Risk, Factor Substitution, and Asset Market Integration *

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Abstract

This paper investigates the effects of asset market integration on the inequality of nations. The standard OLG economy with a neoclassical technology is extended to include a random exogenous technology (Lucas' tree) whose ownership can be traded in the form of paper assets. A world with two such economies is considered which differ only in their initial capital stocks. If asset demand is sufficiently inelastic, the poor country may spend more income in purchasing paper assets than the rich country. In this case, an integration of asset markets alone can prevent capital stocks in the two countries from converging to the same level in the long run, i. e. symmetry breaking occurs. However, if capital and labor are highly substitutable, capital converges to a symmetric steady state.

Keywords: asset market, capital accumulation, rational expectations, symmetry breaking

JEL classification: F43, G11, O11

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1 Introduction

Based on historical and empirical studies, there seems to be a consensus among economists that the first era of modern globalization which took place between the late 19th and the early 20th century has promoted economic growth around the world. Now we have seen an acceleration of globalization again in the last decades. However, economists, starting with Lucas (1990) who pointed out the lack of net capital transfer from the rich to the poor countries, have been disputing the positive effects of globalization this time. Bianchi (1997), Jones (1997) and Quah (1997)) have documented the evidence that the shape of the inter-country income distribution was transformed from a unimodal one in the early 1960s to a bimodal one in 1990s. Durlauf & Johnson (1995) confirm a positive relationship between the starting level of per capita output and subsequent growth rates, implying divergence of income levels during the same period.

It seems an intriguing question whether this trend in inequality of world income distribution is related in fact to globalization. There are many empirical studies which examine the relationship between international financial integration and economic growth (for a recent literature survey see Edison, Klein, Ricci & Slok 2004). Above all, these studies seem not to provide robust evidence for positive effects of international financial integration on economic growth for the last decades. Schularick & Steger (2006) compares the recent globalization era with the first globalization era of late 19th and early 20th century to search for an explanation. They conclude that the first era of financial globalization was accompanied by massive net capital flows from rich to poor economies ("development finance") while today's globalization is marked by high gross flows ("diversification finance") and limited net capital transfers (See also Obstfeld & Taylor (2004) and Schularick (2006)). Their study suggests that while financial integration was highly correlated with domestic capital investment during the first era of globalization this is no longer the case this time. This empirical evidence motivates our theoretical modelling to characterize economic scenarios between countries which may account for the distortion of the relationship between financial market integration and domestic capital investment.

The main objective of the present paper is to investigate the existence of a possible endogenous mechanism which prevents capital stocks of two economies to converge to the same level after their asset markets have been integrated. Krugman & Venables (1995) and Matsuyama (1996) introduced the concept of *symmetry breaking* in eco-

nomic analysis which they define as the appearance of stable asymmetric developments between identical economies after these have been linked by international trade. *Symmetry breaking* occurs whenever, in the long run, two identical economies with identical (symmetric) stable steady states under autarky would exhibit asymmetries of capital and income development caused by the presence of international trading. Thus, formally, the symmetric steady state loses its stability and stable asymmetric steady states come into existence, dividing the world economy into poor and rich nations. Boyd & Smith (1997) and Matsuyama (2004) were the first to show that the presence of imperfections in the international financial market can cause symmetry breaking.

Instead of asking whether symmetry breaking arises due to imperfections in financial markets, the present paper demonstrates the occurrence of symmetry breaking when there are no imperfections in the above sense, but when there are random production shocks which are absorbed through an asset market. In this case, internationally integrated asset markets induce a standard portfolio problem for investors of each country leading to international diversification under uncertainty. As a consequence, risk adjusted returns in the countries matter for portfolio choices and capital accumulation. Without risk a standard no arbitrage argument implies that the equilibrium return from holding assets in two countries would have to be the same implying identical capital stocks and incomes in non trivial interior steady states. Thus, the presence of risk is a necessary condition for an unequalizing mechanism causing the symmetry to break.

The two country model is based on Kikuchi (2006) and Böhm & Vachadze (2006) who identify structural characteristics of economies which cause symmetry breaking. While Kikuchi (2006) exploits the non-concavity of the wage function induced by a quadratic production function, Böhm & Vachadze's 2006 results hinge on a non-monotonicity induced by the CARA utility function. The present paper uses a CES production function and demonstrates that the elasticity of substitution plays a decisive role in symmetry breaking. The paper is organized as follows. Section 2 introduces the model of the closed economy with an asset market and it identifies those conditions when the economy has a unique globally stable steady state. Section 3 then presents the two country model to analyze the implication of asset market integration. Section 4 concludes.

2 Economies in Autarky

The two economies considered are identical in all structural aspects. They are composed of a consumption sector with overlapping generations of two period lived consumers and a production sector. In both economies a single commodity is produced from labor and capital using the same production function. Investment in capital is irreversible, so that capital is embodied in the firm and cannot be consumed. Capital depreciates at the same constant rate in both countries.

In addition to the neoclassical firm, there exists an exogenous random production process[1], whose proceeds are allocated as dividends to the owners of the process. In each period the ownership of the exogenous production process is traded as a paper asset in a competitive market at a uniform market price. Aggregate supply of the paper asset is normalized to unity. Owners of the assets receive the entire random output. There is neither strategic behavior nor any information asymmetry.

The evolution of the two economies will be investigated under rational expectations equilibrium on all markets under stochastically the same random technology. This model combines the basic features of a simple random OLG growth model with that of the dynamic capital asset pricing model (as in Böhm & Chiarella 2005). This implies that asset prices are determined endogenously by the interaction of heterogeneous consumers. At the same time, income as well as capital accumulation are endogenous and interdependent leading to feed backs between the asset pricing process and capital accumulation. Böhm, Kikuchi & Vachadze (2005, 2006) use the same model of a closed economy but with different functional specifications to investigate the role of parameters on welfare and on the dynamics.

Given the complexity of the economic structure it is necessary to assume specific functional forms for preferences, for the production function, and for the random technology to obtain a workable model which allows to carry out a comparative analysis under the two different scenarios (closed vs. open economy). In addition, numerical solutions of the rational expectations dynamics will be used throughout for the comparative analysis.

[1]The proceeds of such a process can be interpreted in the usual way, for example as crops to land or to a so called "Lucas' tree".

2.1 The Production Sector

Aggregate output by the neoclassical firm is produced using a CES production function which is assumed to take the intensive form $y = f(k)$ given in the following assumption.

Assumption 1 *The production technology in described by a CES production function*

$$f(k) := \begin{cases} A(1-\alpha+\alpha k^{\rho})^{\frac{1}{\rho}} & \text{if} \quad \rho \neq 0 \\ Ak^{\alpha} & \text{if} \quad \rho = 0 \end{cases},$$

with $A > 0$, the capital share parameter $\alpha \in (0,1)$, and substitution parameter $\rho \in (-\infty, 1)$, where $\sigma = 1/(1-\rho)$ is the elasticity of factor substitution.

Total output in each period is distributed as wages and capital returns according to the marginal product rule

$$w(k) := f(k) - kf'(k) \quad \text{and} \quad r(k) := f'(k).$$

Once invested, capital is embodied in the firm and it depreciates at a constant rate $\delta \in (0,1]$. Since all capital remains in the firm, next period's capital stock in the economy is determined by non-depreciated capital carried from the previous period plus new investment. Therefore, capital investors receive only the return $r(k)$ per unit.

The exogenous random technology produces $\varepsilon \geq 0$ units of consumption goods each period for which the following assumption is made.

Assumption 2 *The random output ε is an i.i.d. distributed random variable taking the value $d > 0$ with probability $q \in (0,1)$ and 0 with probability $1-q$.*

As a consequence the expected value and the variance of the random output are $\mathbb{E}\varepsilon := qd$ and $\mathbb{V}\varepsilon := q(1-q)d^2$ respectively. The owners of the process (the "Lucas' tree") in any period are entitled to the entire production in the following period. Ownership of the random technology is defined by the acquisition of a paper asset, which young agents may purchase on the market to become shareholders. They can resell their assets when they are old. In other words, ownership is passed on between generations through the re-trading of paper assets.

2.2 The Consumption Sector

The consumption sector consists of overlapping generations of consumers who live for two consecutive periods. For simplicity it is assumed that there is no population growth. Therefore, in any given period, there are always two generations alive referred to as young and old. A typical young consumer of generation t supplies one unit of labor inelastically to the labor market in the first period of his lifetime and receives labor income w. His lifetime utility depends on old age consumption only. He can transfer his wage income to the next period either by investing in capital or by purchasing a retradable paper asset. The young consumer faces the budget constraint

$$b + xp \leq w,$$

where b denotes the amount of investment in capital and x is the number of assets purchased at price p. It is assumed that the young agent can neither take credit in the capital market nor can he take a short position in the asset market in order to finance investment, implying $b \geq 0$ and $x \geq 0$. In the second period of his life time, the old agent receives the rate of return r_1 on his capital investment. In addition, he receives xp_1 units of consumption goods from selling his asset holdings *plus* $\varepsilon_1 x$ units of consumption goods as random dividend payment. The old agent does not leave bequests to future generations and consumes his entire wealth. Therefore, final consumption c_1 is restricted by final wealth as

$$c_1 \leq br_1 + x(p_1 + \varepsilon_1).$$

The young agent's objective is to maximize expected utility of second period consumption.

Assumption 3 *Consumer preferences over second period consumption are described by the CARA utility function*

$$u(c) = 1 - e^{-ac},$$

with risk aversion parameter $a > 0$.

When making the portfolio decision, next period's return on capital and the asset price (r_1, p_1) are unknown for the young consumer. It is assumed that he makes point forecasts (r^e, p^e) for both quantities. The subsequent analysis discusses exclusively the case when consumers form rational expectations, i. e. expectations about these quantities

always coincide with actual realizations. It is assumed that consumers also know the distribution of dividends and that they fully utilize this information when making their portfolio choice. Thus, for given values of $(w, r^e, p^e, p) \geq 0$, the consumer's asset demand $\varphi(w, r^e, p^e, p)$ is defined as

$$\varphi(w, r^e, p^e, p) := \arg\max_{x \geq 0} \left\{ \mathbb{E}[u(r_1 w + x(p_1 + \varepsilon_1 - p r_1))] \,|\, p\, x \leq w \right\},$$

where $\mathbb{E}[\cdot]$ denotes the expectation operator. Due to the functional specifications one obtains the following explicit form of asset demand.

Proposition 1 *If Assumptions 2 and 3 are satisfied, then, for given values of (w, r^e, p^e, p), young consumer's demand of the asset is given by*

$$\varphi(w, r^e, p^e, p) = \begin{cases} 0 & \text{if} \quad \dfrac{p^e + \mathbb{E}\varepsilon}{r^e} \leq p \\ \varphi_m(r^e, p^e, p) & \text{if} \quad \dfrac{p^e + \chi(w, r^e, p^e)}{r^e} < p < \dfrac{p^e + \mathbb{E}\varepsilon}{r^e} \\ \dfrac{w}{p} & \text{if} \quad p \leq \dfrac{p^e + \chi(w, r^e, p^e)}{r^e} \end{cases}$$

where

$$\varphi_m(r^e, p^e, p) := \frac{1}{ad} \ln\left(\frac{q}{1-q} \frac{d - (pr^e - p^e)}{pr^e - p^e} \right), \tag{1}$$

and $\chi(w, r^e, p^e)$ is the unique positive solution for p, of the equation

$$p\, \varphi_m(r^e, p^e, p) = w.$$

Proof: see appendix. □

Equation (1) indicates that the demand for assets is independent of income whenever the consumer demands a mixed portfolio, while income plays a role only when the asset price is sufficiently low so that the entire income is used for purchasing assets. In this case, however, investment in capital is zero.

2.3 Stationary Rational Expectations Equilibrium

Given the simple demographic structure of consumers all assets sold by old consumer are purchased by young investor. Since the total number of available assets in the economy

is constant and normalized to unity, it follows that the asset market clearing price p should satisfy the equation

$$\varphi(w(k), r^e, p^e, p) = 1$$

for any given values of $(k, r^e, p^e) > 0$. Then, the demand function implies that the asset market clearing price is given by

$$p = S(k, r^e, p^e) := \min\left\{w(k), \frac{p^e + \pi}{r^e}\right\} \quad \text{where} \quad \pi := \frac{dq}{q + (1-q)e^{ad}} \ . \tag{2}$$

Equation (2) defines the unique time invariant and deterministic function determining the equilibrium asset price in any period for any wage income and any expectations for the future asset price and future return on capital. Moreover, the sequential structure of the economy implies that any two successive periods are linked only through the asset price, expectations, and capital accumulation. Therefore, in spite of the presence of the random dividends for assets, the dynamics of capital accumulation, wage income, the return on capital, and of the asset price are described by a *deterministic* Markov process. As a consequence, the analysis of Rational Expectations Equilibria becomes particularly simple.

Definition 1 *A Stationary Rational Expectations Equilibrium (SREE) is a pair (k, p), such that*

1. *given $k \in \mathbb{R}_+$, the price $p \in \mathbb{R}_+$ clears the asset market, i.e.*

$$p = S(k, r(k), p);$$

2. *given $p \in \mathbb{R}_+$, capital k is a fixed point of the capital accumulation equation*

$$k = A(k, p) := (1-\delta)k + w(k) - x(k, p)p.$$

2.3.1 Existence and Multiplicity of Rational Expectations Equilibria

Based on the above definition, the equilibrium pair (k, p) must satisfy the two equations

$$p = \frac{\pi}{r(k) - 1} \quad \text{and} \quad k = (1-\delta)k + w(k) - p,$$

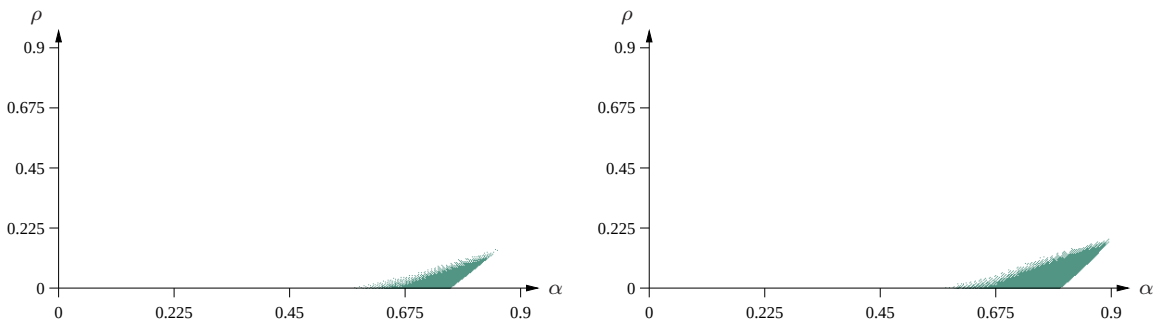
which implies that the capital must satisfy

$$w(k) - \delta k = \frac{\pi}{r(k) - 1}. \tag{3}$$

Let us consider the three cases

- $\rho < 0$, production factors are poorly substitutable,
- $\rho = 0$, production factor substitution is unity (Cobb-Douglas case),
- $\rho > 0$, production factors are highly substitutable

separately. A detailed analysis of equation (3) reveals that the substitution parameter ρ interacts in a complex non linear fashion with the other parameters, in particular with the capital share α. When $\rho < 0$, the economy can have either only one, or two, or three non-negative solutions. However, whenever the economy has a unique solution it is always zero. When $0 < \rho < 1$, the economy can again have either only one, or two, or three positive solutions, but differently from the above case, the unique equilibrium in the economy is always interior. Since the analysis here is directed toward the question of symmetry breaking, a full comparison of the situations of autarky versus international asset market integration is most transparent and convincing when equilibria under autarky are unique *and* interior. Therefore, the case $0 < \rho < 1$ will be analyzed only. In this situation the restrictions on the other parameters can be determined in such a way that the closed economy always has a unique and interior equilibrium[2]. Figure 1 displays the regions (white) in the (ρ, α) parameter space for two different values of the scale parameter A, where the closed economy has a unique interior equilibrium. Multiplicity occurs in the colored/dark region. The figure indicates that there is a large open set of parameters for which the existence of unique stationary rational expectations equilibria can be guaranteed which are interior.



(a) $A = 0.8$ (b) $A = 1.1$

Figure 1: Regions of multiple equilibria; $\delta = 0.50$, $d = 4.00$, $q = 0.90$, $a = 1.40$

[2]The case with $\rho = 0$ has been analyzed in Böhm & Vachadze (2006).

2.3.2 Stability of Stationary Rational Expectations Equilibrium

The local dynamics under rational expectations, using the so called MSV solution, can be obtained using the implicit function theorem[3]. The following proposition states conditions under which a unique interior equilibrium of the closed economy is globally stable.

Proposition 2 *If Assumptions 1, 2, and 3 are satisfied and $\rho \in (0, 1]$, then the unique positive Rational Expectations Equilibrium is globally stable.*

Proof: see appendix. □

The proposition implies that any two identical closed economies converge to the same positive steady state independent of initial conditions. Therefore, under such circumstances in autarky countries converge and consequently have identical income, return on capital, asset price, and welfare in the long run.

3 A Two Country Model

Consider now a world consisting of two economies of the type described in Section 2, which will be denoted by h for home country and by f for foreign country. Production factors, capital and labor, are assumed to be immobile across countries. However, assume now that the market for the paper asset operates internationally paying stochastically the same dividend in each country[4]. Therefore, for young consumers in each country the international asset market is completely homogeneous and they invest either in domestic capital or in the international asset market.

3.1 Temporary Equilibrium in the Asset Market

The overlapping demographic structure of the model implies again that all retradable assets sold by old are bought by the young. The number of all available assets in the international asset market is assumed to be constant and normalized to two. It is straightforward to show that in a stationary rational expectations equilibrium, asset demand in each country must be $\varphi_m(r^e, p^e, p)$. Otherwise equilibrium asset demand

[3]see for example Böhm, Kikuchi & Vachadze (2006)

[4]See Kikuchi (2006) for the interpretation where this structure was first introduced.

implies zero capital in equilibrium. This is in contradiction with rational expectations, since the return on capital tends to infinity as capital goes to zero, while the return on assets is bounded from above. Therefore, for given values of $(k_h, k_f, r_h^e, r_f^e, p^e) > 0$ the market clearing price in a stationary equilibrium must be determined by the equation

$$\varphi_m(r_h^e, p^e, p) + \varphi_m(r_f^e, p^e, p) = 2. \tag{4}$$

Let $p = S(r_h^e, r_f^e, p^e)$ denote the unique positive solution of equation (4). In contrast to the one country model, it is not possible to obtain the price law analytically. However, it can be approximated numerically to determine the symmetric steady state and its stability properties.

3.2 Stationary Rational Expectations Equilibria

Stationary rational expectations equilibria are defined in a straightforward manner.

Definition 2 *A Stationary Rational Expectations Equilibrium in the two country economy is a triple (k_h, k_f, p), such that*

- *given $(k_h, k_f) \in \mathbb{R}_+^2$, the price $p \in \mathbb{R}_+$ clears the asset market, i.e.*

  $$x(k_h, p) + x(k_f, p) = 2,$$

  where $x(k_i, p) := \varphi(w(k_i), r(k_i), p, p)$, $i = h, f$.

- *given $p \in \mathbb{R}_+$, the pair of capital stocks (k_h, k_f) are fixed points of the corresponding capital accumulation equations in each country, i.e.*

  $$k_h = A(k_h, p) \quad \textit{and} \quad k_f = A(k_f, p),$$

  where $A(k_i, p) := (1 - \delta)k_i + w(k_i) - x(k_i, p)p$, $i = h, f$.

The interesting issue to address is whether there exist equilibria in which both countries hold positive assets and positive but different levels of capital, in other words, whether there exist *interior* asymmetric steady states. Suppose that one country holds x assets in equilibrium. Then, asset demand implies that

$$xp = \frac{\pi(x)}{r(k) - 1} \quad \text{and} \quad xp = w(k) - \delta k. \tag{5}$$

where the risk adjusted return $\pi(x)$ of holding x units of assets is defined by

$$\pi(x) = \frac{xdq}{q + (1-q)e^{xad}}. \tag{6}$$

The first equation of (5) is the asset market equilibrium condition and the second one is the capital accumulation equilibrium condition. Eliminating p from equation (5), it follows that for any given value of x, the associated stationary capital stock must satisfy

$$w(k) - \delta k = \frac{\pi(x)}{r(k) - 1}. \tag{7}$$

Proposition 2 implies, that for any given value of x, equation (7) has a unique, non-zero solution. Let $k = \phi(x)$ denote the solution of equation (7) for a given value of $0 \leq x \leq 2$. Then, for any distribution of equilibrium asset holdings $(x, 2-x)$, the steady state distribution of capital, (k_h, k_f), should satisfy

$$k_h = \phi(x) \quad \text{and} \quad k_f = \phi(2-x). \tag{8}$$

Given the distribution of capital (k_h, k_f), there are corresponding prices (p_h, p_f)

$$p_h = \frac{\pi(x)}{(r(\phi(x)) - 1)} \quad \text{and} \quad p_f = \frac{\pi(2-x)}{(r(\phi(2-x)) - 1)},$$

which would clear the asset market in a closed economy. The market clearing price after asset market integration requires

$$p_h = p_f. \tag{9}$$

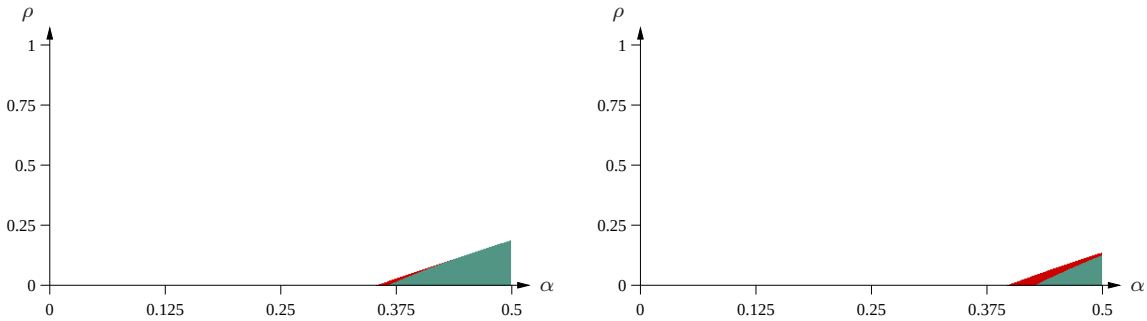
3.2.1 Existence and Multiplicity of Rational Expectations Equilibria

In order to analyze the role of the parameters (ρ, α) for the occurrence of multiple asymmetric steady states, consider a fixed vector of values of the remaining parameters as listed in Table 1 (denoted as the standard parameter set).

A	α	δ	d	q	a
1.1	0.45	0.50	0.50	0.90	1.40

Table 1: Standard parameter set

For the subsequent analysis the values of (ρ, α) will always be restricted in such a way as to guarantee the uniqueness of equilibrium in autarky. Then, figure 2 displays situations



(a) $A = 0.8$ (b) $A = 1.1$

Figure 2: Multiple Stationary Equilibria in the World Economy

with multiple asymmetric equilibria for two values of the parameter A. The green region corresponds to two asymmetric steady states, the red region to four asymmetric steady states, while the white region corresponds to no asymmetric steady states. The enlargement in figure 3 shows that the three possible cases occur by traversing in figure 2 vertically at $\alpha = 0.45$. As ρ takes on the values $\rho = 0.01, 0.0625, 0.1$, the number of asymmetric steady states changes from two to four and then to zero.

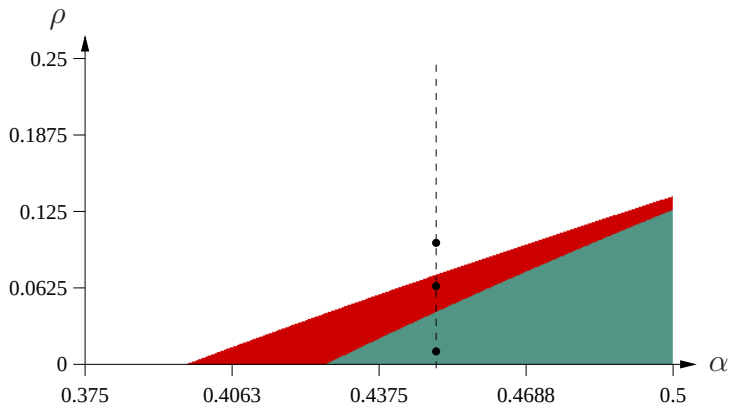
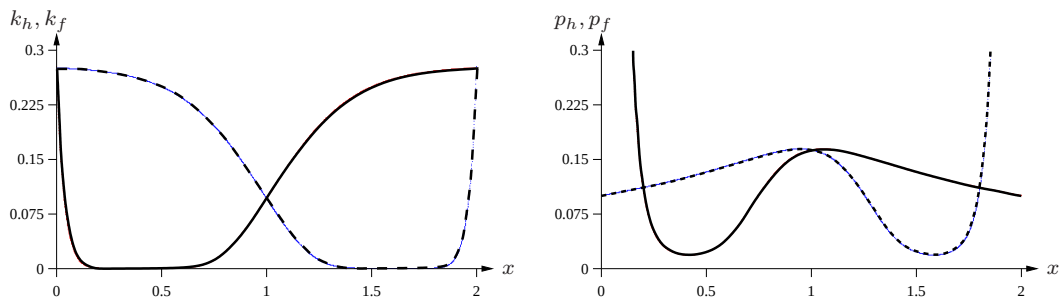


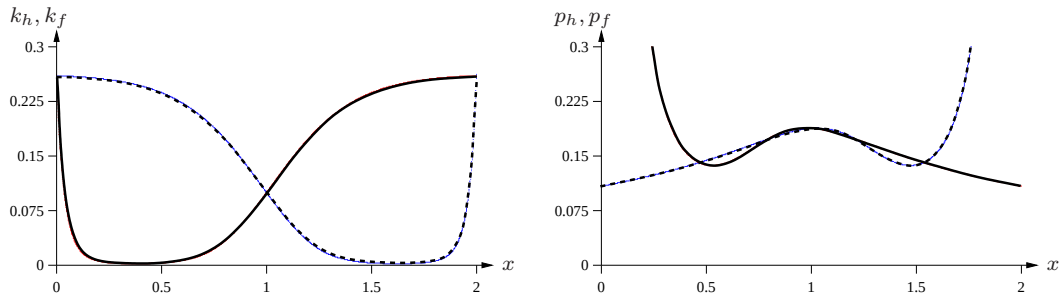
Figure 3: Three configurations of ρ

Figure 4 (a) displays the stationary pair (k^h, k^f) which satisfies equation (8) while figure 4 (b) shows the corresponding stationary asset prices (p_h, p_f) in each economy. The intersection of the two curves in figure 4 (b) is the market clearing price after asset market integration. As shown there always exists a unique symmetric steady state, which coexists with two asymmetric steady states for $\rho = 0.01$ in this case. Figures 5 (a) and (b) show numerical examples when $\rho = 0.065$. In this case four asymmetric steady states coexist with a symmetric one.



(a) Equilibrium k_h, k_f (b) Equilibrium p_h, p_f

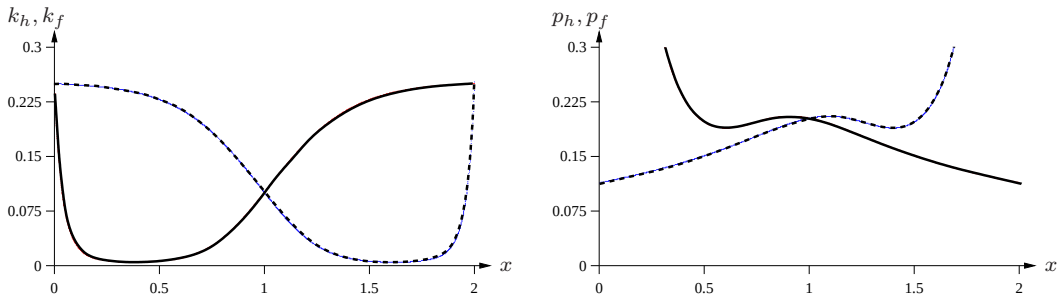
Figure 4: Steady States: $\rho = 0.01$



(a) Equilibrium k_h, k_f (b) Equilibrium p_h, p_f

Figure 5: Steady States: $\rho = 0.065$

Figures 6 (a) and (b) show the results for $\rho = 0.1$. One observes that the asymmetric steady states, which exist for $\rho = 0.065$, disappear. Figures 4, 5, and 6 together reveal that the elasticity of substitution has a decisive effect on the existence of asymmetric steady states. In particular, as factor substitutability increases the asymmetric steady states disappear. The main reason for this result is the functional form of the risk adjusted asset return $\pi(x)$ given in equation (6). One finds that $\pi(0) = 0$ and that $\pi(x)$ is first increasing and then decreasing. This non monotonicity implies that a country with a high asset holding can have a low risk adjusted return in equilibrium. In equilibrium the risk adjusted asset return is equalized to the return on capital in each country. For an asymmetric equilibrium to exist, a rich country which has a low return on capital and a low risk adjusted asset return must coexist with a poor country which has a high return on capital and a high risk adjusted asset return. If the function $\pi(x)$ was increasing throughout, then a high asset holding in equilibrium would imply a high



(a) Equilibrium k_h, k_f (b) Equilibriump_h,p_f

Figure 6: Steady States: $\rho = 0.1$

risk adjusted asset return. This would require a low capital stock in equilibrium. This contradicts the fact that the asset holding is increasing in capital stock. Therefore, the non-monotonicity of the function $\pi(x)$ is the main reason for the existence of asymmetric steady states.

3.2.2 Stability of Symmetric Rational Expectations Equilibria

To analyze the local dynamics of the economy under rational expectations, the standard MSV solution will be calculated which involves the numerical solution of a standard functional equation typical for such models.[5] To show that symmetry breaking occurs requires to demonstrate the instability of the symmetric steady state. This will be done by calculating the eigenvalues of the associated Jacobian matrix at the interior symmetric steady state. These turn out to be real and the larger one crosses unity for a wide range of parameter values. The regions of parameters where symmetry breaking occurs will be displayed.

The dynamic evolution of capital, expectations for returns on capital and for the asset price, and the actual asset price of the two country model $(k_h, k_f, r_h^e, r_f^e, p^e, p)$ is implicitly

[5]See for example, Böhm, Kikuchi & Vachadze (2006) and Wenzelburger (2006a) for a solution of the MSV predictor in particular models, and Böhm & Wenzelburger (2004) and Wenzelburger (2006b) for a general discussion.

given by

$$
\begin{aligned}
p &= S(r_h^e, r_f^e, p^e) \\
k_{h1} &= (1-\delta)k_h + w(k_h) - \varphi_m(r_h^e, p^e, p)p \\
k_{f1} &= (1-\delta)k_f + w(k_f) - \varphi_m(r_f^e, p^e, p)p,
\end{aligned}
\tag{10}
$$

where the first equation is the asset price law defined by equation (4), while the other two are the capital accumulation laws of the two countries. For the MSV solution, predictions and realizations must coincide while the asset price must be a function of the levels of capital in the two countries alone.

Suppose there are two functions $G : \mathbb{R}_+^2 \to \mathbb{R}_+$ and $P : \mathbb{R}_+^2 \to \mathbb{R}_+$ which satisfy

$$
\begin{aligned}
P(x,y) &= S(r(G(x,y)), r(G(y,x)), P(G(x,y), G(y,x))) \\
G(x,y) &= (1-\delta)x + w(x) - \varphi_m\left(r(G(x,y)), P(G(x,y), G(y,x)), P(x,y)\right) P(x,y).
\end{aligned}
$$

With this notation, next period's market clearing asset price can be expressed as $P(k_h, k_f)$, while the home and foreign capital stocks can be expressed as $k_{h1} = G(k_h, k_f)$, $k_{f1} = G(k_f, k_h)$. The function G is symmetric with respect to its arguments. Therefore, one obtains a time one map $H : \mathbb{R}_+^2 \to \mathbb{R}_+^2$ of the economy given by

$$
\begin{aligned}
x_1 &= G(x,y) \\
y_1 &= G(y,x),
\end{aligned}
\tag{11}
$$

with interior symmetric steady state $x > 0$ solving $x = G(x,x)$.

Let P_1, P_2, G_1 and G_2 denote the partial derivatives of the functions $P(.,.)$ and $G(.,.)$ with respect to the first and second argument. Then, one has the following result.

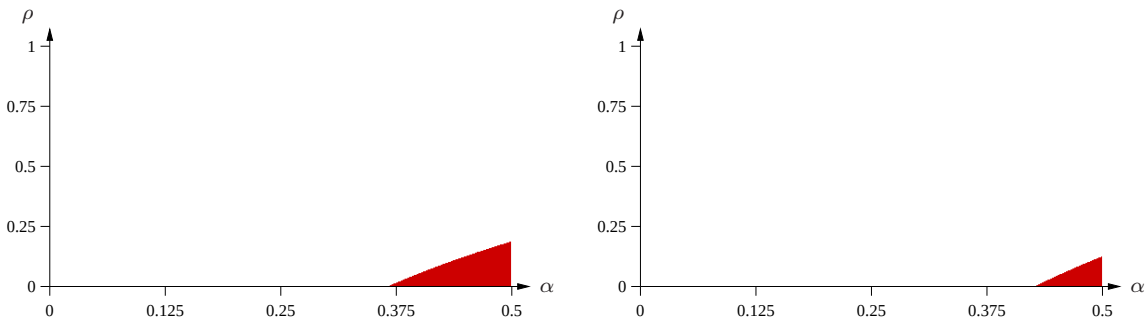
Proposition 3 *In a symmetric steady state $x = y > 0$, the eigenvalues of the Jacobian matrix of (11) are $\lambda_1 = G_1 - G_2$ and $\lambda_2 = G_1 + G_2$, where G_1 and G_2 solve*

$$
\begin{aligned}
P_1 &= -\frac{pr'(G_1+G_1)}{2r-(G_1+G_2)} = P_2 \\
G_1 &= 1-\delta+w' + \eta\left(p^2 r' G_1 - pP_1(G_1+G_2) + rpP_1\right) - P_1 \\
G_2 &= \eta\left(p^2 r' G_2 - pP_1(G_1+G_2) + rpP_1\right) - P_1.
\end{aligned}
\tag{12}
$$

p and r are the values of the asset price and the interest rate, r', and w' are derivatives of the interest rate and the wage functions respectively, and $\eta := \frac{1}{a\pi(d-\pi)}$. All functions are evaluated at the symmetric steady state.

Proof: see appendix. □

Proposition 3 can be used for the numerical analysis of the critical values in parameter space where the symmetric equilibrium looses its stability. In figure 7 the red region displays the values of the parameters (ρ, α) where the symmetric equilibrium is unstable. One observes that the red region in figure 7 corresponds exactly to the green region in figure 2. This means that 1) the unique symmetric steady state is stable, 2) when there exist two asymmetric steady states the symmetric steady state is unstable, and 3) when there exist four asymmetric steady states the symmetric steady state regains its stability.



(a) $A = 0.8$ (b) $A = 1.1$

Figure 7: Stability of Steady States

The above results indicate that for some parameter values two economies with an integrated asset market converge to the symmetric steady state, which is globally stable. For other values of the parameters, the only stable steady states in the economy are asymmetric. In this case, asset market integration induces endogenous inequality of nations in the long run if their initial conditions are not identical. A third possibility arises when a stable symmetric and two stable asymmetric steady states coexist (see figure 8.(c)). In this case two economies may still converge to symmetric steady states after asset market integration when their initial condition are sufficiently similar. This case corresponds to the so called club convergence hypothesis (see Galor 1996).

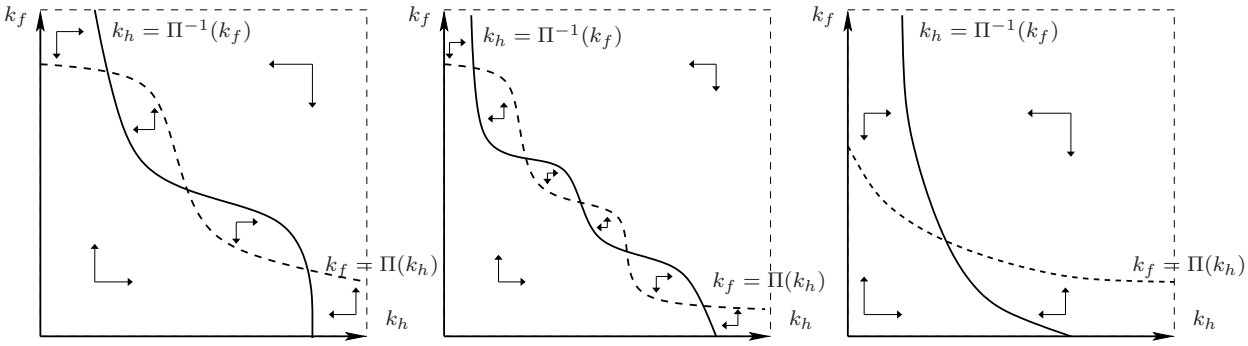
Finally, consider the qualitative properties of the associated phase portraits for the three possible situations with $\rho = 0.01$, 0.0625, 0.1, considered in Section 3.2.1. Given the numerical features of the model, it is possible to calculate the pairwise stationary solutions of each country. That is, for a value of k_h, one can find the stationary value of k_f satisfying

$$k_f = A(k_f, p(k_f, k_h)). \tag{13}$$

Let $k_f = \Pi(k_h)$ denote this solution. Due to the symmetry of the problem $k_h = \Pi^{-1}(k_f)$ will satisfy

$$k_h = A(k_h, p(k_h, k_f)). \tag{14}$$

Figure 8 displays the two curves $\Pi(k_h)$ and $\Pi^{-1}(k_f)$ for three values of ρ.



(a) $\rho = 0.01$ (b) $\rho = 0.0625$ (c) $\rho = 0.1$

Figure 8: Three Possible Phase Portraits

For $\rho = 0.01$ there exist three steady states among which the asymmetric steady states are asymptotically stable. When ρ increases to 0.0625, the number of asymmetric steady states increases to four among which two of them are stable and the others are unstable. If the parameter ρ is increased further, the asymmetric steady states disappear and the unique symmetric steady state becomes globally stable. To summarize, the numerical analysis shows that if production factors are highly substitutable, there exists a unique symmetric steady state which is globally stable. High substitutability of production factors seems to make the assumption of the immobility of capital less severe for the capital stocks in both countries to converge to the same levels.

4 Summary and Conclusions

To explain the divergence of income across countries while global financial markets have decreasingly fewer obstacles to free trade and competition, the conventional literature has focused on imperfections in financial markets. It is commonly assumed that poor countries are exposed to more severe restrictions in financial markets. Hence, imperfect financial markets are taken to restrict productive investment in poor countries. While agents in such models seem to be restricted in their portfolio choice in one way or another,

agents in the present model hold optimal portfolio given only their budget restrictions. Nevertheless, the objective to maximize expected utility does not necessarily equate returns and the level of capital in the two countries.

It is shown that structurally identical economies converge to a unique steady state under perfect competition and rational expectations in a closed economy under certain conditions. However, when asset markets are integrated, asymmetric steady states emerge and the symmetric steady state loses its stability. In this case the initial distribution of capital determines the long run development of otherwise identical economies. This result is in line with the empirical evidence (see Durlauf & Johnson 1995). Whether consumers in the poor country are also worse off from a welfare point of view is an open question.

The causes for symmetry breaking may be explained by an interaction of two effects. First, the concavity of the production function and its diminishing marginal product of capital implies a higher rate of return for lower levels of capital. High capital return attracts funds for productive investment which creates an equalizing force in the world economy. The second effect stems from total spending on international assets. Since investment in the asset market diverts income from production, capital accumulation depends on how much young consumers spend for purchasing assets. If the poor country spends more than the rich country after integration of the asset markets, it creates an un-equalizing force in the world economy. Symmetry breaking may occur in the world economy depending on which force dominates.

Suppose that there is a rich and a poor country in transition under autarky. In this situation the temporary equilibrium asset price is higher in the rich country than in the poor country. If in such a situation the asset markets of the two economies are integrated, the poor country faces a higher price than in autarky. It follows that agents in the poor country decrease the demand of assets, but the question remains how total spending on assets changes. If the demand elasticity of assets is sufficiently small, the poor country's spending on the asset market increases, and the capital stock starts to adjust downward. This will create a downward spiral of low capital/high asset market spending. The same force works in the opposite direction for the rich country. The result is shown to be robust and to occur for a large set of parameters. While the functional specifications of the model have an impact on the results as well (see for example Kikuchi 2006, Böhm & Vachadze 2006), the present paper shows that the elasticity of factor substitution of a CES technology is decisive for the symmetry breaking result under the chosen features.

The paper extends the existing results from Kikuchi (2006) and Böhm & Vachadze (2006) on the causes of symmetry breaking due to the integration of unrestricted asset markets alone. There are no imperfections in the asset market, yet capital remains immobile across countries and foreign direct investment is not allowed. The asset market serves exclusively to diversify risk and to transfer wealth over time freely. At the same time, it withdraws funds from productive investment which is a typical known implication of the creation of asset markets. Therefore, the model captures primarily the *financial aspect* of globalization, namely the effect of the integration of asset markets.

5 Appendix

5.1 Proof of Proposition 1

Let us define the utility of holding x units of equity shares as

$$U(x) := \mathbb{E}[u(c_1)] = qu(wr^e + x(p^e + d - pr^e)) + (1-q)u(wr^e + x(p^e - pr^e)). \tag{15}$$

Differentiating and solving $U'(x) = 0$, one obtains for the middle section of the asset demand function

$$\varphi_m(r^e, p^e, p) := \frac{1}{ad} \ln\left(\frac{q}{1-q}\frac{d-(pr^e - p^e)}{pr^e - p^e}\right). \tag{16}$$

From equation (16) it follows that if, $pr^e \geq p^e + qd$, then asset demand is zero. On the other hand, for any given values of r^e and p^e, if $pr^e \downarrow p^e$, then $\varphi_m(r^e, p^e, p) \to \infty$ and $\frac{w}{p}$ is finite. Since both functions $\varphi_m(r^e, p^e, p)$, and $\frac{w}{p}$ are continuous and monotonically decreasing in the interval $p \in \left(\frac{p^e}{r^e}, \frac{p^e + \mathbb{E}\varepsilon}{r^e}\right)$, it follows that the functions $\varphi_m(r^e, p^e, p)$, and $\frac{w}{p}$ cross each other exactly once at $p = \frac{p^e + \chi(w, r^e, p^e)}{r^e}$, where $\chi(w, r^e, p^e) \in (0, \mathbb{E}\varepsilon)$. This implies that for sufficiently low price, $pr^e < p^e + \chi(w, r^e, p^e)$, young agents will invest all their income in the paper asset. □

5.2 Proof of Proposition 2

Under rational expectations dynamics, the MSV price and capital accumulation functions satisfy the following system of functional equations

$$P(k) = \frac{P(G(k)) + \pi}{r(G(k))} \tag{17}$$

$$G(k) = (1-\delta)k + w(k) - P(k). \tag{18}$$

Step 1. Let us show that G is monotonic. Suppose $G(k_1) = G(k_2)$ for any $k_1 \neq k_2$. Then, equation (17) implies that $P(k_1) = P(k_2)$. However, $G(k_1) = G(k_2)$ and $P(k_1) = P(k_2)$ is in contradiction with equation (18) because if $k_1 \neq k_2$, $(1-\delta)k_1 + w(k_1) \neq (1-\delta)k_2 + w(k_2)$.

Step 2. Suppose that G is monotonically decreasing. Then equation (17) implies that P is monotonically increasing. However, if P is increasing and G is decreasing, $\frac{P(G(k))+\pi}{r(G(k))}$ is decreasing, which is a contradiction.

Step 3. We show that $G(0) > 0$. Suppose that $G(0) = 0$. Then $P(0) = \frac{P(0)+\pi}{r(0)}$ which implies $P(0) = \frac{\pi}{r(0)-1} = 0$. This is a contradiction since $(1-\delta)0 + w(0) > 0$.

Step 4. $\lim_{k\to\infty} \frac{G(k)}{k} < \lim_{k\to\infty} \frac{w(k)}{k} = 0$. Hence, if there exists a unique steady state it is globally stable. □

5.3 Proof of Proposition 3

Since expectations of home and foreign rates of return coincide in the symmetric steady state, $r_h^e = r_f^e = r^e$, one can easily evaluate the sensitivity of the asset price with respect to r^e and p^e. From the asset demand function (1) one obtains the following partial derivatives

$$\frac{\partial \varphi_m(r^e, p^e, p)}{\partial r^e} = -\frac{p}{a(pr^e - p^e)(d - (pr^e - p^e))}, \tag{19}$$

$$\frac{\partial \varphi_m(r^e, p^e, p)}{\partial p^e} = \frac{1}{a(pr^e - p^e)(d - (pr^e - p^e))}, \tag{20}$$

$$\frac{\partial \varphi_m(r^e, p^e, p)}{\partial p} = -\frac{r^e}{a(pr^e - p^e)(d - (pr^e - p^e))}. \tag{21}$$

Equations (19), (20), and (21) imply that

$$\frac{\partial S(r^e, r^e, p^e)}{\partial r^e} = -\frac{p}{2r^e} \quad \text{and} \quad \frac{\partial S(r^e, r^e, p^e)}{\partial p^e} = \frac{1}{2r^e}. \tag{22}$$

From the price law it follows that the sensitivity of the asset price with respect to home and foreign capital in a symmetric steady state is given by

$$\begin{aligned} P_1 &= \frac{\partial S}{\partial r^e} r'(G_1 + G_2) + \frac{\partial S}{\partial p^e}(P_1 G_1 + P_2 G_2) \\ P_2 &= \frac{\partial S}{\partial r^e} r'(G_1 + G_2) + \frac{\partial S}{\partial p^e}(P_1 G_2 + P_2 G_1). \end{aligned} \tag{23}$$

Equation (23) implies that the relationship between P_1, P_2, G_1 and G_2 in the interior symmetric equilibrium is

$$\begin{aligned} P_1(2r - G_1) - P_2G_2 &= -pr'(G_1 + G_2) \\ -P_1G_2 + P_2(2r - G_1) &= -pr'(G_1 + G_2), \end{aligned} \tag{24}$$

implying the first relation given in equation (12).

Let

$$x(k_i, k_{-i}) := \varphi_m \left(r(G(k_i, k_{-i})), P(G(k_i, k_{-i}), G(k_{-i}, k_i)), P(k_i, k_{-i})\right)$$

denote the asset demand and

$$s(k_i, k_{-i}) := x(k_i, k_{-i})P(k_i, k_{-i})$$

the total spending in the international asset market by the agent of country i, when the capital stock in his own country is k_i. Then, the properties of the asset demand function φ_m imply that the derivatives of asset demand with respect to home and foreign country capital stocks in a symmetric equilibrium are given by

$$\begin{aligned} x_1 &= \frac{\partial \varphi}{\partial r^e} r' G_1 + \frac{\partial \varphi}{\partial p^e}(P_1G_1 + P_2G_2) + \frac{\partial \varphi}{\partial p} P_1 \\ x_2 &= \frac{\partial \varphi}{\partial r^e} r' G_2 + \frac{\partial \varphi}{\partial p^e}(P_1G_2 + P_2G_1) + \frac{\partial \varphi}{\partial p} P_2. \end{aligned}$$

Equations (19), (20) and (21) imply that in the interior symmetric equilibrium

$$\begin{aligned} x_1 &= \eta \left(-pr'G_1 + (P_1G_1 + P_2G_2) - rp_1\right) \\ x_2 &= \eta \left(-pr'G_2 + (P_1G_1 + P_2G_2) - rp_2\right), \end{aligned} \tag{25}$$

where $\eta = \frac{1}{a\pi(d-\pi)}$. Using Equations (24) and (25), we can evaluate the derivative of $s(k_i, k_{-i})$ with respect to its arguments to be

$$\begin{aligned} s_1 &= x_1 p + P_1 \\ s_2 &= x_2 p + P_2. \end{aligned} \tag{26}$$

The capital accumulation equation implies that

$$G_1 = (1 - \delta) + w' - s_1 \quad \text{and} \quad G_2 = -s_2. \tag{27}$$

Equations (24), (25), (26) and (27) imply the second and third relations given in equation (12). Since the diagonal and out of diagonal entries of the Jacobian matrix are G_1 and G_2 respectively, it follows that eigenvalues of the Jacobian matrix are $\lambda_1 = G_1 + G_2$ and $\lambda_2 = G_1 - G_2$. □

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