Tacit collusion with price-matching punishments

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Abstract

Tacit collusion is explored under a particularly simple continuous intertemporal reaction function, in which, loosely speaking, firms match the lowest price set by any firm in the previous period. Conditions are provided under which the proposed strategy supports collusive outcomes in a subgame perfect equilibrium. In contrast to traditional results, the highest sustainable collusive price is always lower than the monopoly price. We derive some interesting properties of this highest collusive price, including that it corresponds to the unique Nash equilibrium price of the supergame under strategies in which both upward and downward price deviations are matched.

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JEL : L11, L12, L13, L41

1 Introduction

Following Friedman (1971), the vast majority of models involving tacit collusion assume players follow trigger strategies in which punishments involve either reversion to the one-shot Nash equilibrium following any firm defecting from the collusive agreement (so-called Nash reversion) or optimal punishment strategies (following Abreu, 1986, 1988). One unappealing aspect of these particular punishment rules is that they imply the degree of

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punishment chosen is entirely divorced from the crime. A small cut in prices results in the same severe punishment as does a large cut in prices, even if a small cut in prices has little impact on the rival's profit (e.g. if products are differentiated). As Friedman and Samuelson (1994, p.56) note "*In many circumstances strategies associating severe penalties with arbitrarily small deviations are implausible.*"

A different line of literature (starting with Friedman, 1968) has explored whether tacit collusion can arise when firms instead adopt continuous intertemporal reaction functions each period, so that a firm's price this period is a continuous function of what the rival did in the previous period. Stanford (1986) initially found the answer to be no; subgame perfection failed if strategies conditioned only on the rival's previous price.[1] Samuelson (1987) and Friedman and Samuelson (1990, 1994) allowed strategies to condition as well on the firm's own previous price and gave an affirmative answer. However, their strategies are complicated, involving a kind of continuous approximation to standard trigger strategies. Recognizing this, Friedman and Samuelson (1990, p.323) note "*... our results have been obtained in a general class of abstract games and we have paid for this abstraction in the form of complex strategy formulations.*"

The type of strategies firms use to support collusive outcomes are likely to depend on the structure of the industry and the type of information the firms can exchange. Harrington (1991, p.1089) has noted "*it is quite natural to think of a punishment strategy as being an industry norm with respect to firm conduct ... Thus, even though the norm might not be the best in some sense (for example, it might not be a most severe punishment strategy), firms might choose to maintain it if it seems to work.*" Related to this, Friedman and Samuelson (1990, p.323) ask "*Do games exist whose structure can be exploited to obtain especially simple, continuous reaction function equilibria?*" This paper follows Harrington's suggestion and answers the question posed by Friedman and Samuelson by studying a particularly simple candidate for a possible industry norm, one not previously analyzed.

In an environment in which firms are symmetric and compete in observable prices, we analyze whether the strategy of starting with a common collusive price and thereafter matching the lowest of the prices actually set by firms in the previous period and this collusive price (but never below the one-shot Nash equilibrium price) can still support a

[1] Kalai and Stanford (1985) showed, however, ε-subgame perfection is obtained provided there is sufficiently little discounting of the future.

collusive outcome in a subgame perfect equilibrium of a standard infinitely repeated stage game. We call this strategy "price-matching punishments".

An important feature of the price-matching punishments is that the punishment a firm faces from defecting depends on the size of the deviation. An implication of this feature is that under homogenous price competition, a firm can obtain the maximum one-period profit from deviating, while facing an arbitrarily small punishment, by lowering its price by an infinitesimal amount when defecting. As a result, collusive outcomes can only be supported in this case when firms do not discount future profits. In the more realistic setting in which a firm cannot obtain the whole market by lowering its price by an infinitesimal price for a single period, collusive outcomes will be supported with positive discounting. We model such situations by assuming the firms' products are imperfect substitutes.

Tacit collusion will require firms to be more patient to support a given collusive outcome under price-matching punishments compared to traditional Nash reversion or optimal punishments. This reflects the fact that a defecting firm can always set the same price as it would in the standard analysis, and face a smaller punishment given rivals simply match its price rather than further undercut it. In addition, the defecting firm can do even better by restricting the amount it deviates from the original agreement, thereby further reducing the severity of the punishment. As a result, firms will be more tempted to defect from any collusive agreement. In particular, the monopoly price is never sustainable when firms discount future profits.

The highest collusive price supportable under price-matching punishments has particularly desirable properties. It is easy to characterize, being determined by a simple fixed point condition. It is the unique Nash equilibrium outcome in the supergame in which players play a strategy of matching price increases as well as price decreases. Based on a linear demand example we show that for any discount factor it predicts a unique collusive price which is typically substantially below the monopoly price. This price monotonically decreases from the monopoly price to marginal cost as the degree of product substitutability increases from the case with independent products to that with perfect substitutes. The price also monotonically increases from the one-shot Nash equilibrium price to the monopoly price as the discount factor increases between zero and one. This contrasts with the Nash reversion setting, where the highest collusive price predicted by the model is no longer always a differentiable function of the parameters of the model, is stuck at the monopoly price for most realistic discount factors and levels of product substitutability, and is non-monotonic in the degree of product substitutability for intermediate values of the discount factor. These results demonstrate that the form of firms' collusive strategies is not purely one of modeling convenience.[2]

The rest of the paper proceeds as follows. Section 2 discusses the related literature, including some of the empirical evidence support for continuous punishment strategies and price-matching. Section 3 outlines our basic theory of tacit collusion with price-matching punishment strategies, and presents some general results on when collusion is supported. Section 4 applies the general analysis to a linear model of demand, deriving a simple closed form expression for the maximum sustainable collusive price and re-examining the relationship between tacit collusion and product substitutability in this new framework. Various generalizations are considered in Section 5. Finally, Section 6 concludes with some discussion of possible competition policy implications.

2 Related literature and evidence

There has been a long and rich tradition in oligopoly theory of focusing on continuous reaction functions (most notably Bowley, 1924 and Fellner, 1949). In a repeated game setting, continuous intertemporal reaction functions were first studied by Friedman (1968). One reason for this long-held interest lies in the intuitive appeal of continuous intertemporal reaction functions. Stanford (1986) writes "*However, for the author at least, continuous reaction function models continue to hold an intuitive appeal which is hard to resist*". Friedman and Samuelson (1990, p. 307) write "*there are some circumstances in which continuous strategies are intuitively plausible and discontinuous strategies are not*". Such circumstances may be when there are a few differentiated but symmetric firms competing in observable prices, and when firms do not have the opportunity for pre-game discussion and agreement but instead learn over time by observing each other's reactions. Then simple continuous intertemporal reaction functions such as the price-matching punishment strategies we study may be the type of equilibrium strategies that firms would naturally gravitate to. Even where some communication is possible, provided the scope for communication is limited, there may be grounds for firms to adopt such strategies.

Indeed there is some anecdotal support for the idea firms, at least in certain settings,

[2]Some policy implications of these differences are discussed in Section 6.

use continuous punishment strategies involving matching in practice. Scherer (1980, p.167) reports, based on interviews with business executives and the testimony recorded in numerous antitrust investigations, that a paramount consideration deterring price cutting was "the belief that cuts will be matched". This belief seems to have applied in the U.S. steel industry from about 1900 to 1958, during which U.S. Steel set prices and other domestic producers followed with identical prices, and according to Kalai and Sattherhwaite (1994, p.31) "Each smaller firm certainly believed that if it quoted lower prices than U.S. Steel, then U.S. Steel would either match the lower prices or quote a revision that the smaller firm would then match." Slade (1990, p.532), in her study of data from various price wars episodes, reports that for the Vancouver retail gasoline market "the data reveal a high degree of (lagged) price-matching during the period of the war." Slade (1987, p.515) uses the data to test between different supergame price-war models, and concludes "the evidence seems to point to the reaction-function example as the most appropriate for this market" (see also Slade, 1992). Similarly, in her discussion of several other industries, Slade (1989, p. 304) writes "In all of these industries, behaviour during price wars resembles tit-for-tat. This suggests that our example with continuous intertemporal-reaction functions has substantial descriptive power." Genesove and Mullin (2001) contains a detailed examination of collusive practices from the weekly meeting notes of the 1930s sugar-refining cartel in the U.S., noting moderate deviations from the collusive agreement were generally punished with matching, in degree and in kind. In particular, they note (p. 391) "When one firm openly lowered its rate for rail shipments to the lower water-barge rate, other firms would respond by lowering their rail rates to the same level. When the Pacific refiners gave a freight allowance on certain contracts, American announced that it would match it. The punishment was indeed 'tailored to fit the crime'."

While none of this evidence can be taken as conclusive, it is suggestive that price-matching punishments may be a reasonable description of the strategies used to support collusive outcomes in some industries in which a small number of imperfectly competitive firms compete in observable prices and face roughly symmetric demands and costs. Perhaps, more telling, is that to date there is no solid empirical basis for believing firms use any other particular types of strategies to support collusive outcomes. Thus, for some market structures at least, we think it is worthwhile to explore strategies based on price-matching. A theoretical justification for why firms would suffice with particular strategies such as these, or any others, however, remains an important direction for future research on the topic of collusion in repeated games.[3]

Our work relates to two other lines of work in industrial organization that consider similar types of pricing. Price-matching punishments relates quite directly to the old theory of a kinked demand curve equilibrium (Hall and Hitch, 1939, and Sweezy, 1939). The kinked demand curve theory tries to explain why prices sometimes remain constant at some focal price under oligopoly. It is argued that firms believe that if they increase prices above the focal price, no one will follow, while if they lower prices, everyone else will do the same. In this setting there is no incentive for any firm to change its price. Our model provides game theoretic foundations for this story which, unlike earlier attempts, does not depart from standard timing assumptions to do so.[4] In our setting, single period demand is generally not kinked. Initially, any firm that raises its price loses demand, while any firm which lowers its price gains demand. However, taking into account demand over more than one period, the fact that a price increase will not be matched, while a price decrease will be matched in the subsequent periods, implies a kink in the demand schedule at the initial collusive price, when measured over multiple periods.

Price-matching as a punishment strategy in tacit collusion is distinct from firms offering price-matching guarantees, whereby if a customer receives a better price offer from another seller, the current seller will match that price. These are sometimes also called "meeting-competition" clauses. The main difference with our approach is that the literature on meeting-competition clauses assumes matching happen instantaneously (Kalai and Satterthwaite, 1994), whereas we allow that matching is not instantaneous and future profits are discounted.[5] This explains why the meeting-competition literature does not

[3]Further empirical or experimental evidence on the strategies actually adopted to support collusive outcomes would also be helpful in this regard. In particular, the "strategy method" could be used in experiments to directly elicit subjects' strategies from repeated oligopoly games. Selten et al. (1997) use this approach and conclude that subjects try to achieve cooperation by a "measure-for-measure policy," although their experiment differs in that each subject faces each other subject in a tournament of Cournot duopoly games.

[4]Existing game theoretic treatments of the kinked demand curve theory rely on non-standard timing assumptions of the dynamic oligopoly game. Maskin and Tirole (1988) assume short-run commitments that arise if firms alternate in setting their prices and follow Markov strategies. Bhaskar (1988) and Kalai and Satterthwaite (1994) rely on firms having instantaneous reactions to price decreases.

[5]Nevertheless, statements about "matching rivals' prices" or "will never be beaten on price" could be used to help coordinate on the type of price-matching punishment strategies we examine in this paper. In this respect, our framework may also appear superficially similar to MacLeod's (1985) theory of conscious parallelism. However, his focus is on how signalling (through price announcements) can select a particular

rely on an infinitely repeated stage game, whereas for the usual reasons of unraveling due to backward induction, our theory does.

3 Theory of price-matching punishments

The framework we use to analyze tacit collusion is standard, other than the strategies considered. Consider symmetric duopolists that compete each period in prices for an infinite number of periods. (The cases of asymmetric firms, more than two firms, and quantity competition are discussed in Section 5). Firm i sets a price p_i and faces demand $q_i(p_i, p_j)$ in each period. Time subscripts are not used except where necessary, given that the stage game remains constant throughout time. Single period profits are denoted by some scaler valued payoff function $\pi_i(p_i, p_j)$, where $\pi_i(p_i, p_j) = p_i q_i(p_i, p_j) - C(q_i)$, and C represents some unspecified cost function. Firms discount the future at the constant discount factor $0 < \delta < 1$. Given symmetry, we denote each firm's profit in the case in which both firms set the same price p as $\pi(p)$, so $\pi(p) = \pi_i(p, p)$. The firms seek to sustain some common collusive price denoted p^c, which is also the common initial price. Denote the unique one-shot Nash equilibrium price of the stage game p^n. Define p^m to be the monopoly price, the unique price that maximizes $\pi(p)$.

A **price-matching punishment strategy** is defined so that in the initial period 0 each firm sets its price equal to the common collusive price p^c, and from period 1 onwards each firm sets its price equal to the minimum of the prices set by all firms in the previous period and p^c, provided this is no lower than p^n (in which case each firm prices at p^n). Denoting firm i's price in period t as $p_{i,t}$, each firm i's price-matching punishment strategy can be stated succinctly as:

$$\begin{aligned} p_{i,0} &= p^c \\ p_{i,t+1} &= \max\left(p^n, \min\left(p_{i,t}, p_{j,t}, p^c\right)\right). \end{aligned} \tag{1}$$

We will say a price p^c is **supportable by price-matching punishment strategies** if the price-matching punishment strategy profile (i.e. (1) for $i = 1, 2$) is a subgame perfect equilibrium.

In defining the price-matching punishment strategy in (1) we have been careful to describe what firms should do for any price history. With the standard Nash reversion

collusive outcome. While his announcement game does rely on price-matching, he continues to rely on Nash reversion to support collusion.

punishment strategy, any deviation from the initial collusive price leads to a breakdown in collusion and the reversion to the one-shot Nash equilibrium price forever. In case of (1), a deviation in price does not necessarily lead collusion to break down, since collusion can be reestablished at the new lower price in case a firm deviates by lowering its price from p^c by a moderate amount. However, for prices set below the one-shot Nash equilibrium such matching no longer seems reasonable. This is why in the strategy proposed, deviations below the one-shot Nash equilibrium price still cause a breakdown in collusion with both firms setting their price at the one-shot Nash equilibrium thereafter (as with Nash reversion). Since a deviation to a price below one-shot Nash equilibrium would never arise even for an initial price $p^c > p^n$ which is not supportable by price-matching punishment strategies, this particular assumption plays no role in determining the range of prices for which collusion is sustainable. Nevertheless some such specification is needed in order for the proposed strategy profile to define a subgame perfect equilibrium since firms would not want to continue to match prices below the one-shot Nash equilibrium in case such a price history arises. The particular specification adopted also ensures strategies remain continuous intertemporal reaction functions, the requirement Friedman and Samuelson (1990, 1994) and others wished to maintain.

The strategy profile proposed in (1) also handles cases of simultaneous deviations by both firms, which result in both prices being set above p^c. Since only unilateral deviations have to be prevented in equilibrium, the definition of the pricing strategy is arbitrary for such price histories, except in so far as they matter for subgame perfection. This is the reason the specified strategy involves firms never matching prices above p^c, even for a price history in which both prices ended up above p^c. An alternative approach would be to define price-matching punishment strategies recursively so that for each firm i:

$$\begin{aligned} p_{i,0} &= p_0^c = p^c \\ p_{i,t+1} &= p_{t+1}^c = \max\left(p^n, \min\left(p_{i,t}, p_{j,t}, p_t^c\right)\right), \end{aligned} \tag{2}$$

in which only downward deviations from the equilibrium strategy are matched. For any price history, this would lead to identical prices to the strategies in (1), other than price histories that result from simultaneous deviations by both firms. All of our analysis equally applies to (2). However, unlike (1), the strategy in (2) is no longer a continuous intertemporal reaction function (i.e. each firm's price is no longer a continuous function of the lagged prices of both firms; indeed it no longer just depends on prices in the immediately proceeding period given it also depends on the reference point p_t^c which is a function of all previous actions). This is the reason we focus on (1).

We first note that collusive prices cannot be supported by (1) under homogeneous Bertrand competition.

Proposition 1 *When the firms' products are perfect substitutes and they face constant marginal costs (which we denote c), no price above p^n is supportable by price-matching punishment strategies.*

Proof. Let market demand be denoted $Q(p)$**.** If firms stick to the initial price they each receive $\pi_C = (p^c - c) Q(p^c) / (2(1-\delta))$. If either defects, setting a price of $p^d = p^c - \varepsilon$, it will get profits of $\pi_D = (p^c - c - \varepsilon) Q(p^c - \varepsilon) + \delta (p^c - c - \varepsilon) Q(p^c - \varepsilon) / (2(1-\delta))$ if the firms thereafter follow (1). Since ε can be chosen to be arbitrarily small, π_D can always be made higher than π_C for any $\delta < 1$ unless $p^c = c$. ■

With homogenous Bertrand competition, each firm can make the punishment from it undercutting so as to take the whole market arbitrarily small by undercutting by a sufficiently small amount. This undermines any attempt to sustain collusive outcomes. In more realistic models, where a firm cannot obtain the entire market demand by lowering its price by an infinitesimal amount for a single period, collusive outcomes will be attainable. For this reason, for the remainder of the paper we focus on the case in which the firms' products are imperfect substitutes.[6]

Assume price competition in the stage game is well-behaved. Specifically, sufficient (although not necessary) conditions for our results are that in the stage game, the symmetric firms' profit functions are twice continuously differentiable, with

$$\frac{\partial^2 \pi_i}{\partial p_i^2} < -\left|\frac{\partial^2 \pi_i}{\partial p_i \partial p_j}\right|. \tag{3}$$

Note (3) is a sufficient condition for the best reply mapping to be a contraction (Vives, 2001), which in turn guarantees uniqueness of the one-shot Nash equilibrium (Friedman, 1977). (3) also implies each firm's profit function is strictly concave in its own price (i.e. $\partial^2 \pi_i / \partial p_i^2 < 0$). This, together with the assumption that $\pi_i(p_i, p_j)$ is twice continuously differentiable, guarantees the existence of the one-shot Nash equilibrium in pure strategies

[6] Another way to restore some collusion would be to require prices be set in discrete units, which is typically the case in experimental settings.

so that p^n is well defined.[7] In addition we assume

$$\frac{\partial^2 \pi_i}{\partial p_i \partial p_j} > 0, \tag{4}$$

which says that prices are strategic complements. This together with $\partial^2 \pi_i / \partial p_i^2 < 0$ implies that each firm's best response function is upward sloping, so that given (3), starting from any common price p^c above p^n, in the stage game each firm's best response would be to lower its price towards p^n. Together these assumptions ensure that the stage game is a well-behaved differentiated Bertrand competition game.

These assumptions also ensure $\pi(p)$ is strictly concave in p, since

$$\frac{d^2 \pi(p)}{dp^2} = \left[\frac{\partial^2 \pi_i}{\partial p_i^2} + \frac{\partial^2 \pi_i}{\partial p_j^2} + 2 \frac{\partial^2 \pi_i}{\partial p_i \partial p_j} \right]_{p_i = p_j = p} < 0.$$

Hence, p^m is also well defined, with $p^m > p^n$.

Given (1), the present discounted value of deviation profits is

$$\pi_i(p, p^c) + \frac{\delta}{1-\delta} \pi(p). \tag{5}$$

Proposition 2 *The monopoly price p^m is not supportable by price-matching punishment strategies.*

Proof. The gain from defecting from the monopoly price and setting a lower price p given thereafter firms follow (1) is $\max_{p \leq p^m} \left[\pi_i(p, p^m) + \frac{\delta}{1-\delta} \pi(p) - \frac{1}{1-\delta} \pi(p^m) \right]$. When evaluated at $p = p^m$, $\partial \pi_i(p, p^m) / \partial p < 0$ and $d\pi(p)/dp = 0$, so a sufficiently small decrease in price below the monopoly price is always profitable. ■

This result reflects the fact that starting from the monopoly price, a small price decrease which is matched in subsequent periods has no first order impact on subsequent collusive profits (given collusive profits are flat at the monopoly price), but does generate a first order increase in profits for the defection period. The result also contrasts with the Bhaskar (1988) and Kalai and Satterthwaite (1994) theory of kinked demand curves, which predicts the monopoly price as the equilibrium price in a symmetric setting.

[7] These assumptions are stronger than the usual continuity and strict quasi-concavity assumptions used for existence of the Nash equilibrium (e.g. see Theorem 2.1 in Vives, 2001 and Theorem 7.1 in Friedman, 1977). We are implicitly assuming that we can restrict attention to a compact set of prices, which will be true provided profits decrease in a firm's own price beyond some finite price — for example, because demand vanishes when prices are set too high. The same is assumed with respect to the collusive profit function $\pi(p)$.

We now explore whether some lower collusive price (but still above p^n) is supportable by price-matching punishment strategies. The answer is affirmative.

Proposition 3 *For any given* $0 < \delta < 1$*, there exists some maximum collusive price* $\overline{p}^c(\delta)$ *supportable by price-matching punishment strategies. This price is the solution to (6) and satisfies* $p^n < \overline{p}^c(\delta) < p^m$*. Any collusive price* p^c *such that* $p^n < p^c \leq \overline{p}^c(\delta)$ *is also supportable by price-matching punishment strategies.*

Proof. From the one-stage deviation principle (Fudenberg and Tirole, 1991, pp.108-110) it suffices to check whether there are any histories up to some stage t where one player can gain by deviating for one period from the actions prescribed by his strategy at time t and conforming to his strategy thereafter. The only relevant history is the prices in the immediately preceeding period. Denote these prices as (p_1, p_2). If $p_i \leq p^n$ for some i, then the strategy profile in (1) trivially defines a Nash equilibrium in the subgame. Otherwise, the subgame that follows is identical to a history in which both had set a common price $\min(p_1, p_2, p^c) \in (p^n, p^c]$. To prove subgame perfection, it suffices to prove that if a firm cannot increase profit by deviating once from a common price p^c, then it will also not want to deviate once from any lower price between p^n and p^c.

Suppose we consider some collusive price $p \in (p^n, p^c]$ and consider firm i setting the same or lower price $p_i \in (p^n, p]$. From (5) define

$$\Delta\pi(p) = \left[\frac{\partial \pi_i(p_i, p)}{\partial p_i} + \frac{\delta}{1-\delta}\frac{d\pi(p_i)}{dp_i}\right]_{p_i = p}.$$

The fact that a firm does not want to deviate once from p^c is equivalent to $\Delta\pi(p^c) \geq 0$, otherwise a firm can lower its price and increase its profit while still satisfying the constraint that $p_i \leq p^c$. Suppose we consider some lower collusive price p such that $p^n < p < p^c$. We wish to show that $\Delta\pi(p) \geq 0$ for any $p \in (p^n, p^c)$. Differentiating $\Delta\pi(p)$ with respect to p yields

$$\frac{d}{dp}(\Delta\pi(p)) = \left[\frac{\partial^2 \pi_i(p_i, p)}{\partial p_i^2} + \frac{\partial^2 \pi_i(p_i, p)}{\partial p_i \partial p} + \frac{\delta}{1-\delta}\frac{d^2\pi(p_i)}{dp_i^2}\right]_{p_i = p}.$$

Assumption (3) implies the result that $d(\Delta\pi(p))/dp < 0$. Hence, we have that $\Delta\pi(p) > 0$ for $p \in (p^n, p^c)$. Thus, a firm also does not want to deviate once from any common price between p^n and p^c.

Given $\Delta\pi(p^n) > 0$, $\Delta\pi(p^m) < 0$, and $d(\Delta\pi(p))/dp < 0$, there exists a unique price denoted $\overline{p}^c$ and satisfying $p^n < \overline{p}^c < p^m$, such that

$$\Delta\pi(\overline{p}^c) = \left[\frac{\partial \pi_i(p_i, \overline{p}^c)}{\partial p_i} + \frac{\delta}{1-\delta}\frac{d\pi(p_i)}{dp_i}\right]_{p_i = \overline{p}^c} = 0. \tag{6}$$

$\overline{p}^c$ is the maximum supportable collusive price given (1). It follows from the above analysis, any initial collusive price p^c in the range $p^n < p^c \leq \overline{p}^c$ is supportable by price-matching punishment strategies. To emphasize that $\overline{p}^c$ depends on δ, we write the solution as $\overline{p}^c(\delta)$.

■

Price-matching punishment strategies ensure that the condition for a firm to want to deviate from the punishment subgame is the same as the condition for a firm to want to deviate from the initial collusive phase, except that prices in the punishment subgame are lower. Since under standard properties of the underlying competition game (such as those we assumed), it is more profitable to defect from a high collusive price than one closer to the one-shot Nash equilibrium price, this implies provided firms do not want to deviate once from the initial price, firms will not want to deviate from any punishment subgame.

We know from Proposition 2 that the highest collusive price $\overline{p}^c(\delta)$ given (1) must be strictly less than the monopoly price p^m. More generally we have

Proposition 4 *The range of collusive prices that are supportable by price-matching punishment strategies is a strict subset of the range of prices supportable by Nash reversion (or optimal punishments).*

Proof. The highest common price $p^c > p^n$ at which collusion is sustainable under Nash reversion (and therefore also under optimal punishments) is strictly higher than $\overline{p}^c(\delta)$ since the profits from following the equilibrium strategies are the same (i.e. setting p^c every period), but the profit under the best possible deviation from any common price p^c under Nash reversion (i.e. the best response to p^c in the stage game, denoted $p^r(p^c)$) is always strictly less than the profit under the best possible deviation under price-matching punishment strategies (i.e. the defecting firm can always deviate by also setting $p^r(p^c)$ and face strictly less punishment than under Nash reversion given that $p^r(p^c) > p^n$, and indeed may do even better by deviating by a lesser amount). ■

Tacit collusion will require firms to be more patient to sustain a given collusive outcome under price-matching punishments compared to traditional Nash reversion. This reflects the fact that under price-matching punishments, a defecting firm can always set the same price as it would in the standard analysis, and face a smaller punishment given rivals simply match its price rather than further undercut it. In addition, the defecting firm can do even better by restricting the amount it deviates from the original agreement, thereby further reducing the severity of its punishment. As a result, firms will be more tempted to defect from any collusive agreement. This finding provides a reason as to why tacit collusion may be less effective than predicted by standard theory (based on grim or optimal punishments), in addition to other possible explanations such as the fear of getting caught by authorities or the difficulty of monitoring and coordinating on a collusive agreement. Indeed, such considerations may help explain why price-matching punishment strategies come about in the first place given statements about matching each other's lower prices may signal the adoption of such strategies without being considered illegal.

The maximum collusive price $\overline{p}^c(\delta)$ defined by (6) has the property that the unconstrained maximum of (5) equals p^c. This fixed point property makes $\overline{p}^c(\delta)$ particularly easy to characterize. It also implies another property of interest.

Define a **price-matching** strategy, in which firms match price increases as well as price decreases. The definition is recursive. Let $p_0 = p^c$ be the common (collusive) price in the initial period. Define p^c_{t+1}, the common price both firms set in period $t+1$ to equal p^c_t unless a firm deviated in period t (i.e. it chose a different price from p^c_{t-1}) in which case p^c_{t+1} equals this deviation price. Note we ignore simultaneous deviations by both firms (the definition of the strategy in this case is arbitrary since such deviations do not need to be considered in defining a Nash equilibrium).

Proposition 5 *The collusive price $\overline{p}^c(\delta)$ is not only the firms' preferred equilibrium price under price-matching punishment strategies, but it is the only price for which the price-matching strategy of matching higher and lower prices defines a Nash equilibrium of the supergame.*

Proof. Firm 1's profit at period 1 from defecting is $\pi_1(p_1, p_0) + \delta\pi_1(p_2, p_1) + \delta^2\pi_1(p_3, p_2) + ...$, where (with a slight abuse of notation) p_1 is firm 1's optimal deviation price in period 1, p_2 is its optimal deviation price in period 2, and so on. Define

$$V(p_t) = \max_{p_{t+1}, p_{t+2}, \ldots} \left(\pi_i(p_{t+1}, p_t) + \delta\pi_i(p_{t+2}, p_{t+1}) + \delta^2\pi_i(p_{t+3}, p_{t+2}) + ...\right).$$

Then we have the sequence of Euler equations defined by

$$\frac{\partial\pi_i(p_{t+1}, p_t)}{\partial p_{t+1}} + \delta\frac{\partial\pi_i(p_{t+2}, p_{t+1})}{\partial p_{t+1}} = 0 \tag{7}$$

for all $t \geq 0$. This can be written as

$$\frac{\partial\pi_i(p_{t+1}, p_t)}{\partial p_{t+1}} + \frac{\delta}{1-\delta}\left(\frac{\partial\pi_i(p_{t+1}, p_t)}{\partial p_{t+1}} + \frac{\partial\pi_i(p_{t+2}, p_{t+1})}{\partial p_{t+1}}\right) = 0.$$

To have $p_t = p_0$ for all $t \geq 0$, the following condition has to be satisfied:

$$\left[\frac{\partial \pi_i (p_{t+1}, p_t)}{\partial p_{t+1}} + \frac{\delta}{1-\delta}\left(\frac{\partial \pi_i (p_{t+1}, p_t)}{\partial p_{t+1}} + \frac{\partial \pi_i (p_{t+2}, p_{t+1})}{\partial p_{t+1}}\right)\right]_{p_t=p_{t+1}=p_{t+2}=p_0} = 0.$$

Since

$$\frac{d\pi (p)}{dp} = \frac{d\pi_i (p, p)}{dp} = \frac{\partial \pi_i (p, p)}{\partial p_i} + \frac{\partial \pi_i (p, p)}{\partial p_j},$$

the condition is actually the same as

$$\left[\frac{\partial \pi_i (p_i, p_0)}{\partial p_i} + \frac{\delta}{1-\delta}\frac{d\pi (p_i)}{dp_i}\right]_{p_t=p_0} = 0,$$

which is the same as (6). ■

The result provides a further reason why $\overline{p}^c (\delta)$ may be the focal price for any collusive agreement under price-matching punishments. Not only is it the best the firms can achieve under price-matching punishment strategies, but it is the unique price at which each firm has no incentive to increase or decrease its price, even if the increase (as well as the decrease) in its price is matched.

More generally, the set of Nash equilibrium collusive prices in the supergame can be expressed as $\left[\underline{p}^c (\delta), \overline{p}^c (\delta)\right]$ where $\underline{p}^c (\delta)$ varies from p^n to $\overline{p}^c (\delta)$ as the proportion of any price increase that is matched goes from no matching (under the price-matching punishment strategy) to complete matching (under simple price-matching). Thus, $\overline{p}^c (\delta)$ is the only price that remains a Nash equilibrium outcome regardless of the proportion of any price increase that is matched.

4 A linear demand example

We illustrate and complement the above analysis by considering competition in the stage game being modeled by a standard differentiated Bertrand competition model with linear demands. We do this for three reasons. First, the linear demand model of product differentiation is widely used in applications (including to tacit collusion) and so it is useful for comparing our findings with the existing literature. Second, this case gives rise to a closed form expression for the maximum sustainable collusive price that has appealing properties. Third, we show that collusion always gets easier to sustain as the degree of product differentiation increases, which contrasts with the non-monotonic comparative static found under Nash reversion which is also derived under linear demands.

Inverse demand functions are given by $p_i = \alpha - \beta(q_i + \gamma q_j)$, where $0 < \gamma < 1$ serves as a measure of the degree of product substitutability. The closer is γ to one, the more substitutable are the firms' products. These demand functions can be derived from utility maximization of a representative consumer facing a quadratic utility function (Singh and Vives, 1984). Firms are assumed to have constant marginal costs c.

The implied direct demand functions are kinked, reflecting that for $p_i \leq (-\alpha(1-\gamma)+p_j)/\gamma$, firm i will capture the entire market and face the monopoly demand function $q_i = (\alpha - p_i)/\beta$, while conversely when $p_i > \alpha(1-\gamma)+\gamma p_j$ firm i will face no demand. For prices in between these two extremes, firm i's demand function is found by inverting the firms' inverse demand functions so $q_i = (\alpha(1-\gamma) - p_i + \gamma p_j)/(\beta(1+\gamma)(1-\gamma))$. In deriving the results stated below, we have been careful to take into account these kinks in each firm's demand function.

It is straightforward to check that the monopoly price is $p^m = (\alpha + c)/2$, which does not depend on γ, and that the one-shot Nash equilibrium price is $p^n = (\alpha(1-\gamma)+c)/(2-\gamma)$. As argued earlier, the highest collusive price defined by (6) is the focal price predicted by our theory. For the linear demand example, (6) has the closed form solution

$$\overline{p}^c(\delta) = \frac{\alpha(1-\gamma)+c(1-\delta\gamma)}{2-(1+\delta)\gamma}, \tag{8}$$

which applies for $0 < \gamma < 1$. Despite the kinks in the underlying demand function, this is a simple continuously differentiable function of all the parameters of interest.[8] The collusive price in (8) varies between the one-shot Nash equilibrium price when firms do not care about future profits and the monopoly price when they are infinitely patient, and is everywhere increasing in the discount factor. The price also varies between the monopoly price and marginal cost as the degree of product substitutability varies between no substitutability and perfect substitutability, and is everywhere decreasing in the substitutability parameter γ. These results contrast to the Nash reversion setting, where the maximum sustainable collusive price is the monopoly price for a wide range of realistic parameter values, and more generally is a very complicated function of δ and γ making it less amenable to applications than the present formula.

An existing literature has studied the question of whether collusion is easier or harder to sustain when products are closer substitutes. The question has been addressed in the same context as the present model with price competition and linear demands, assuming Nash reversion punishment. Deneckere (1983), Albaek and Lambertini (1998) and Ross

[8]This is because under price matching punishment strategies, deviating to a price lower than $(-\alpha(1-\gamma)+p^c)/\gamma$ is always strictly worse than deviating to the price $(-\alpha(1-\gamma)+p^c)/\gamma$. Thus, the equation defining the highest collusive price is the same for any $0 < \gamma < 1$.

(1992) find a non-monotonic relationship between the degree of product substitutability and the critical discount factor that is necessary to sustain collusion. The non-monotonic relationship found under Nash reversion reflects the fact that an increase in product substitutability increases both the one period gain to defecting on an agreement and also the degree of subsequent punishment that can be imposed. In our setting, an increase in product substitutability allows the gain from cheating to be realized with a smaller reduction in prices, which therefore induces a smaller punishment for the defecting firm. Put differently, greater product substitutability not only increases the one-period gain from defecting, as with Nash reversion, but it also reduces the long-term loss resulting from punishment, which is not the case under Nash reversion. By inverting (8) and differentiating with respect to γ, in contrast to the results under Nash reversion (i.e. from the earlier literature), we have that the critical discount factor is always increasing in γ for any collusive price between the one-shot Nash equilibrium price and the monopoly price, meaning collusive outcomes are always harder to support when products become closer substitutes.

The highest collusive price defined by (8) is decreasing in γ, both since p^n is decreasing in γ and since the collusive price moves closer to the one-shot Nash equilibrium (as opposed to the monopoly price) as γ increases. To see the latter point, note (8) can be rewritten as $\overline{p}^c(\delta) = p^n + \theta(p^m - p^n)$, where $\theta = 2\delta(1-\gamma)/(2-\gamma(1+\delta))$ measures how far the highest collusive price is above the one-shot Nash equilibrium price relative to the monopoly price. Note θ is decreasing in γ, which implies higher product substitutability reduces the scope for collusion in this model.

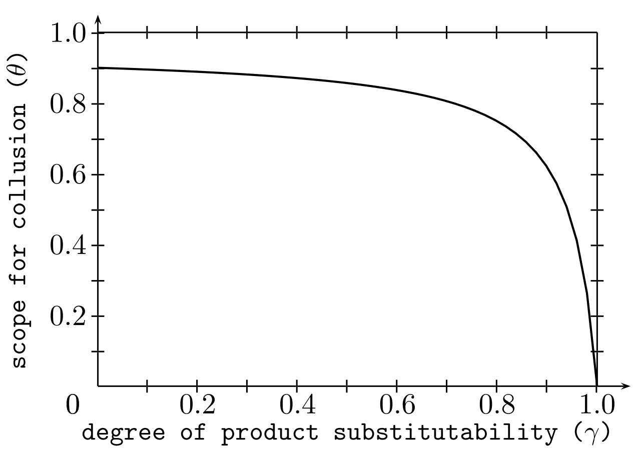

Figure 1: ($\alpha = 2, c = 0$ and $\delta = 0.9$).

Figure 1 plots θ as a function of γ. Parameter values are chosen such that the monopoly price is equal to unity. Since with a discount factor of 0.9, the monopoly price can always be sustained under Nash reversion, the vertical axis in figure 1 also indicates the degree of collusion under price-matching punishment strategies as a proportion of that under Nash reversion. The figure shows that the scope for collusion may be substantially reduced under price-matching punishments, particularly if products are highly substitutable. Figure 1 also illustrates the monotonic relationship between the scope for collusion and the degree of product substitutability. Interestingly, it shows that even as the one-shot Nash equilibrium prices approaches the monopoly price (i.e. as $\gamma \to 0$, so goods become close to being independent), the highest collusive price under price-matching punishments is still strictly between the one-shot Nash price and the monoply price (with $\theta \to \delta$), consistent with Proposition 2.

5 Generalizations

The analysis in this paper extends in a straightforward manner to n symmetric firms by generalizing the strategy in (1) so that for each firm i:

$$\begin{aligned} p_{i,0} &= p^c \\ p_{i,t+1} &= \max\left(p^n, \min\left(p_{1,t}, ..., p_{n,t}, p^c\right)\right). \end{aligned}$$

Under such a strategy, any firm deciding whether to defect assumes all other firms will subsequently match, while any firm deciding whether to match following a defection assumes all the other firms will do so. Then the matching or defecting decisions remain the same as analyzed in Section 3, except that the single rival is now replaced by $n-1$ identical rival firms. Since the gains to a single firm that undercuts a collusive price p^c are increasing in the number of competing firms, collusion will become harder to sustain as the number of competitors increases.

The extension to quantity competition is also straightforward. Quantity-matching punishment strategies would involve firms starting with a common fixed quantity which is between the monopoly quantity and the one-shot Nash equilibrium quantity, and thereafter match the highest quantity set by any firm in the previous period (provided these quantities are no higher than the one-shot Nash equilibrium quantity and no lower than the initial collusive quantity). Apart from the fact higher quantities are now matched (rather than lower prices), and replacing our assumption that the actions in the stage game are strategic complements with that they are strategic substitutes, our analysis continues to apply.

Assuming punishments last forever is perhaps unrealistic. However, our analysis continues to apply when the strategy under consideration is that the punishment only lasts for T periods. The proposed strategy, which we call the T-period price-matching punishment strategy, is defined as follows. There is a collusive phase and a punishment phase. The game begins in the collusive phase with $p_{i,0} = p^c$. Define $p_t \equiv \max(p^n, \min(p_{i,t}, p_{j,t}, p^c))$.[9] In the collusive phase both firms price at p^c until one or more firms lower their price below p^c (i.e. $p_t < p^c$). Such a lower price triggers a punishment phase. Denote the period in which such a lower price occurs by s. In the punishment phase, which starts in period $s+1$ and lasts for T periods, both firms price at p_s each period and thereafter revert back to the collusive phase (unless one or more firms lowers their price below p_s during the punishment phase so that $p_{s+\tau} < p_s$, in which case the punishment phase starts again, with the new punishment price $p_{s+\tau}$). The analysis of such strategies is not very different from our benchmark case with infinitely lived punishments. In the appendix we establish that the equivalent of each of our propositions continues to hold, as well as showing how the linear demand example of Section 4 extends.

Another generalization we have considered is allowing for asymmetries between firms. The strategies in (1) can be naturally generalized to such a setting. Consider the line connecting the one-shot Nash equilibrium prices (p_1^n, p_2^n) and some initial collusive prices (p_1^c, p_2^c) that are to be supported (such that $p_i^c > p_i^n$ for $i = 1, 2$). Then "matching" is defined with respect to this line. If the line is written as $p_i = R_i(p_j)$, then for each firm i, generalized price-matching punishment strategies are:

$$\begin{aligned} p_{i,0} &= p_i^c \\ p_{i,t+1} &= \max\left(p_i^n, \min\left(p_{i,t}, R_i\left(p_{j,t}\right), p_i^c\right)\right), \end{aligned}$$

where

$$R_i\left(p_{j,t}\right) = \frac{p_j^c p_i^n - p_i^c p_j^n + \left(p_i^c - p_i^n\right) p_{j,t}}{p_j^c - p_j^n}.$$

When firms are symmetric and $p_1^c = p_2^c$, the specified line would be the 45-degree line so that $R_i(p_{j,t}) = p_{j,t}$, and we have (1) as studied in the paper. With asymmetric firms, the line could be steeper or flatter than before and only price combinations along this line would be supported by generalized price-matching punishment strategies. According to these strategies, if either firm deviates to set a higher price, it will not be matched. If

[9] We use subscripts t, s, τ to denote periods and 1, 2, i, j to denote firms.

either firm deviates to a lower price, the other firm will lower its price by choosing the corresponding (lower) price from the specified line.[10]

There are several ways our analysis can be extended to allow for common demand or cost shocks. For stationary demand or cost shocks along the lines of Rotemberg and Saloner (1986), one can still determine the set of collusive prices that are sustainable under our price-matching punishment strategies. For instance, in a setting in which there is a fixed probability each period of the demand being high or low, the initial collusive price must be set low enough that it is still sustainable even starting from a low demand state. Our framework can also handle shocks that represent structural changes, following Slade (1989), in which demand (or cost) is subject to infrequent random shocks. When a shock occurs, the change in demand is such that firms optimally act as if they will always be playing the current game forever. In our linear demand specification, this property holds when parameter changes are white noise. Then rather than adopting a learning approach as in Slade, one could take a more standard approach (e.g. Rotemberg and Saloner, 1986) that after a shock has occurred, firms will coordinate on the new equilibrium which generates the highest expected present discounted value of profits.

6 Concluding remarks

In this paper we have studied tacit collusion when firms employ a price-matching punishment strategy. Firms start with a common collusive price which is between the one-shot Nash equilibrium price and the monopoly price, and thereafter match the lowest of the prices actually set by firms in the previous period and this collusive price (provided these prices are no lower than the one-shot Nash equilibrium price). Under this price-matching punishment strategy, no profit above the one-shot Nash equilibrium level can be sustained if products are homogeneous. When products are imperfect substitutes, profits from the one-shot Nash equilibrium level up to a maximum level can be sustained. The corresponding highest collusive price has a nice fixed-point characterization which corresponds to the

[10]Using this approach we have considered the linear demand example given in Section 4, but with the demand intercept α allowed to vary across firms. For the numerical example in figure 1, we find the monotonic relationship between the scope for collusion and product substitutability shown in figure 1 is robust, at least for small amounts of asymmetry (i.e. $\alpha_1 \neq \alpha_2$). We also find that increasing α_1 and decreasing α_2 by an equal amount, initially increases the scope for collusion for firm 1 and decreases it for firm 2, but eventually decreases the scope for collusion for both firms.

unique Nash equilibrium price of the supergame under strategies in which both upward and downward price deviations are matched. Based on a linear demand example we showed this highest collusive price monotonically decreases from the monopoly price to marginal cost as the degree of product substitutability increases from the case with independent products to that with perfect substitutes. It also monotonically increases from the one-shot Nash equilibrium price to the monopoly price as the discount factor increases from zero to one. This contrasts with the Nash reversion setting, where the highest collusive price predicted by the model is no longer always a differentiable function of the parameters of the model, is stuck at the monopoly price for most realistic discount factors and levels of product substitutability, and is non-monotonic in the degree of product substitutability for intermediate values of the discount factor.

Some competition policy implications follow from these results. For instance, to the extent the use of price-matching punishments facilitate collusion, we would expect the scope for collusion to be greater when firms products are less substitutable. This conclusion seems to be somewhat at odds with the traditional view of antitrust authorities, which is that homogeneity is considered a factor making collusion more likely to occur. For example, Section 2.11 in the "Horizontal Merger Guidelines" issued by the U.S. Department of Justice and the Federal Trade Commission in 1992 states:

> "Market conditions may be conducive to or hinder reaching terms of coordination. For example, reaching terms of coordination may be facilitated by product or firm homogeneity and by existing practices among firms, practices not necessarily themselves antitrust violations, such as standardization of pricing or product variables on which firms could compete."

However, this traditional view may reflect more the ease of reaching an agreement under homogenous conditions (as suggested by Stigler, 1964), than the sustainability of collusion thereafter. In this regard, one has to be careful to distinguish between the symmetry of firms (including the fact they sell like products and are not vertically differentiated), which presumably makes it easier for them to reach an agreement in the first place (and in our view makes price-matching punishments more likely to arise and be sustainable), and the strength of competition in any given period. For instance, brand loyalty, switching costs, transportation costs, or consumer search may all decrease effective product substitutability in terms of our model without requiring an asymmetry between firms. Our results suggest authorities should not put less weight on the possibility of collusion in industries where product substitutability is relatively weak; for example, between gasoline stations, mobile phone operators, or retail banks where switching costs may be high.

Price-matching punishments may reduce the need for communication or overt collusion between firms. Vague statements about matching each other's lower prices may signal the adoption of such a strategy. Such statements should therefore be viewed with some suspicion — they may be facilitating tacit collusion. Alternatively, if through trial and error, firms arrive at a point where they set equal prices, price-matching punishments may arise naturally. However, in such cases the need for communication may not be eliminated altogether. Firms may still need to coordinate their strategies, or, reset their strategies in the face of a structural change in the industry. This suggests that evidence consistent with price-matching (such as near parallelism in pricing and the use of statements about matching a rival's prices) may provide a useful screen for authorities that are considering whether to investigate an industry for possible overt collusion.

Finally, our findings may also help reconcile the apparent dichotomy between the approach taken by competition authorities in coordinated-effects merger cases, in which only mergers that leads to symmetric industry structures will tend to be disallowed on the grounds of facilitating tacit collusion, and the fact that prosecuted cartels (in which collusion is overt) often involve asymmetric firms (see Davies and Olczak, 2008). In the former case, collusion may only be possible through some rule-of-thumb, which in the case of price-matching punishment strategies is much more likely to arise under symmetric conditions.

7 Appendix

Under T-period price-matching punishment strategies (as defined in Section 5), the present discounted value of deviation profits by setting a deviation price $p < p^c$ is

$$\pi_i(p, p^c) + \frac{\delta - \delta^{T+1}}{1-\delta}\pi(p) + \frac{\delta^{T+1}}{1-\delta}\pi(p^c). \tag{A.1}$$

Clearly, the punishment in T-period strategies is weaker than in (1). As a result, Propositions 1 and 2 trivially extend for such strategies, as does Proposition 4. It remains only to show that the equivalent of Propositions 3 and 5 hold for the T-period case.

Proposition 6 *For any given $0 < \delta < 1$, there exists some maximum collusive price $\overline{p}^c(\delta, T)$ supportable by T-period price-matching punishment strategies. This price is the* *solution to (A.3) and satisfies* $p^n < \overline{p}^c(\delta, T) < p^m$*. Any collusive price* p^c *such that* $p^n < p^c \leq \overline{p}^c(\delta, T)$ *is also supportable by* T*-period price-matching punishment strategies.*

Proof. Again, we use the one-stage deviation principle. For any price pair (p_1, p_2) in the previous period in which $\min(p_1, p_2) \leq p^n$, the T-period price matching punishment strategies define a Nash equilibrium in the subgame for the same reason as do T-period Nash reversion strategies. If $p_i > p^n$ for $i = 1, 2$, we need to distinguish two cases: (i) the current period is in a collusive phase, (ii) the current period is the kth ($k = 1, 2, 3, ..., T$) period in a punishment phase.

Case (i): This case follows exactly in the same way as the proof of Proposition 3, but with δ replaced by $\left(\delta - \delta^{T+1}\right) / \left(1 - \delta^{T+1}\right)$ so that

$$\Delta\pi(p) = \left[\frac{\partial \pi_i(p_i, p)}{\partial p_i} + \frac{\delta - \delta^{T+1}}{1 - \delta}\frac{d\pi(p_i)}{dp_i}\right]_{p_i = p} \tag{A.2}$$

and

$$\Delta\pi(\overline{p}^c) = \left[\frac{\partial \pi_i(p_i, \overline{p}^c)}{\partial p_i} + \frac{\delta - \delta^{T+1}}{1 - \delta}\frac{d\pi(p_i)}{dp_i}\right]_{p_i = \overline{p}^c} = 0. \tag{A.3}$$

It follows that the proposed strategies define a Nash equilibrium in the subgame for any initial collusive price p^c for which $p^n < p^c \leq \overline{p}^c$. To emphasize that $\overline{p}^c$ now depends on δ and T we write the solution of (A.3) as $\overline{p}^c(\delta, T)$.

Case (ii): If both firms follow the proposed strategy, then the price in the current period and the next $T - k$ periods will be $\widetilde{p} = \min(p_1, p_2, p^c) \in (p^n, p^c]$. Given the rival firm follows the proposed strategy, firm i's continuing profit if it also follows the strategy is

$$\frac{1 - \delta^{T-k+1}}{1 - \delta}\pi(\widetilde{p}) + \frac{\delta^{T-k+1}}{1 - \delta}\pi(p^c).$$

Clearly, firm i will not set a price higher than $\widetilde{p}$ given it will not be matched. If instead firm i sets a lower price $p_i < \widetilde{p}$, it will lead to a further T periods of punishment and its continuing profits are

$$\pi_i(p_i, \widetilde{p}) + \frac{\delta - \delta^{T+1}}{1 - \delta}\pi(p_i) + \frac{\delta^{T+1}}{1 - \delta}\pi(p^c).$$

We claim that if the initial price $\widetilde{p}$ is sustainable in the collusive phase, then firm i also has no incentive to set the price $p_i < \widetilde{p}$ in the punishment phase. The fact that $\widetilde{p}$ is sustainable in the collusive phase implies that

$$\max_{p_i < \widetilde{p}}\left[\pi_i(p_i, \widetilde{p}) + \frac{\delta - \delta^{T+1}}{1 - \delta}\pi(p_i)\right] + \frac{\delta^{T+1}}{1 - \delta}\pi(\widetilde{p}) < \frac{1}{1 - \delta}\pi(\widetilde{p}).$$

It follows that

$$\begin{aligned}
\max_{p_i<\widetilde{p}}\left[\pi_i\left(p_i,\widetilde{p}\right)+\frac{\delta-\delta^{T+1}}{1-\delta}\pi\left(p_i\right)\right]+\frac{\delta^{T+1}}{1-\delta}\pi\left(p^c\right) &< \frac{1}{1-\delta}\pi\left(\widetilde{p}\right)+\frac{\delta^{T+1}}{1-\delta}\left(\pi\left(p^c\right)-\pi\left(\widetilde{p}\right)\right)\\
&< \frac{1}{1-\delta}\pi\left(\widetilde{p}\right)+\frac{\delta^{T-k+1}}{1-\delta}\left(\pi\left(p^c\right)-\pi\left(\widetilde{p}\right)\right)\\
&= \frac{1-\delta^{T-k+1}}{1-\delta}\pi\left(\widetilde{p}\right)+\frac{\delta^{T-k+1}}{1-\delta}\pi\left(p^c\right).
\end{aligned}$$

This shows that firm i has no incentive to set a price $p_i<\widetilde{p}$ in the punishment phase. Hence, it suffices to show that the price $\widetilde{p}$ is sustainable in the collusive phase, which we have already established from (i) given $p^n<\widetilde{p}\leq p^c\leq\overline{p}^c\left(\delta,T\right)$. ■

The collusive price $\overline{p}^c\left(\delta,T\right)$ defined in the above proposition is not only the firms' preferred equilibrium price under T-period price-matching punishment strategies, but like the result in Proposition 5, it is also the only price for which the price-matching strategy of matching higher and lower prices for T-periods defines a Nash equilibrium of the supergame. The present discount value of deviation profits under T-period price-matching strategies with the initial price p^c is given by (A.1). Define $\Delta\pi(p)$ as in (A.2). It is then clear that $\overline{p}^c\left(\delta,T\right)$ is the unique price such that $\Delta\pi(p)=0$.

Finally, the maximum sustainable collusive price in the linear demand example of Section 4 in the text can be easily obtained by replacing δ in (8) with $\left(\delta-\delta^{T+1}\right)/\left(1-\delta^{T+1}\right)$. Apart from the fact that the effective discount factor is lower, all our previous results continue to apply. In particular, increased product substitutability still always reduces the scope for collusion. Compared to the highest collusive price under infinite punishments $\overline{p}^c\left(\delta\right)$, T-period punishments may considerably further reduce the scope for collusion. For instance, with the parameter values used in figure 1 and with $\gamma=0.9$, the range of feasible collusive prices is from $p^n=0.18$ to $p^m=1$. The highest collusive price under Nash reversion is p^m, while under (1) it is $\overline{p}^c\left(\delta\right)=0.69$. Under Nash reversion for T-periods, the highest collusive price is still equal to the monopoly price for any $T=2,3,..$ (only with $T=1$ is the highest collusive price lower than monopoly, and equal to 0.58). In contrast, under T-period price-matching punishment strategies, then $\overline{p}^c\left(\delta,1\right)=0.30$, $\overline{p}^c\left(\delta,2\right)=0.38$, $\overline{p}^c\left(\delta,3\right)=0.43$, $\overline{p}^c\left(\delta,4\right)=0.48$, $\overline{p}^c\left(\delta,5\right)=0.51$, indicating substantially less scope for collusion.

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