q; e)$ is strictly increasing (cf. Lemma 8) and $\{q_t\}_{t\geq 0}$ converges monotonically, $\mathcal{R}(q_t; e) \leq R_{t_0} := \mathcal{R}(q_{t_0}; e) < 1$ for all $t \geq t_0$. Thus, $0 \leq \lim_{t\to\infty} b_t \leq b_{t_0} \lim_{t\to\infty} R_{t_0}^{t-t_0} = 0$ and $\lim_{t\to\infty} p_t = \lim_{t\to\infty} q_t = \bar{q}$.
(iii) If $\mathcal{R}(\bar{q}; e) > 1$, the same arguments as in (ii) yield existence of $t_0 \geq 0$ such that $\mathcal{R}(q_t; e) \geq R_{t_0} := \mathcal{R}(q_{t_0}; e) > 1$ for $t \geq t_0$. Thus, $\lim_{t\to\infty} b_t \geq b_{t_0} \lim_{t\to\infty} R_{t_0}^{t-t_0} = \infty$ and $p_t = q_t + b_t > b_t$ for all t gives $\lim_{t\to\infty} p_t = \infty$. ■

A.5 Proof of Lemma 4

(i) We first show that $\bar{\mathbb{Q}}$ is self-supporting. Let $q \in \bar{\mathbb{Q}}$ be arbitrary. By Lemma 2(ii), the monotonicity properties of f and (18a–c) we have for each $e^y \in \mathcal{E}$:

$$\bar{q}_{\min} = f(\bar{q}_{\min}; e^y_{\max}) \leq f(\bar{q}_{\min}; e^y) \leq f(q; e^y) \leq f(\bar{q}_{\max}; e^y) \leq f(\bar{q}_{\max}; e^y_{\min})) = \bar{q}_{\max}. \tag{A.7}$$

Thus, $f(q; e^y) \in \bar{\mathbb{Q}}$. To prove that $\mathbb{Q}$ is self-supporting, let $q \in \mathbb{Q}$ and $e^y \in \mathcal{E}^y$ be arbitrary. The case $q \in \bar{\mathbb{Q}}$ is evident, so suppose first that $q \in (\bar{q}_{\max}, \bar{\bar{q}}_{\min})$. Then, by (18a–c), $\bar{q}(e^y) \leq \bar{q}_{\max} < q < \bar{\bar{q}}_{\min} \leq \bar{\bar{q}}(e^y)$ which implies, by Lemma 2(ii) and monotonicity of f that $\bar{q}(e^y) < f(q; e^y) < q$. Thus, $f(q; e^y) \in \mathbb{Q}$. Conversely, let $q \in (0, \bar{q}_{\min})$. By (18a–c), $0 < q < \bar{q}_{\min} \leq \bar{q}(e^y)$ which implies $q < f(q; e^y) < \bar{q}(e^y)$ by Lemma 2(ii) and monotonicity of f. Thus, $f(q; e^y) \in \mathbb{Q}$ again.
(ii) Let $q_0 \in \mathbb{Q}$ be arbitrary. Define the sequences $\{\overline{q}_t\}_{t\geq 0}$ and $\{\underline{q}_t\}_{t\geq 0}$ by setting $\overline{q}_0 = \underline{q}_0 = q_0$ and $\overline{q}_{t+1} := f(\overline{q}_t; e^y_{\min})$ and $\underline{q}_{t+1} := f(\underline{q}_t; e^y_{\max})$ for each $t \geq 0$. Then, by the monotonicity properties of f, $\underline{q}_t \leq q_t \leq \overline{q}_t$ for all $t \geq 0$ and the claim follows from $\lim_{t\to\infty} \underline{q}_t = \bar{q}(e^y_{\max}) = \bar{q}_{\min}$ and $\lim_{t\to\infty} \overline{q}_t = \bar{q}(e^y_{\min}) = \bar{q}_{\max}$. ■

A.6 Proof of Theorem 2

Lemma 4 ensures that $q_t \in \mathbb{Q}$ for all $t \geq 0$. By (1) and (9), $b_0 \geq 0$ implies $b_t \geq 0$ and, by (5), $p_t \geq q_t > 0$ for all $t \geq 0$. ■

A.7 Proof of Theorem 3

(i) Suppose $R_{\max} < 1$. Then, $\mathcal{R}(q;e) \leq R_{\max} < 1$ for all $q \in \bar{\mathbb{Q}}$ and $e \in \mathcal{E}^y$. Let $\hat{R}_{\max}$ be a number between $R_{\max}$ and 1. By continuity of $\mathcal{R}$, we can choose an open neighborhood $\hat{\mathbb{Q}} \subset \mathbb{Q}$ of $\bar{\mathbb{Q}}$ such that $\mathcal{R}(q;e) < \hat{R}_{\max}$ for all $q \in \hat{\mathbb{Q}}$ and $e \in \mathcal{E}^y$. Let $q_0 \in \mathbb{Q}$ be arbitrary. By Lemma 4(ii), there exists $t_0 > 0$ such that $q_t \in \hat{\mathbb{Q}}$ for all $t > t_0$. Hence, $R_t < \hat{R}_{\max} < 1$ for all $t > t_0$ and it follows that $0 \leq \lim_{t\to\infty} b_t \leq \lim_{t\to\infty} b_{t_0}\big(\hat{R}_{\max}\big)^{t-t_0} = 0$. This and (5) imply $\lim_{t\to\infty} |p_t - q_t| = \lim_{t\to\infty} |b_t| = 0$.
(ii) Suppose $R_{\min} > 1$. Similar to the previous part, choose a number $\hat{R}_{\min}$ between 1 and $R_{\min}$ and an open neighborhood $\hat{\mathbb{Q}} \subset \mathbb{Q}$ of $\bar{\mathbb{Q}}$ such that $\mathcal{R}(q;e) > \hat{R}_{\min} > 1$ for all $q \in \hat{\mathbb{Q}}$ and $e \in \mathcal{E}^y$. Let $q_0 \in \mathbb{Q}$ be arbitrary. By Lemma 4(ii), there exists $t_0 > 0$ such that $q_t \in \hat{\mathbb{Q}}$ for all $t > t_0$. Hence, $R_t > \hat{R}_{\min} > 1$ for all $t > t_0$ and it follows from (1) that $\lim_{t\to\infty} b_t \geq \lim_{t\to\infty} b_{t_0}\big(\hat{R}_{\min}\big)^{t-t_0} = \infty$. Since q_t remains uniformly bounded, the limit of the sequence $\{p_t\}_{t\geq 0}$ follows from (5). ■

B Technical results

B.1 Properties of the time-one map f

Lemma 5
Suppose $\rho \geq 0$ and $\alpha < 1$. Then, for each $e = (e^y, e^0) \in \mathcal{E}$ the map $f = f(\cdot;e)$ defined above is continuously differentiable with derivative $f'(q) > 0$ for all $q < e^y$.

Proof: Since $F_{q_1}(q_1, q; e) = -H'(q_1) < 0$ by (7) and (A.5) and F is continuously differentiable, so is the implicit function f by the Implicit Function Theorem. The partial derivative of (7) with respect to q computes

$$F_q(q_1, q; e) = u'(e^y - q) - qu''(e^y - q) = (e^y - q)^{-\alpha}\frac{e^y - (1-\alpha)q}{e^y - q} > 0. \tag{A.8}$$

By the implicit function theorem $f'(q) = -\frac{F_q(q_1,q;e)}{F_{q_1}(q_1,q;e)} > 0$ where $q_1 = f(q;e)$. ■

B.2 Properties of the fixed-point map G

Lemma 6
Suppose $\rho \geq 0$ and $\alpha < 1$. Then, for each $e = (e^y, e^0) \in \mathcal{E}$ satisfying $e^y + e^o > \kappa$ the map $G = G(\cdot;e)$ defined in (12) is a strictly convex function and the derivative satisfies the boundary behavior $\lim_{q\to e^y} G'(q) = -\lim_{q\to\kappa-e^o} G'(q) = \infty$.

Proof: By (12), the function G can be written as $G(q) = D(q) - H(q)$ with H being defined as in (A.2) and $D(q) := qu'(e^y - q) = q(e^y - q)^{-\alpha}, \quad q < e^y$.
Consider first the behavior of the function D whose derivatives satisfy

$$D'(q) = \frac{e^y - (1-\alpha)q}{(e^y - q)^{1+\alpha}} > (1-\alpha)\frac{e^y - q}{(e^y - q)^{1+\alpha}} = \frac{1-\alpha}{(e^y - q)^{\alpha}} > 0 \tag{A.9}$$

$$D''(q) = \frac{\alpha}{(e^y - q)^{2+\alpha}}\Big(2e^y - (1-\alpha)q\Big) > \frac{\alpha(1-\alpha)}{(e^y - q)^{2+\alpha}}\Big(e^y - q\Big) > 0. \tag{A.10}$$

The second inequality shows that D is a strictly convex function while the first one implies that D is strictly increasing with boundary behavior $\lim_{q\to e^y} D'(q) = \infty$. As shown in the proof of Lemma 1, the derivative of H is given by (A.5) and, therefore, satisfies $H'(q) > 0$ and $\lim_{q\to\kappa-e^o} H'(q) \geq (1-\alpha)\lim_{q\to\kappa-e^o} v_c(e^o - \kappa + q, 1) = \infty$. We claim that H' is a strictly decreasing function implying that $-H$ is strictly convex. The first term in (A.5) is strictly decreasing by strict concavity of v. It therefore suffices to show that $c \mapsto -v_{cc}(c,1)$ is decreasing as well. Defining g as in (2), direct calculations reveal that the second derivative of v can be written as

$$\begin{aligned} -v_{cc}(c,1) &= \frac{v_c(c,1)}{c}\left[1 - \rho - (1-\rho-\alpha)\frac{\beta c^\rho}{g(c,1)^\rho}\right] \\ &= v_c(c,1)\left[\frac{\alpha}{c^{1-\rho}}\frac{\beta}{g(c,1)^\rho} + \frac{1-\rho}{c}\cdot\frac{(1-\beta)}{g(c,1)^\rho}\right]. \end{aligned} \tag{A.11}$$

Recalling that $1 - \rho \geq 0$, all three terms in (A.11) are positive and strictly decreasing functions of c which implies that $c \mapsto -v_{cc}(c,1)$ is decreasing as claimed. Thus, $-H$ is a strictly convex function as claimed and G being the sum of two (strictly) convex functions is also strictly convex. The boundary behavior of G' follows directly from the limits computed above and the monotonicity properties of D and $-H$. ■

Lemma 7
Let the hypotheses of Lemma 3 be satisfied for each $e^y \in \mathcal{E}^y$. Then,

(i) For each $q > 0$ the map $e^y \mapsto f(q; e^y)$ is continuously differentiable (on the interior of $\mathcal{E}^y$) and strictly decreasing.

(ii) The fixed point maps $e^y \mapsto \bar{q}(e^y)$ and $e^y \mapsto \bar{\bar{q}}(e^y)$ are both continuously differentiable. Moreover, $\bar{q}(\cdot)$ is strictly decreasing while $\bar{\bar{q}}(\cdot)$ is strictly increasing.

Proof: (i) The proof of Lemma 5 revealed that $F_{q_1}(q_1, q; e) < 0$ with F defined in (7). Since $F_{e^y}(q_1, q; e) = q\,u''(e^y - q) < 0$, $q > 0$, the claim follows from the

Implicit Function Theorem.
(ii) Recall that fixed points are solutions to $G(q;e) = F(q,q;e) = 0$. By (i), $G_{e^y}(q;e) = F_{e^y}(q,q;e) < 0$. As $\lim_{q\to\kappa-e^o} G(q;e) = \lim_{q\to e^y} G(q;e) = \infty$ implies $G_q(\bar{q};e) < 0 < G_q(\bar{\bar{q}};e)$, the claim follows again from the Implicit Function Theorem. ■

Lemma 8
Let the hypotheses of Lemma 3 be satisfied for each $e^y \in \mathcal{E}^y$. Then, the map $\mathcal{R}$ defined in (9) is continuously differentiable with partial derivatives $\mathcal{R}_{e^y}(q;e^y) < 0 < \mathcal{R}_q(q;e^y)$ for all $e^y > 0$ and $q < e^y$.

Proof: The claim follows directly by taking the partial derivatives of (9) and using Lemmata 5 and 7(i). ■

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