Multihoming and oligopolistic platform competition*

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Abstract

We provide a general framework to analyze competition between two-sided platforms, in which buyers and sellers can multihome, and platforms compete on transaction fees charged on both sides. We show how the implications of increased platform competition (e.g. entry) change dramatically depending on whether buyers multihome or not. Increased platform competition shifts the fee structure in favor of buyers if buyers are singlehoming, but shifts the fee structure in favor of sellers if buyers are multihoming. We generalize these results to allow for partial-multihoming, and discuss the economic implications for payment cards and ride-hailing.

JEL classification: L11, L13, L4

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1 Introduction

A growing number of two-sided platforms intermediate transactions between buyers and sellers of products and services. Payment card platforms (MasterCard and Visa), ride-hailing platforms (Uber and Lyft), hotel booking platforms (Booking.com and Expedia), and e-commerce marketplaces (eBay and Amazon marketplace) are among the best-known examples. Key features of the markets in which these platforms operate are: (i) the fees charged by platforms to each side are transaction based; and (ii) platform users can choose to (and often do) join more than one of the multiple competing platforms, a phenomenon known in the literature as “multihoming”. This paper investigate a model of oligopolistic platform competition that matches these features.

Our interest in studying these markets stems from the observation that transaction-based platform businesses typically face competition. For example, American Express, Discover, MasterCard and Visa compete in the U.S. to attract consumers and merchants to adopt and use their payment cards; Uber and Lyft compete in the U.S. to attract riders and drivers to adopt and use their ride-hailing services; Booking.com, Expedia and TripAdvisor compete internationally to attract hotel guests and hotels to sign up to and use their platforms. A natural question is: how does such competition affect equilibrium fees? This question is well-studied in the context of one-sided markets, but less so for two-sided markets whereby the two user sides exhibit cross-group externality and each side can be charged a different fee. With these additional features, it is not immediately obvious how competition among two-sided platforms affects the total fees (the sum of fees charged on both user sides) and the fee structure (the allocation of fees across the two user sides).

Moreover, following advancements in technology that make it easier for buyers to compare the options across multiple platforms, there has been a substantial shift in the capability and willingness on the buyer-side to multihome on platforms. For example, in the ride-hailing market, advancements in mobile phone technology and fare-comparison “metasearch” aggregators such as Google Maps, BellHop and RideGuru, allow more riders to easily compare fares across different apps when hailing a ride, resulting in more active multihoming by riders. Similar aggregators have also become quite widely used for hotel booking platforms, such as Kayak and Trivago. What are the implications of this type of shift in multihoming behaviour of buyers? And how does such a shift in buyer multihoming interact with changes in the extent of platform competition?

To address these questions, we adapt the canonical two-sided market framework pioneered by Rochet and Tirole (2003). Users (buyers and sellers) have heterogenous valuations over interaction benefits and platforms charge users on each side per-transaction fees. Platforms are differentiated from the buyers’ perspective, but identical from the sellers’ perspective. We generalize Rochet and Tirole’s framework by allowing for more
than two platforms and by considering different configurations of multihoming behaviour: one in which only sellers can multihome, and another one in which both sellers and buyers can multihome. Using this framework we investigate the impact on the equilibrium total fee and fee structure of: (i) buyer-multihoming, (ii) the number of platform competitors (e.g. entry), and (iii) the interaction of these two factors.

Our first major result shows that buyer multihoming increases buyer fees, decreases seller fees, and decreases the total fee paid by the two types of users. Intuitively, when buyers are singlehoming, each platform can act as a monopolist with respect to multihoming sellers in providing access to their exclusive buyers. In contrast, when buyers are multihoming, platforms lose this monopoly power because there are now multiple platforms through which sellers can access each buyer. Sellers can quit a high-fee platform so as to divert buyers to transact using other platforms that set lower seller fees. Buyer multihoming therefore leads to more elastic seller-side demand and drives down the seller fee in equilibrium, thus restoring competition to the previously monopolistic seller side. The lower seller fee then implies a smaller profit margin on the seller side from attracting buyers, so that platforms increase their buyer fees in response. While our result resembles the classic “competitive bottleneck” result in the membership fee framework of Armstrong (2006), Armstrong and Wright (2007) and Belleflamme and Peitz (2019), the underlying mechanism is very different given that sellers are always multihoming in our model. Furthermore, our result is obtained in a setting in which user heterogeneity is with respect to transaction benefits and fees are per transaction, rather than these being membership based as in the previous literature looking at the implications of multihoming.

Our second major result addresses when increased platform competition increases the fees charged on one of the sides by providing sufficient conditions for the buyer or seller fees to increase with platform entry. We find that while the total fee always goes down with entry, the fee structure shifts in favor of buyers when buyers are singlehoming, but tends to shift in favor of sellers when buyers are multihoming. Intuitively, when buyers are singlehoming, the monopoly power of platforms over the seller side means that the buyer side is the only side that is subject to competition. Therefore, platform entry intensifies only buyer-side competition (by making platforms more substitutable for buyers) resulting in a decrease in buyer fees, and in response to this, an increase in seller fees. In contrast, buyer multihoming opens up the seller side to competition, so that platform entry intensifies competition in both the buyer and seller sides. The entry effect on the seller side reflects that when platforms become more substitutable for buyers, sellers can divert more buyers to use low seller-fee platforms when it quits a high seller-fee platform. Therefore, the seller-side demand becomes more elastic as a result of entry. We show that the seller-side competition tends to dominate for a fairly wide class of distribution functions for users’ valuations, so that more platform competition
(i.e. entry) increases buyer fees and decreases seller fees when buyers are multihoming.

To unify the extreme cases of pure singlehoming and pure multihoming buyers, we extend our analysis to a more general environment of partial-multihoming buyers, whereby some fraction of buyers are allowed to multihome while the remaining buyers cannot. In this case, our first main result, that holding the number of platforms fixed, an increase in the fraction of multihoming buyers decreases the total fee and shifts the fee structure in favor of sellers, continues to hold. With stronger distributional assumptions on users’ valuations, our second main result, that platform entry shifts the fee structure in favor of buyers (sellers) if the fraction of multihoming buyers is sufficiently low (high), also continues to hold. This extension strengthens the empirical relevance of our findings given that in practice some buyers tend to singlehome and others multihome.

Our theoretical results are limited to the effect on platform fees. To provide some welfare implications we calibrate the parameters in our model using real world data from ride-hailing services in the United States. Based on the calibrated model, we find that platform entry: (i) decreases fares and increases rider surplus, (ii) decreases driver per-trip earnings and surplus, and (iii) decreases platform profit and increases total welfare. Meanwhile, an increase in the extent of multihoming by riders: (i) increases fares and decreases rider surplus, (ii) increases driver per-trip earning and surplus, and (iii) decreases platform profit and increases total welfare. For the scenario considered, these results show that the side which faces an increase in platform fee also ends up with a reduction in user surplus, illustrating that our theoretical results on platform fees can also be relevant for understanding how surpluses change.

Our findings echo the general view in the two-sided market literature that one-sided logic may be misleading in making inferences in two-sided markets (e.g. Wright, 2004a). First, seemingly buyer-friendly measures that facilitate multihoming and switching across platforms, such as the growing popularity of metasearch aggregators, may not necessarily benefit buyers once the effect on the seller side is taken into account. Second, in a two-sided market context, high fees charged on one of the user side (either buyers or sellers) do not necessarily imply a lack of competition. Indeed, our analysis reveals the reverse possibility that platform competition drives up the fees (on one side). For example, an increase in platform competition can potentially result in higher fares when a sufficiently large fraction of riders are using metasearch aggregators to multihome across ride-hailing services.

The rest of the paper proceeds as follows. Section 1.1 surveys the relevant literature. Section 2 lays out the model of buyer-singlehoming and buyer-multihoming. Section 3 analyzes and compares these two scenarios. Section 4 investigates the impact of platform entry. Section 5 extends our analysis to allow for partial-multihoming buyers. Section 6 interprets our results in the case of payment cards and ride hailing, and calibrates the
model based on data from ride-hailing services. Finally, Section 7 concludes. All proofs and omitted derivations are relegated to the Appendix.

1.1 Relevant literature

The literature on two-sided markets starts with the seminal papers by Caillaud and Jullien (2003), Rochet and Tirole (2003, 2006), and Armstrong (2006), which provide a basic foundation for studying pricing schemes by monopoly and duopoly platforms.\(^1\) In developing and investigating a model of oligopolistic platform competition, our study relates closely to the recent contribution by Tan and Zhou (2018) that presents a model of oligopolistic multi-sided platform competition rooted in the membership pricing model of Armstrong (2006). They provide important insights on the impact of platform entry and on the extent of excessive or insufficient platform entry.\(^2\) However, their framework does not consider heterogeneity in interaction benefits, transaction fees, or multihoming by users, which are the focus of our setting.

As mentioned in the Introduction, our result on how buyer-multihoming shifts the fee structure in favor of buyers resembles the classic “competitive bottleneck” result obtained by Armstrong (2006) and Armstrong and Wright (2007), and recently revisited by Belleflamme and Peitz (2019). These studies typically start with a configuration of two-sided singlehoming and show that multihoming on one side leads to a competitive bottleneck, whereby platforms no longer need to compete for the multihoming side due to the monopoly power over providing exclusive access to each (singlehoming) user on the other side. Thus, in these studies, buyer-side multihoming would shift the fee structure in favor of sellers by shutting down competition on the buyer side. In contrast, we specify that sellers are always multihoming, and show that allowing for buyer-side multihoming still shifts the fee structure in favor of sellers, but for a different reason: by restoring competition on the previously monopolistic seller side, without directly influencing the existing competition on the buyer side. To this end, one can interpret our result as saying, in a transaction fee environment, that buyer-side multihoming “removes” the competitive bottleneck initially faced by the seller side.\(^3\)

\(^1\)Subsequent developments in the two-sided market literature extend the canonical two-sided framework in various directions. Among others, Weyl (2010) provides a more general model of a monopoly two-sided platform and examines the source of welfare distortions in platform pricing; Hagiu (2006) considers platform pricing and commitment issues when two sides of the market do not participate simultaneously; White and Weyl (2016) consider general nonlinear tariffs that are conditional on participation decisions of customers on all platforms; Jullien and Pavan (2019) consider platform pricing under dispersed information; Karle et al. (forthcoming) explore how the phenomenon of platform market tipping relates to the presence of seller competition on platforms.

\(^2\)In addition, their model allows for an arbitrary number of user sides with a very general form of cross-side and within-side externalities.

\(^3\)In a slightly different vein, Bakos and Halaburda (2019) compare two-sided multihoming with two-sided singlehoming in Armstrong’s framework, showing that two-sided multihoming can eliminate platforms’ incentive to cross-subsidize. However, they do not compare the resulting fee levels and fee struc-
While we aim to provide insights on various general forms of two-sided markets, our main illustrative examples throughout are the payment card market and ride-hailing services. This relates our paper to the long-standing theoretical literature on payment card markets (Rochet and Tirole, 2002; Wright, 2004b, Guthrie and Wright, 2007; and Bedre-Defolie and Calvano, 2013). With the exception of Guthrie and Wright (2007), most of these contributions focus on monopoly card platforms and explore the policy issue of determining the socially-optimal interchange fees (that is, how a social planner should optimally allocate the burden of credit card fees across consumers and merchants), and how equilibrium interchange fees may fail to be socially-optimal. Guthrie and Wright consider a model of duopolistic payment card platforms, and show that platform competition increases interchange fees if cardholders are singlehoming but decreases interchange fees if cardholders are multihoming. Given this, our result — that buyer-multihoming can reverse the impact of platform entry — can be seen as generalizing Guthrie and Wright’s insight to a broader two-sided market environment beyond the specific context of payment card platforms, and to allow for more than two platforms.

At a more general level, our analysis on the impact of user multihoming behaviour and its interaction with platform entry relates to several recent papers in the media literature that investigate similar issues (Ambrus et al. 2016; Athey et al. 2018; Anderson et al. 2019). Ambrus et al. and Athey et al. show that multihoming by media consumers can either increase or decrease the equilibrium number of ads that platforms admit, depending on the correlation of consumers preference and the extent to which advertisements generate negative externalities on consumers. Anderson et al. consider a model of multihoming media consumption based on the Salop (1979) circular city model, deriving the interesting property of “incremental value pricing” whereby platform entry has no effect on consumers but harms advertisers. While these results bear some resemblance to ours, the market they analyze and the underlying economic reasoning behind their results is quite different.

2 Model setup

There is a set of $N = \{1, ..., n\}$, $n \geq 1$ platforms which compete for a continuum of buyers and a continuum of sellers, both of measure one. Buyers and sellers wish to “interact” or “transact” with each other to create economic value. If we consider any buyer/seller pair, then we can assume without loss of generality that each such pair corresponds to

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4 Most of the existing theoretical studies on ride-hailing services focus on the aspect of dynamic pricing by monopoly platforms (Castillo et al., 2017). Of particular relevance is the recent investigation by Bryan and Gans (2019), which consider how the multihoming behaviour by users (riders and drivers) affect pricing by duopolistic platforms.
one transaction. Such a transaction can occur directly or through one of the platforms. The transaction can occur through platforms only if there exists at least one platform that both sides join and are willing to trade on.

**Buyers.** Platforms are heterogeneous from buyers' perspective. Each buyer obtains a draw of gross per-transaction surplus from interacting with each seller through platform \( i \), which is denoted as \( \epsilon_i \) for \( i \in N \). We adopt the standard discrete choice formulation by assuming that all \( \epsilon_i \in [\xi, \bar{\epsilon}] \) (where \( \xi \geq -\infty \) and \( \bar{\epsilon} \leq \infty \)) are identically and independently drawn across buyers and platforms from a common distribution \( F \) with log-concave density \( f \). We use \( \epsilon_0 \in [\xi, \bar{\epsilon}] \) to represent each buyer's draw of surplus from the outside option of transacting with a given seller directly, which is identically and independently drawn across buyers from the distribution \( F_0 \) with log-concave density \( f_0 \). The cost of buyers' outside option is \( p_0 \), which we normalize to zero without loss of generality.

**Sellers.** Each seller obtains a draw of per-transaction surplus, which is denoted \( v \), while the seller surplus for no transaction on any platform is normalized to zero. Following Rochet and Tirole (2003), we assume that seller surpluses do not vary across platforms. Specifically, we assume \( v \in [\underline{v}, \bar{v}] \) (where \( \underline{v} \geq -\infty \) and \( \bar{v} \leq \infty \)) is identically and independently drawn across sellers from the distribution \( G \) with density function \( g \), in which \( 1 - G \) is strictly log-concave. Finally, we also assume that \( \epsilon_0, \epsilon_i \) and \( v \) are independently drawn. The modelling choices for buyers and sellers capture the fact that in many two-sided markets, sellers view competing platform as more or less homogenous, while buyers usually have idiosyncratic preferences for using particular platforms over others.

**Platforms.** For each transaction facilitated, platform \( i \in N \) charges fees \( p_i = (p^b_i, p^s_i) \) to buyers and sellers. We allow negative transaction fees, as is commonly seen in certain markets such as payment cards (negative buyer fees in the form of rewards) and ride-hailing apps (negative seller fees in the form of a payment per ride). In accordance with our motivating examples, we assume that sellers observe both buyer fees and seller fees set by platforms, while buyers observe only buyer fees but not seller fees.\(^5\) We specify that buyers hold passive beliefs (Hart and Tirole, 1990) on the unobserved seller fees, meaning buyers believe these fees are equal to their equilibrium levels even if they observe an off-equilibrium buyer fee. Facilitating each transaction involves marginal cost \( c \), which is assumed to be constant and symmetric across all platforms. We focus on the transactional aspect of platforms and abstract from any participation benefits (or costs)\(^7\)

\(^5\)As pointed out by Janssen and Shelegia (2015), in practice the vertical arrangement between sellers and platforms are typically confidential, meaning that in various applications it may not be realistic to assume that buyers are informed of the seller fees set by platforms. For example, the exact commission rate set by ride-sharing platform for drivers is often unknown to riders. Similarly, the merchant fees collected by payment card platforms are also not typically observed by consumers. Hagiu and Halaburda (2014) and Belleflamme and Peitz (forthcoming) analyze the implications of this informational assumption for pricing in two-sided markets.
and fees. Therefore, platform \( i \)'s profit is written as

\[
\Pi_i(p_i; p_{-i}) = \left( p_i^b + p_i^s - c \right) Q_i(p_i; p_{-i}),
\]

where \( p_{-i} \) is the fees set by all other platforms excluding \( i \) while \( Q_i \) is the total volume of transactions facilitated by platform \( i \), which will be determined in Section 3.

**Multihoming.** Buyers choose which platform(s) to join, and after joining, choose a channel to transact with each seller (i.e. through one of the platforms they have joined or directly). On the other hand, sellers only choose which platform(s) to join. Sellers are always allowed to multihome (join multiple platforms). We consider two possible scenarios for buyers with respect to multihoming: (i) buyers are restricted to singlehome (they cannot join more than one platform); (ii) buyers always multihome on all platforms. Note if both sides multihome, the choice of which of these platform to use for a transaction is a-priori indeterminate. Following Rochet and Tirole (2003) and our motivating examples, we assume that, whenever a seller is available on multiple platforms, the buyer chooses the platform on which the transaction takes place.\(^6\)

**Timing and equilibrium.** The timing of the game is summarized as follows: (1) \( n \) platforms simultaneously set their transaction fees with platform \( i \)'s fee being \( p_i = (p_i^b, p_i^s) \); (2) Given the fee profile \( p = (p_1, ..., p_n) \), buyers and sellers observe their realized per-transaction surplus and simultaneously decide which platform(s) to join.\(^7\) Buyers do not observe the fees set on the seller side; (3) Buyers choose the platform(s) on which their transactions takes place. Our equilibrium concept is pure-strategy perfect Bayesian equilibrium (PBE) with symmetric fees, whereby all platforms set the same fees. As a tie-breaking rule, we assume that, whenever a user is indifferent between joining and not joining a platform, she breaks tie in favor of joining.

**Discussion.** While we have exogenously specified buyers’ multihoming behaviour, one can easily micro-found our specification by alternatively assuming that buyers obtain some stand-alone participation benefit from joining at least one platform and then incur a cost (or benefit) \( \psi \) for each additional platform joined. The case of singlehoming buyers arises endogenously when \( \psi > 0 \). Specifically, provided that buyers expect each seller either multihomes on all platforms or joins no platform (which we will show to be true in equilibrium), a buyer does not expect to gain additional access to sellers by joining more than one platform. Given that additional participation by buyers is costly and buyers cannot observe any deviation in the fees set to sellers (or sellers joining decisions) when deciding which platform(s) to join, a buyer will only join the platform that gives

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\(^6\)For example, cardholders choose the payment card among those accepted by a merchant to use for payment; buyers choose the e-commerce marketplace to purchase products from sellers; consumers choose the booking platform to use to book a hotel; riders choose the ride-hailing app to use to get drivers.

\(^7\)Note that buyers only have to decide which platform to join in the case of singlehoming.
the highest per-transaction surplus $\epsilon_i - p_i^b$. The case of multihoming buyers then arises when $\psi \leq 0$ (i.e. there is some stand-alone benefit from joining additional platforms). Given (i) buyers get to choose the channel to make their transactions on and (ii) buyers obtain a stand-alone benefit from joining each platform, it will be a weakly dominant strategy for each buyer to join all platforms (and strictly dominant if $\psi < 0$) regardless of the observed $p_i^b$ and their draw of $\epsilon_i$. Our tie-breaking rule then implies all buyers would multihome on all platforms in equilibrium in this case.

**Illustrative examples.** To illustrate the model setup, consider payment cards and ride-hailing services. With payment cards, each buyer wants to purchase a bundle of goods from each different merchant (or a fixed proportion of them). A “transaction” through a platform if a buyer pays by cards instead of using another payment instrument (say, cash). Therefore, terms $\epsilon_i - \epsilon_0$ and $v$ correspond to differences in utility of the buyer and merchant respectively when the buyer pays by card $i$ rather than cash. In ride-hailing platforms, each rider wants to order a driver (and car) to travel to their desired location. A “transaction” through a platform takes place if and only if a rider orders for a ride using the ride-hailing app instead of using some alternative form of transport (e.g. a conventional taxi). Therefore, $\epsilon_i - \epsilon_0$ corresponds to the difference in utility for the rider when she uses the ride-hailing app $i$ instead of the conventional taxi. Likewise, $v$ captures differences in utility of the driver between driving and idling, which is typically negative due to the effort spent driving (recall that $v$ can be negative). Other possible examples that fit our setting include hotel booking websites and food delivery platforms.

### 3 Equilibrium analysis

In this section, we first analyze the scenario in which buyers have to singlehome (Section 3.1), and then the scenario in which buyers are allowed to multihome (Section 3.2). After that, we compare and contrast the two scenarios to demonstrate the implications of buyer multihoming while holding fixed the number of platforms (Section 3.3).

To facilitate our subsequent exposition, it is useful to define the function

$$B_i(\Theta)(p_i^b, p_{-i}^b) \equiv \Pr \left( \epsilon_i - p_i^b \geq \max_{j \in \Theta} \{ \epsilon_j - p_j^b, \epsilon_0 \} \right) \text{ for any } \Theta \subseteq N,$$

which can be seen as the mass of buyers who obtain the highest net surplus from using platform $i$ for transaction, among the set of alternatives $\Theta$ and the outside option. Note that (1) does not necessarily equal to the mass of buyers who join $i$.  

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8Alternatively, we could allow the first participation to also cost $\psi > 0$, in which case a buyer joins at least one platform if $\max_i \{ (\epsilon_i - p_i^b - \epsilon_0) S_i \} \geq \psi$. In the symmetric equilibrium where $S_i = S$ is the same across all platforms $i \in N$, the analysis and results remain the same after replacing the value of the buyer’s outside option with $\epsilon_0 + \frac{\psi}{S}$. 

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3.1 Singlehoming buyers

In this scenario, buyers cannot join more than one platform. If a buyer joins platform $i$, she gets the option of using the platform to transact with all sellers that are available on the platform. The buyer’s total utility from participating and being able to transact with sellers through platform $i$ is

$$U_i^b = \max \{ \epsilon_i - p_i^b, \epsilon_0 \} S_i + \epsilon_0 (1 - S_i),$$

(2)

where $S_i = S_i(p_i, p_{-i})$ denotes the number of sellers on platform $i$ for any given fee profile $(p_i, p_{-i})$ set by platform $i$ and platforms other than $i$. For each seller on platform $i$, the buyer uses platform $i$ to transact if and only if $\epsilon_i - p_i^b \geq \epsilon_0$; for each seller not on platform $i$, the buyer can only transact directly. Likewise, for a seller of type $v$ that has joined a set of platforms $\Theta^v$ (which can be empty), its total utility is the summation over the product of net transaction surplus and the number of transactions on each platform:

$$U_s(\Theta^v) = \sum_{i \in \Theta^v} (v - s_i^v) B_i,$$

(3)

where $B_i = B_i(p_i, p_{-i})$ denotes, for any given fee profile, the mass of buyers that actually use platform $i$ to transact with each seller.

We denote the symmetric fee equilibrium under singlehoming buyers as $\tilde{p} = (\tilde{p}_b, \tilde{p}_s)$. To determine the equilibrium condition for $\tilde{p}$, we first determine the demand and profit of a deviating platform $i$, that sets $p_i = (p_i^b, p_i^s) \neq \tilde{p}$ while all the remaining platforms set $\tilde{p}$.

Consider first seller participation. Given that sellers are free to multihome and that buyers singlehome, it is easy to see that each seller cares only about the net surplus she gains from each transaction generated from the platforms, and so she will join all platforms $i$ that satisfies $v \geq p_i^s$. This joining decision is independent of what happens to the buyer side. Thus the number of sellers joining platform $i$ (or seller quasi-demand) is

$$S_i(p_i, \tilde{p}) = 1 - G(p_i^s).$$

(4)

Given that each buyer can only join a single platform, a buyer will join the platform that yields the highest expected utility, i.e. $U_i^b \geq \max_{j \in N} U_j^b$. Since buyers do not observe seller fees, they take $p_i^s$ as fixed at the equilibrium level $\tilde{p}_s$ (passive beliefs), which is the same across all platforms. Given this and the result in (4), buyers expect the same set of sellers on each platform. Consequently, from (2) the condition $U_i^b \geq \max_{j \in N} U_j^b$ can be

\footnote{Here we implicitly assume that a buyer joins at least one platform even when the realization of $\epsilon_0$ is so high such that the buyer knows she will only use the direct channel for transactions. The assumption simplifies the exposition but does not otherwise affect the analysis and results.}
rewritten as $\epsilon_i - p^b_i \geq \max_{j \in N} \{\epsilon_j - \tilde{p}^b\}$. Therefore, a buyer joins platform $i$ if and only if $\epsilon_i - p^b_i \geq \max_{j \in N} \{\epsilon_j - \tilde{p}^b\}$. After joining platform $i$, the buyer uses it for a transaction (with each seller) if $\epsilon_i - p^b_i > \epsilon_0$. Therefore, the total mass of buyers using platform $i$ for transactions (or buyer quasi-demand) is

$$B_i(p_i, \tilde{p}) = B_i^{(N)}(p_i, \tilde{p}) = \Pr \left( \epsilon_i - p^b_i \geq \max_{j \in N} \{\epsilon_j - \tilde{p}^b, \epsilon_0\} \right) \equiv \int_\epsilon^\tilde{\epsilon} \int_\epsilon^\tilde{\epsilon} \left(1 - F \left( \max \{\epsilon - \tilde{p}^b, \epsilon_0\} + p^b_i \right)\right) dF(\epsilon)^{n-1} dF_0(\epsilon_0).$$

Recall that the profit of the deviating platform $i$ is

$$\Pi_i = (p^b_i + p^s_i - c) Q_i(p^b_i, p^s_i; \tilde{p})$$

where $Q_i(p^b_i, p^s_i; \tilde{p})$ denotes the volume (or proportion) of transactions facilitated by platform $i$. Given that each buyer-seller pair corresponds to one transaction, $Q_i$ is equal to the product of the buyers’ and sellers’ quasi-demands; i.e.,

$$Q_i(p^b_i, p^s_i; \tilde{p}) = (1 - G(p^s_i)) B_i^{(N)}(p^b_i, \tilde{p}).$$

To proceed, we define the buyer-side inverse semi-elasticity function: for any arbitrarily given (symmetric) equilibrium buyer fee $p^b$,

$$X(p^b; n) \equiv \frac{B_i^{(N)}}{\partial B_i^{(N)} / \partial p^b_i}_{p^b_i=p^s_i=p^b} = \int_\epsilon^\tilde{\epsilon} \int_\epsilon^\tilde{\epsilon} \left(1 - F \left( \max \{\epsilon, \epsilon_0 + p^b\} \right)\right) dF(\epsilon)^{n-1} dF_0(\epsilon_0).$$

Here, $X(p^b; n)$ is a measure of the extent of platform differentiation from buyers’ perspective. The numerator is the equilibrium number of buyers that transact using platform $i$, while the denominator is the total mass of marginal buyers. Thus, $X(p^b; n)$ captures the competitive markup a platform can extract from buyers in a given equilibrium with buyer fee $p^b$ and $n$ competing platforms. The following lemma shows two useful properties of $X(p^b; n)$.

**Lemma 1** Buyer-side inverse semi-elasticity $X(p^b; n)$ defined in (5) is decreasing in $n$ and $p^b$.

Lemma 1 was shown by Zhou (2017) when the outside option is non-random. Here we show that the result is true even when the outside option is random. The first part of Lemma 1 reflects the standard intuition that platforms are more substitutable for buyers
when buyers have more platforms to choose from, while the second part of the lemma is a consequence of the log-concavity assumption on the density functions $f$ and $f_0$.

### 3.1.1 Equilibrium fees with singlehoming buyers

It is easily verified that our assumptions on the distribution functions $F$, $F_0$, and $G$ imply that the platform’s profit function is log-concave with respect to $(\tilde{p}_b, \tilde{p}_s)$. This implies that the profit-maximizing fees can be characterized by the usual first-order conditions.

We can calculate the standard demand derivative terms at the symmetric equilibrium as:

$$-\frac{Q_i(\tilde{p}; \tilde{p})}{dQ_i(\tilde{p}; \tilde{p})/dp_j} = \frac{1 - G(\tilde{p}^s)}{g(\tilde{p}^s)} \quad \text{and} \quad -\frac{Q_i(\tilde{p}; \tilde{p})}{dQ_i(\tilde{p}; \tilde{p})/dp_k} = X(\tilde{p}^b; n).$$

Then, the equilibrium fees on both sides can be characterized as follows.

**Proposition 1** (Buyer-singlehoming equilibrium) A pure symmetric pricing equilibrium is characterized by all platforms setting $\tilde{p} = (\tilde{p}_b, \tilde{p}_s)$ that solves

$$\tilde{p}_b + \tilde{p}_s - c = X(\tilde{p}^b; n) = \frac{1 - G(\tilde{p}^s)}{g(\tilde{p}^s)}. \quad (6)$$

There exists a unique solution $\tilde{p}$ to (6).

The equilibrium condition (6) can be intuitively understood as the intersection of equilibrium conditions for competition in the buyer-side and seller-side markets. To see this, we first denote $P^b(p^s)$ as a function defined implicitly by

$$P^b = (c - p^s) + X(P^b; n).$$

It represents the standard oligopoly pricing equilibrium (in setting the buyer fee) with competitive markup $X$, except that per-transaction cost is replaced by the effective cost $c - p^s$ because the platform’s marginal cost $c$ is compensated by the seller fee collected. If we take as given the (common) seller-side fee, then $P^b(p^s)$ can be understood as a curve that maps out the (one-sided) equilibrium buyer fee for each arbitrarily given seller fee. Likewise, we can denote $P^s(p^b)$ as a function defined implicitly by

$$P^s = (c - p^b) + \frac{1 - G(P^s)}{g(P^s)}.$$

It is the sum of the effective cost discussed earlier, plus the standard monopoly pricing (in setting seller fee) with markup $\frac{1 - G(P^s)}{g(P^s)}$. The monopoly markup reflects that, with singlehoming buyers, any seller that quits platform $i$ will lose access to all buyers who are associated with platform $i$. Therefore, even though there are multiple platforms,
each platform still exercises monopoly power over providing access to their singlehoming buyers for the multihoming sellers and hence extracts the full monopolist markup on the seller side. If we take as given the (common) buyer-side fee, then \( P^s(p^b) \) is a curve that maps out the (one-sided) equilibrium seller fee for each arbitrarily given buyer fee.

Then, the equilibrium (6) is simply the intersection of the \( P^s(p^b) \) and \( P^b(p^s) \) curves, which each represents the equilibrium condition on each side of the market. Moreover, our assumptions on demand distributions imply log-concave quasi-demands of buyers and sellers, so that it is easily checked that both curves have negative slopes with gradient less than unity. This implies a unique intersection point \((\hat{p}^b, \hat{p}^s)\), and so a unique symmetric equilibrium.

Equilibrium (6) is similar to the pricing formula obtained by Rochet and Tirole (2003, Proposition 3) in the case of singlehoming buyers. Our result shows that their pricing formula generalizes to oligopolistic platforms and relates the formula to the underlying distribution of buyers’ and sellers’ valuations over interaction benefits rather than expressing the formula in terms of elasticities of reduced-form demand functions. In the special case of \( n = 1 \), (6) becomes the familiar monopoly pricing rule extended to the two-sided market setting:

\[
\hat{p}^b + \hat{p}^s - c = \frac{1 - F(\hat{p}^b)}{f(\hat{p}^b)} = \frac{1 - G(\hat{p}^s)}{g(\hat{p}^s)}.
\]

The market configuration of singlehoming buyers and multihoming sellers analyzed here is similar to the competitive bottleneck setup considered by Armstrong (2006) and Armstrong and Wright (2007). Even though those papers focused on user membership fees instead of the transaction fees considered here, they obtained similar equilibrium outcomes as in (6) in which platforms compete exclusively for buyer participation and exercise monopoly power over sellers. Nonetheless, as noted by Belleflamme and Peitz (2019), this logic does not necessarily imply that sellers will face a higher fee than buyers (in our model and theirs). In particular, if the seller transaction surplus is very low or even negative (as is the case in the ride-hailing application), then the seller fee can possibly be lower than the buyer fee in equilibrium even though sellers are the only users that multihome.

### 3.2 Multihoming buyers

Note that if \( n = 1 \) then the cases of multihoming buyers and singlehoming buyers trivially coincide. For this reason, we focus on \( n \geq 2 \) in what follows. We denote the symmetric fee equilibrium under multihoming buyers as \( \hat{\mathbf{p}} = (\hat{p}^b, \hat{p}^s) \). We consider a platform \( i \) which deviates from the equilibrium and sets \( \mathbf{p}_i = (p^b_i, p^s_i) \neq \hat{\mathbf{p}} \). Whenever convenient, we use \( \mathbf{N}_{-i} \equiv \mathbf{N} \setminus \{i\} \) to denote the set of all platforms excluding \( i \).
In this scenario, buyers are assumed to join all \( n \) platforms. When a buyer wishes to transact with a type-\( v \) seller that has joined a set of platforms \( \Theta_v \), the buyer is willing to use platform \( i \in \Theta_v \) if \( \epsilon_i - p^b_i > \max_{j \in \Theta_v} \{ \epsilon_j - p^b_j, \epsilon_0 \} \). Therefore, the mass of buyers who are willing to use \( i \) to transact with seller \( v \) is exactly

\[
\Pr \left( \epsilon_i - p^b_i \geq \max_{j \in \Theta_v} \{ \epsilon_j - p^b_j, \epsilon_0 \} \right) = B_i^{(\Theta_v)}(p^b_i, \hat{p}^s).
\]  

(7)

From (7), note that a seller, by selecting the platform(s) she wants to join, can restrict the set of platforms that buyers can choose from to make their transactions.\(^{10}\)

Sellers’ participation profile generally depends on how the seller fee set by platform \( i \) compares to other platforms. To derive the equilibrium fees, it suffices for us to focus on an upward deviation by platform \( i \), that is, \( p^s_i \geq \hat{p}^s \). We derive in detail the case of a downward deviation in the Appendix.

**Lemma 2** Suppose \( p^s_i \geq \hat{p}^s \). Define

\[
\hat{v} = \hat{v}(p_i, \hat{p}) \equiv \frac{p^s_i B_i^{(N)} - \hat{p}^s \sum_{j \in N-i} \left( B_j^{(N-i)} - B_j^{(N)} \right)}{B_i^{(N)} - \sum_{j \in N-i} \left( B_j^{(N-i)} - B_j^{(N)} \right)}.
\]

A seller of type \( v \) joins no platform if \( v < \hat{p}^s \), joins all platforms \( j \neq i \) if \( \hat{p}^s \leq v < \hat{v} \), and joins all platforms including \( i \) if \( v \geq \hat{v} \). Moreover, \( \hat{v} = \hat{p}^s \) when \( p^s_i = \hat{p}^s \).

Lemma 2 comes from a usual indifference condition. Given \( p^s_i \geq \hat{p}^s \) and that all platforms \( j \neq i \) set the lowest seller fee \( \hat{p}^s \), it is easy to see that a seller either joins no platforms, joins all platforms except \( i \) (i.e. \( N-i \)), or joins all platforms including \( i \) (i.e. \( N \)). The net surplus from joining all platforms \( j \in N-i \) for a seller of type-\( v \) is

\[
\sum_{j \in N-i} (v - \hat{p}^s) B_j^{(N-i)}.
\]  

(8)

The seller will join all platforms \( j \in N-i \) as long as \( v \geq \hat{p}^s \) so that (8) is positive. Meanwhile the net surplus from joining all platforms including \( i \) is

\[
(v - p^s_i) B_i^{(N)} + \sum_{j \in N-i} (v - \hat{p}^s) B_j^{(N)}.
\]  

(9)

Equating (8) and (9) yields the cutoff \( \hat{v} \) in Lemma 2.

The comparison between (8) and (9) reflects a key trade-off faced by a typical seller in the participation decision. By joining (the more expensive) platform \( i \), the seller gains

\(^{10}\)Similar to the utility expressions in (2) and (3) for the case of singlehoming buyers, we can write down a multihoming buyer’s total utility as: \( U^b = \int_{\Theta^b} \max_{i \in \Theta^b} \{ \epsilon_i - p^b_i, \epsilon_0 \} dG(v) \). Likewise the total utility of a type-\( v \) seller is \( U^s(\Theta^v) = \sum_{i \in \Theta^v} (v - p^s_i) B_i^{(\Theta^v)} \).
access to additional buyers who would have used the outside option if platform i were not available. At the same time, the seller diverts some of its existing buyers away from transacting using other platforms \( j \in N_{-i} \) that charge lower seller fees. In particular, by joining platform i, the seller is obliged to pay platform i’s higher seller fee \( p^s_i > \hat{p}^s \) for these diverted transactions. For sellers with relatively low \( v \), the gain from obtaining extra buyers is less than the loss associated with paying higher seller fees, so that only sellers with sufficiently high \( v \) will join platform i.

Recall \( \Pi_i = (p^b_i + p^s_i - c) Q_i(p^b_i, p^s_i; \hat{p}) \), where the volume of transactions facilitated by platform i is

\[
Q_i(p^b_i, p^s_i; \hat{p}) = (1 - G(\hat{v})) B^{(N)}(p^b_i, \hat{p}) \quad \text{if} \quad p^s_i \geq \hat{p}^s
\]

by Lemma 2. To proceed, we define the buyer loyalty index function: for any arbitrarily given (symmetric) equilibrium buyer fee \( p^b \),

\[
\sigma(p^b; n) = \frac{\sum_{j \in N} B^{(N)}_j - \sum_{j \in N_{-i}} B^{(N_{-i})}_j}{B^{(N)}_i | p^t_i = p^b_j = \hat{p}}
\]

(11)

In (11), the numerator represents the (absolute) change in “total market coverage” when a platform i is no longer available, while the denominator is the number of existing buyers on platform i. Hence, the buyer loyalty index \( \sigma \in [0, 1] \) can be understood as the proportion of buyers who switch to use the direct channel for transactions when one of the platforms, say i, ceases to be available (among those who were transacting through platform i). If \( \sigma \) is close to zero, it means that platforms are highly substitutable to buyers, so that many existing buyers on platform i simply shift to the remaining platforms \( j \neq i \) when platform i is not available. If \( \sigma \) is close to one, it means that few of the existing buyers on platform i will shift to other platforms when platform i is not available so that buyers are very “loyal” to platform i. Note that \( \sigma \leq 1 \) because \( B^{(N)}_j \leq B^{(N_{-i})}_j \).

The loyalty index \( \sigma(\hat{p}^b) \) plays a significant role in understanding how sellers react to changes in seller fees set by platforms. We first note from Lemma 2 that

\[
\frac{d\hat{v}}{dp^i} | p^t_i = p^b_j = \hat{p} = \frac{1}{\sigma(p^b; n)},
\]

so that \( 1/\sigma \) measures the sensitivity of seller participation to increases in \( p^t_i \). Recall that with singlehoming buyers, each platform has monopoly power over the seller side such that only sellers with \( v < p^s_i \) quit platform i. When buyers are multihoming, we see from (7) that a seller can restrict buyers who want to transact with it to choose among the set
of platforms Θ. If platform $i$ sets a higher seller fee than its rivals, sellers with $v \geq p^s_i$ may find it profitable to quit platform $i$ in order to divert buyers to use other platforms with lower seller fees.$^{11}$ From the point of view of the sellers, if buyers are highly loyal to platform $i$ (so that $1/\sigma$ is low) then the profitability of such diversion is low, so that sellers are less likely to quit platform $i$ following an increase in $p^s_i$. Conversely, if buyers are less loyal to platform $i$ (so that $1/\sigma$ is high) then sellers are more likely to quit platform $i$ following an increase in $p^s_i$. In sum, platforms no longer exercise full monopoly power over sellers when buyers are multihoming, resulting in a demand system $Q_i\left(p^b_i, p^s_i; \hat{p}\right)$ that is more price-elastic with respect to seller fees.

The following lemma shows two useful properties of $\sigma(p^b; n)$.

**Lemma 3** The buyer loyalty index $\sigma(p^b; n)$ defined in (11) is decreasing in $n$ and increasing in $p^b$.

The first part of Lemma 3 states that buyers become less loyal towards each platform when $n$ is higher, reflecting that platforms become more substitutable for buyers when buyers have more platforms to choose from. The second part of the lemma states that when transacting through platforms becomes less attractive for buyers relative to direct transactions (i.e. a higher $p^b$), buyers become more loyal — in the sense that when one of the platform ceases to be available, many existing buyers on this platform will shift to use the outside option for transactions instead of using other platforms.

### 3.2.1 Equilibrium with multihoming buyers

In what follows, we assume that the profit function $\Pi_i$ is quasi-concave in $(p^s_i, p^b_i)$. $^{12}$ In Section B of the Online Appendix, we show that a sufficient condition for quasi-concavity is to have $F$ and $F_0$ correspond to Gumbel distribution and $G$ to correspond to uniform distribution.$^{13}$ Then, the first-order conditions for optimal pricing is standard and the

$^{11}$Conversely, if platform $i$ undercuts its rivals with a lower seller fee, some sellers may find it profitable to quit the other platforms to divert buyers to platform $i$.

$^{12}$We note that our derivation thus far has focused on the case of an upward deviation $p_i^s \geq \hat{p}^s$. A similar but more technically involved analysis, which we relegate to the Appendix, shows that a similar derivation is applicable in the case of a downward derivation $p_i^s < \hat{p}^s$. In general, this leads to a piece-wise defined demand function $Q_i\left(p^b_i, p^s_i; \hat{p}\right)$ that takes different function forms depending on whether $p_i^s \geq \hat{p}^s$ or $p_i^s < \hat{p}^s$. In Section B of the Online Appendix, we verify that $Q_i\left(p^b_i, p^s_i; \hat{p}\right)$ is always continuous, and that platforms cannot profitably deviate from the equilibrium in (12) by slightly decreasing its seller fee $p_i^s < \hat{p}^s$.

$^{13}$Beyond these specific distributional assumptions, the functional form of $Q_i\left(p^b_i, p^s_i; \hat{p}\right)$ when $p^s_i < \hat{p}^s$ is complicated, and we could not obtain a more general result on the quasi-concavity of $\Pi_i$. We numerically check that the profit function is indeed quasi-concave over a wide range of parameter values and distribution functions suggesting that quasi-concavity of the profit function may indeed be a reasonable assumption.
usual demand derivative terms can be calculated as

\[- \frac{Q_i(\hat{p}; \hat{p})}{dQ_i(\hat{p}; \hat{p})/dp_i} = X(\hat{p}; n),\]

where \(X(.; n)\) is defined by (5), and

\[- \frac{Q_i(\hat{p}; \hat{p})}{dQ_i(\hat{p}; \hat{p})/dp_i} = \frac{1}{g(\hat{p}^s)} \left( \frac{d\hat{v}}{dp_i^s} \right)^{-1} = \frac{1}{g(\hat{p}^s)} \sigma(\hat{p}; n).\]

This leads to the following characterization of equilibrium fees on both sides.

**Proposition 2** (Buyer-multihoming equilibrium) A pure symmetric pricing equilibrium is characterized by all platforms setting \(\hat{p} = (\hat{p}^b, \hat{p}^s)\) that solves

\[\hat{p}^b + \hat{p}^s - c = X(\hat{p}^b; n) = \frac{1}{g(\hat{p}^s)} \sigma(\hat{p}; n). \tag{12}\]

Moreover, the solution \(\hat{p}\) to (12) is unique.

Similar to the discussion after Proposition 1, the equilibrium condition (12) can be understood as the intersection of two curves, corresponding to the equilibrium conditions in the buyer side and the seller side respectively. From (12), the curve for the buyer side \(P^b(p^s)\) continues to be defined by \(P^b = c - p^s - X(P^b; n)\), reflecting that buyer multihoming does not qualitatively alter the competition for the buyer side. This is because, even though platforms no longer compete for buyer participation when buyers multihome, platforms still need to compete for usage by buyers, given that buyers make the final choice of which platform to use for a transaction with a seller.

Meanwhile, the curve for the seller side \(\bar{P}^s(p^b)\) is now defined by

\[\bar{P}^s = (c - p^b) + \frac{1}{g(\bar{P}^s)} \sigma(p^b; n)\]

instead. The presence of the loyalty index term \(\sigma(p^b; n) \leq 1\) discounts the monopoly markup term by a factor that is less than one. This reflects that buyer-multihoming qualitatively alters the seller side by causing firms to compete for sellers. The intensity of this competition for sellers depends on how easily sellers can divert buyers to transact through different platforms, which in turns depends on the buyer loyalty index.

In contrast to its counterpart in the case of singlehoming buyers (Section 3.1), we note that \(\bar{P}^s(p^b)\) need not be a downward sloping curve in general because \(\sigma(p^b; n)\) increases with \(p^b\). Nonetheless, in the proof of Proposition 2, we show that \(\bar{P}^s(p^b)\) must be downward sloping with gradient less than unity whenever it intersects with the \(P^b(p^s)\) curve. Therefore, the two curve intersects as most once, so the equilibrium \((\hat{p}^b, \hat{p}^s)\) pinned down by (12) must be unique.
The equilibrium characterization in Proposition 2 nests the counterpart in the duopolistic model of Rochet and Tirole (2003, Proposition 3) as a special case if \( n = 2 \). A key distinction is that our micro-founded approach can relate the buyer loyalty index to the underlying distribution of buyers’ valuations, so that we can pin down how \( \sigma(p^b; n) \) changes with \( \tilde{p}^b \) and \( n \) as stated Lemma 1. This feature allows us to obtain a sharp comparative static result with respect to the effect of intensified platform competition on fees, which we explore in Section 4.

We also note that the equilibrium condition (12) is still applicable when \( n = 1 \) even though the analysis of the buyer-multihoming case assumed \( n \geq 2 \). When \( n = 1 \), from (11) we have \( \sigma = 1 \) so that equilibrium conditions (12) and (6) coincide, reflecting that the cases of singlehoming buyers and multihoming buyers coincide when there is a monopoly platform.

### 3.3 Singlehoming versus multihoming buyers

We now compare the buyer-singlehoming equilibrium in (6) and the buyer-multihoming equilibrium in (12). Recall that the only difference between (6) and (12) is the term \( \sigma \leq 1 \) that discounts the seller markup, so that the comparison of interest boils down to a comparative static exercise with respect to changing \( \sigma = 1 \) to \( \sigma < 1 \). This exercise can be approached graphically, as in Figure 1:

![Figure 1: Buyer-singlehoming equilibrium and buyer-multihoming equilibrium](image)

Starting with the case buyers are singlehoming, the equilibrium fees \((\tilde{p}^b, \tilde{p}^s)\) are given by the intersection of the \( P^b(p^s) \) and \( P^s(p^b) \) curves. When buyers multihome, we have \( \sigma < 1 \) so that the equilibrium seller fee, at each given buyer fee, falls. This fall reflects the intensified competition for the seller side, given that platforms can no longer exer-
cise their full monopoly power over sellers. In particular, given $\sigma < 1$, each seller now has the ability to quit the more expensive platforms to divert some buyers to transact through other cheaper platforms. Consequently, the corresponding curve for the seller side shifts downward from $P^s(p^b)$ to a new curve $\bar{P}^s(p^b)$, resulting in a new equilibrium with multihoming buyers, defined by $(\hat{p}^b, \hat{p}^s)$.

Notably, buyer multihoming has two effects: it reduces the equilibrium seller fee directly (by shifting the seller-side curve, as discussed in the previous paragraph), and it increases the equilibrium buyer fee indirectly (through the movement along the $P^b(p^s)$ curve). The latter effect is the well-known seesaw principle in the two-sided market literature (Rochet and Tirole, 2006) — any change that makes it conducive to have a lower fee on one side will call for a higher fee on the opposite side (and vice-versa). The economic intuition for the seesaw principle at work in our setting is as follow. When seller-side competition becomes intensified (due to buyer multihoming), the seller fee falls. A lower seller fee implies a higher effective marginal cost of servicing buyers $c - p^s$, meaning that transactions generated from attracting buyer participation become less valuable to the platforms. Platforms compete less intensely for buyer participation so that they now charge buyers more. Nonetheless, the log-concavity assumption on density function $f$ implies a passthrough rate lower than one, so that the gradient of $P^b(\cdot)$ curve is less than one (in absolute terms). Therefore, the increase in buyer fee is always smaller than the decrease in seller fee, and so the total fee $p^b + p^s$ must decrease overall.

To summarize our discussion above:

**Proposition 3 (Effect of buyer multihoming)** Compared to the equilibrium under buyer singlehoming, buyer multihoming decreases total fees ($\hat{p}^b + \hat{p}^s < \tilde{p}^b + \tilde{p}^s$), decreases seller fee ($\hat{p}^s < \tilde{p}^s$), and increases buyer fee ($\hat{p}^b > \tilde{p}^b$).

Proposition 3 is analogous to Proposition 5.3 of Rochet and Tirole (2003), but there are three important differences. First, their result focuses on competing associations (each that maximizes the volume of transactions) whereas our result considers proprietary platforms (that maximize profit). Second, their result is stated in terms of an exogenous increase in the loyalty index parameter while holding constant the elasticity of buyer-side quasi-demand. Our approach, by directly comparing the cases of singlehoming buyers and multihoming buyers does not impose specific requirements on how the loyalty index and the demand elasticity change. This approach allows us to link to the competitive

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14 The standard pass-through rate refers to how the equilibrium buyer fee changes when there is a one unit increase in the effective marginal cost (see e.g. Weyl and Farbinger, 2013). In our context here, a passthrough rate less than one means $\left|dP^b(p^s)/dp^b\right| \in [0, 1]$.

15 In their primary example of an extended linear Hotelling model, such an exogenous increase in loyalty index corresponds to an increase in the marginal transportation cost of buyers for distances in the noncompetitive hinterland of the rival platform while holding constant the transportation cost of all other segments of the Hotelling line.
bottleneck theory (Armstrong, 2006; Armstrong and Wright, 2007) in that buyer-side multihoming can be seen as “removing” the competitive bottleneck initially faced by the seller side in a per-transaction fee environment. Third, given the discrete choice microfoundation we have, our result does not rely on demand linearity and can accommodate an arbitrary number of platforms.

It is also useful to discuss the relation of Proposition 3 with the result obtained in the Hotelling duopoly platform model of Armstrong. Specifically, Belleflamme and Peitz (2019) show that in Armstrong’s model, moving from an environment of two-sided singlehoming to an environment in which one side multihomes (a competitive bottleneck environment) may sometimes lower the fee on the multihoming side even though each platform exercises monopoly power over these multihoming users. As pointed out by Belleflamme and Peitz, their somewhat counter-intuitive result is a consequence of a particular feature of the Hotelling model, in which a “firm” faces a more elastic demand from consumers when the consumers’ outside option is a constant (monopoly case) rather than the competitor’s offer (duopoly case). When this demand elasticity effect is sufficiently large such that the monopolistic fee is higher than the duopolistic fee, the side that shifts to multihoming can face a lower equilibrium fee as a result. Notably, this complication does not arise in our discrete choice-based framework, explaining why in our model buyer multihoming always increases the fee charged to buyers.

4 Impact of increased platform competition

In this section we explore how increased platform competition (i.e. entry) affects the platforms’ equilibrium total fee and fee structure, and in particular, how this interacts with whether buyers multihome or not.

4.1 Singlehoming buyers

To analyze how equilibrium prices change with platform entry, we utilize the approach developed in Section 3.3 by examining how the $P_b(\cdot)$ and $P_s(\cdot)$ curves change with a change in $n$.

The equilibrium with singlehoming buyers is characterized by (6). The only component that explicitly depends on $n$ is the buyer-side competitive markup $X = X(p^b; n)$, which decreases with $n$ from Lemma 1. Consequently, the buyer-side curve $P^b(p^s)$ depends on $n$, and we write $P^b(p^s; n)$ to make explicit this dependency on $n$. Meanwhile, the seller-side curve $P^s(p^b)$ is independent of $n$. This asymmetry reflects that platforms engage in oligopolistic competition in the buyer side, while they exercise monopoly power on the seller side (where $n$ has no direct impact).
As illustrated in Figure 2, when the number of platforms increases from $n_1$ to $n_2$, the equilibrium buyer fee decreases due to a downward shift in the buyer-side curve from the solid line $P^b(p^s; n_1)$ to the dotted line $P^b(p^s; n_2)$. Then, the equilibrium seller fee increases due to the movement along the $P^s(p^b)$ curve. The movement along the curve reflects the seesaw principle (as in the explanation for Figure 1), whereby a lower buyer fee implies a higher effective cost of servicing sellers $c - p^b$ so that platforms will have an incentive to charge sellers more. Therefore, with singlehoming buyers, the effect of higher $n$ on competitive markups shifts the fee structure in favor of buyers, in the sense that it induces a lower buyer fee and a higher seller fee. Finally, the log-concavity assumption on $1 - G$ implies the passthrough rate is less than one for $P^s(\cdot)$, so that the increase in seller fee is always smaller than the decrease in buyer fee. Consequently, the total fee $\tilde{p}^s + \tilde{p}^b$ must decrease with an increase in $n$. Formally, we have:

\textbf{Proposition 4} (Increased platform competition with buyer-singlehoming) In the equilibrium characterized by Proposition 1, an increase in $n$ (i.e. platform entry) decreases the total fee $\tilde{p}^s + \tilde{p}^b$, decreases the buyer fee $\tilde{p}^b$, and increases the seller fee $\tilde{p}^s$.

\subsection{Multihoming buyers}

The equilibrium fees with multihoming buyers are characterized by (12). The components that explicitly depend on $n$ are the competitive markup $X = X(p^b; n)$ and the buyer loyalty index $\sigma = \sigma(p^b; n)$. Therefore, both the $P^b(p^s)$ curve and the $\tilde{P}^s(p^b)$ curve now depend on $n$ so we can explicitly write $P^b(p^s; n)$ and $\tilde{P}^s(p^b; n)$.

We know from Lemma 3 that the buyer loyalty index $\sigma$ decreases with $n$, reflecting that platforms become more substitutable for buyers when there are more platforms. Hence, an increase in $n$ reduces buyer loyalty. Suppose, for the moment, we ignore the
effect of $n$ on the competitive markup $X$. Then, the reduced buyer loyalty from a higher $n$ means that a seller will be less-concerned about losing access to buyers when it quits a platform, and this effect implies platforms will enjoy a lower seller-side markup. All else being equal, this shifts the seller-side curve downward from the solid line $\bar{P}_s(p^b; n_1)$ to the dotted line $\bar{P}_s(p^b; n_2)$, as shown in Figure 3. The shift results in an immediate decrease in the equilibrium seller fee, and an indirect increase in the equilibrium buyer fee through the movement along the $P^b(p^s)$ curve. Consequently, we say that the effect of a higher $n$ on buyer loyalty shifts the fee structure in favor of sellers, in the sense that it induces a lower seller fee and a higher buyer fee.

![Figure 3: Reduced buyer loyalty due to platform competition ($n_1 < n_2$) ignoring shifts in $P^b(p^s)$ curve](image)

Once we take into account the effect of a higher $n$ on the competitive markup $X$, we see that with multihoming buyers, an increase in $n$ affects the equilibrium fees via two effects: reduced buyer markup (Lemma 1) and reduced buyer loyalty (Lemma 3). Hence, there is a simultaneous downward shift in both $P^b(p^s; n)$ and $\bar{P}_s(p^b; n)$ curves. An immediate impact of these two effects is a decrease in both the buyer-side and the seller-side markups that platforms earn in equilibrium, so that it is not surprising that the equilibrium total fee decreases with $n$ (see Proposition 5 below).

However, these two effects shift the fee structure in opposite directions — the reduced buyer markup favors buyers while the reduced buyer loyalty favors sellers. For this reason, the overall comparative statics on the fee structure will generally depend on the relative magnitude of these two effects. Nonetheless, we find that under weakly decreasing densities, the reduced buyer loyalty dominates, leading to the following formal result:

**Proposition 5** (Increased platform competition with buyer multihoming) In the equilibrium characterized by Proposition 2, an increase in $n$ (i.e. platform entry) decreases the total fee.
total fee $p^s + p^b$. Furthermore, an increase in $n$ decreases the seller fee $\hat{p}^s$ if the density $f$ is a weakly decreasing function, and increases buyer fee $\hat{p}^b$ if in addition the density $g$ is a weakly decreasing function.

The condition of decreasing densities, as stated in the second part of 5, is satisfied by some commonly used distributions such as the uniform distribution and the exponential distribution, and the generalized Pareto distribution (for certain range of parameter values). Indeed, the uniform distribution (linear demand) has often been used in the related literature (e.g. Rochet and Tirole, 2003; Armstrong, 2006) to obtain results. Moreover, we note that the stated conditions are sufficient and definitely not necessary. In particular, in Section C of the Online Appendix we consider the case in which $F$ and $F_0$ correspond to the same Gumbel distribution, whereby the buyer quasi-demand follows the standard multinomial logit form. Even though the density of Gumbel distribution is not always decreasing, we still find that Proposition 5 continues to hold with a less stringent condition on $g$.

### 4.3 Summary

To summarize our discussion above succinctly, we use the fee difference $p^b - p^s$ to represent the platform fee structure. A comparison between Propositions 4 and 5 leads to the following summary.

**Proposition 6** (Increased platform competition) An increase in $n$ (i.e. platform entry) always reduces the total fee. Moreover:

1. If buyers are singlehoming, an increase in $n$ shifts the fee structure in favor of buyers ($\hat{p}^b - \hat{p}^s$ decreases with $n$).

2. If buyers are multihoming and the densities $f$ and $g$ are weakly decreasing functions, an increase in $n$ shifts the fee structure in favor of sellers ($\hat{p}^b - \hat{p}^s$ increases with $n$).

Proposition 6 highlights the key finding of our paper: even though increased platform competition always reduces the total fee charged to the two sides, whether it shifts the fee structure in favor of buyers or sellers depends on whether buyers are singlehoming or multihoming. When buyers singlehome, platforms have monopoly power over providing access to their buyers for the multihoming sellers, so increased platform competition induces platforms to compete more intensely for buyers rather than for sellers. In contrast, when buyers multihome, platforms lose their monopoly power over sellers. In this case, increased platform competition induces platforms to compete more intensely for sellers. We discuss the economic implication of this result for specific markets in Section 6.
5 Extension: partial-multihoming buyers

In our benchmark setting, we have so far assumed that either all buyers singlehome or all buyers multihome. We now relax this restriction by assuming that an exogenous fraction $\lambda$ of buyers are multihoming while the remaining fraction $1 - \lambda$ of buyers are singlehoming.\(^\text{17}\) We show that all our main findings continue to hold under this extended model. To keep the exposition brief, we focus on presenting the main insights in this section and relegate further details and formal proofs of the propositions to Section D of the Online Appendix.

The derivation for this partial-multihoming model largely follows those of the full-multihoming model in Section 3.2. The notable difference is that the presence of some singlehoming buyers means sellers, whenever they quit one of the platforms, divert less buyers to other platforms for transactions. Sellers thus have less incentive to quit a deviating platform that charges higher seller fees than other platforms. Therefore, seller quasi-demand becomes less elastic from the point of view of platforms, allowing platforms to exercise a greater market power over sellers.

We can define the counterpart of the buyer loyalty index (11) for this environment:

\[
\sigma_\lambda(p^b; n) \equiv \lambda \int_{\bar{\epsilon}}^{\hat{\epsilon}} \frac{\left[ F(\epsilon_0 + p)^{n-1} - F^n(\epsilon_0 + p)^n \right] dF_0(\epsilon_0)}{\frac{1}{n} \int_{\bar{\epsilon}}^{\hat{\epsilon}} [1 - F(\epsilon_0 + p)^n] dF_0(\epsilon_0)} + 1 - \lambda
\]

Notably, if $\lambda = 1$ then $\sigma_\lambda$ defined here corresponds to the definition for the setting of multihoming buyers, while if $\lambda = 0$ then $\sigma_\lambda = 1$ as in the setting of singlehoming buyers. Holding all else equal, an increase in $\lambda$ decreases the buyer loyalty index, reflecting that a higher fraction of multihoming buyers naturally means that buyers are less loyal to the platforms.

Then, a pure symmetric pricing equilibrium can be characterized by all platforms choosing $\hat{p} = (\hat{p}^b, \hat{p}^s)$ that uniquely solves

\[
\hat{p}^b + \hat{p}^s - c = X(\hat{p}^b; n) = \frac{1 - G(\hat{p}^s)}{g(\hat{p}^s)} \sigma_\lambda(p^b; n).
\]

Given that increasing $\lambda$ decreases the buyer loyalty index, the argument behind Proposition 3 immediately implies the following result:

**Proposition 7 (Effect of buyer-multihoming)** In the equilibrium with partial-multihoming buyers characterized by (13), a higher fraction of multihoming buyers ($\lambda$) increases the

---

\(^17\)Following the micro-foundation in Section 2, this extended model can be interpreted as buyers having heterogenous cost $\psi$ of joining additional platforms. That is, a fraction $\lambda$ of buyers have $\psi \leq 0$ while the remaining buyers have $\psi > 0$.  

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buyer fee $\hat{p}_b$, decreases the seller fee $\hat{p}_s$, and decreases the total fee $\hat{p}_b + \hat{p}_s$.

Finally, the insights from Proposition 6 remain valid in this extended model of partial-multihoming buyers.

**Proposition 8** *(Increased platform competition)* In the equilibrium with partial-multihoming buyers characterized by (13), an increase in $n$ (i.e. platform entry) always decreases the total fee.

1. If $\lambda \to 0$, an increase in $n$ shifts the fee structure in favor of buyers ($\hat{p}_b - \hat{p}_s$ decreases with $n$).

2. If $\lambda \to 1$ and densities $f$ and $g$ are weakly decreasing functions, an increase in $n$ shifts the fee structure in favor of sellers ($\hat{p}_b - \hat{p}_s$ increases with $n$).

In the case where $\epsilon_0$ and all $\epsilon_i$ are drawn from Gumbel distribution, we can obtain a stronger version of Proposition 8 with a unique cutoff on $\lambda$:

**Remark 1** If $\epsilon_i$ for $i = 0, 1, ..., n$ follows Gumbel distribution, then there exists a unique cutoff $\lambda^*$ such that:

1. If $\lambda < \lambda^*$, an increase in $n$ shifts the fee structure in favor of buyers ($\hat{p}_b - \hat{p}_s$ decreases with $n$).

2. If $\lambda \geq \lambda^*$ and density $g$ is a weakly decreasing function, an increase in $n$ shifts the fee structure in favor of sellers ($\hat{p}_b - \hat{p}_s$ increases with $n$).

### 6 Discussion and implications

In this section, we illustrate the implications of our analysis using our motivating examples of payment card (Section 6.1) and ride-hailing platforms (Section 6.2) based on the analytical results derived thus far. To facilitate exposition, we focus on the baseline cases of all buyers singlehoming ($\lambda = 0$) and all buyers multihoming ($\lambda = 1$), while noting that the general qualitative insights remain the same for cases between these two extremes ($\lambda \in (0, 1)$). Then, in section 6.3 we calibrate the model parameters using real world data from ride-hailing services in order to present some welfare implications from our analysis.

#### 6.1 Payment card platforms

Payment card platforms typically offer card holders (buyers) a variety of card-usage benefits e.g. interest-free periods, cash rebates and loyalty rewards. Platforms then make money by charging transaction fees on merchants (sellers), so that $p^s > 0 > p^b$ in practice.
Here, the negative value of $p^b$ represents the various card-usage benefit, $p^s$ is the merchant fee (or interchange fee assuming the acquiring side is perfectly competitive), and $p^b + p^s$ is the profit margin earned by card issuers. Figure 4 numerically illustrates this application, assuming that $c = 0.1$, $F$ and $F_0 \sim Gumbel(\mu, \gamma)$ with location parameter $\mu = 0$ and scale parameter $\gamma = 1$, while $G \sim Normal(\mu_G, \sigma^2)$ with mean $\mu_G = 3$ and variance $\sigma^2 = 2$.

![Figure 4: Payment card market](image)

**Platform competition and interchange fees.** Policymakers in some jurisdictions, including Australia, Europe, and United Kingdom, have claimed that payment card platforms set interchange fees too high. As summed up by Guthrie and Wright (2006), these authorities appear to view the lack of competition between platforms as a possible cause of high interchange fees. As can be seen from Figure 4 however, this view by the authorities is true only when most of the cardholders are multihoming, whereby increasing inter-platform competition indeed helps to reduce the interchange fee. Interestingly, the reverse view is true when the fraction of singlehoming cardholders is sufficiently large, whereby increasing inter-platform competition drives up the interchange fee instead, which seems to match the empirical evidence better (Rysman and Wright, 2015).

A caveat to this discussion is that our analysis has abstracted away from the possibility of merchants adjusting their product prices when the merchant fee increases. Some payment card platforms employ no-surcharge rules which prevent merchants from charging higher prices to their card-users compared to users paying with other payment instruments (such as another card, or cash). Under the no-surcharge rule, if a platform increases its merchant fee, merchants may respond by increasing both the price paid by the users of this platform as well as the price paid by users of other platforms or cash. In terms of our framework, this means that an increase in seller fee may decrease the transactional surplus of buyers across all other platforms including the outside option. Hence, from a platform’s perspective it may be profitable to deviate from the equilibrium.

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fee we characterize by further increasing fee to sellers and decreasing it to buyers to attract more buyers. As shown by Edelman and Wright (2015), the profitability of such a deviation can increase the more platforms compete, suggesting an additional reason for why platform competition may actually increase interchange fees.

6.2 Ride-hailing platforms

In the context of ride-hailing platforms, for each trip the riders (buyers) enjoy benefits while the drivers (sellers) incur efforts, so that \( p^b > 0 > p^s \) in practice. Here, \( p^b \) is the fare set by the platforms, the negative value of \( p^s \) is the per-ride driver gross earning (or wage), and \( p^b + p^s \) is the net commission that platforms earn from each ride. Multihoming riders are those who compare and choose between multiple apps whenever they call for a ride, while singlehoming riders are those who do not do so. Figure 5 numerically illustrates this application, where the model parameters and distribution functions follow from the calibration exercise in Section 6.3.

![Figure 5: Ride-hailing market](image)

**Figure 5:** Ride-hailing market

**Rider-multihoming.** Recent advancements in mobile phone technology and fare-comparison “aggregator” services allow more riders to seamlessly compare across different ride-hailing apps, leading to a growing number of multihoming riders.\(^{18}\) Figure 5 shows that, holding constant the number of platforms, such a switch to multihoming is predicted to increase fares and wages. Our analysis thus suggests the advent of fare-comparison services that seemingly benefits riders may not necessarily do so due to the higher fare. This somewhat counter-intuitive observation can be understood as follows. As riders become more likely to switch across apps, each driver can choose to quit the low-wage platforms without worrying about losing too many riders. Platforms thus compete more

\(^{18}\)Among the notable examples are Google Maps, BellHop and RideGuru, see https://www.wired.com/story/uber-and-lyfts-never-ending-quest-to-crush-price-comparison-apps/
intensely to sign up drivers by increasing the wage. The increased wage then gets passed through to the fare charged to riders.

It should be noted that the discussion above merely focuses on one impact of fare-comparison services and by no means captures all the possible impacts of a shift to multihoming buyers. In particular, in our two-sided framework buyer multihoming affects the seller-side competition but has no direct impact on the buyer-side competition. If, for example, one constructs a model based on search frictions faced by riders, then the fare-comparison services, by reducing search friction, would directly intensify buyer-side competition. In that case, there would be a simultaneous shift in both curves in Figure 1, so that the overall comparative static would generally depend on the relative magnitudes of these two shifts in curves.

**Platform competition.** Buyer multihoming profoundly reverses the dynamics of platform competition. When riders are singlehoming, existing ride-hailing platforms respond to entry by cutting the fare to attract riders, and then reoptimize by offering less to drivers. However, when riders are multihoming, if the incumbent platforms naively continue to respond by cutting fares and driver wages, then some drivers will simply quit the lower-wage incumbents, knowing that they can still access a large portion of riders through other higher-wage platforms. Instead, our analysis suggests that the response in equilibrium would be the reverse: platforms increase wages to attract drivers, and then reoptimize the fare by charging more. The possibility of a fare increase following entry is in contrast to the conventional one-sided logic that high final product prices (in this case, rider fares) are caused by a lack of competition. This highlights the importance of taking into account the two-sided nature of ride-hailing applications in the analysis of market power.

**Platform merger and exit.** The industry of ride-hailing services has witnessed several high profile merger cases in recent years, including Didi-Uber in China (2016), Yandex-Uber in Russia (2017), Grab-Uber in South East Asia (2018), and Careem-Uber in Middle East (2019). Notably, each of these mergers has resulted in one of the platforms exiting the market entirely. Based on analyzing what happens when $n$ decreases by one, our analysis suggests that the effect of these mergers on the platform fee structure may go in opposite directions depending on the level of rider-multihoming. This provides an empirical implication: even in the absence of any cost-efficiency gain from the merger, it is possible for such a merger to result in lower fares for riders (if the extent of rider-multihoming is high) or higher earnings for drivers (if the extent of rider-multihoming is low). Regardless of the level of rider-multihoming, however, our model also predicts the total fee charged to the two sides will increase.

19 Therefore, these merger cases are different from standard horizontal mergers involving differentiated products, where the merged entity would continue operating both of the original brands so as to maximize their joint profit.
6.3 Calibrated model and welfare analysis

So far, our analysis and discussion have focused on the effect of buyer multihoming and increased platform competition on prices. In this section, we explore the corresponding welfare implications based on calibrating our model to ride-hailing services.

6.3.1 Reinterpreting the model

For computational simplicity, we assume that \( F, F_0 \sim \text{Gumbel}(\mu_F, \gamma_F) \) and \( G \sim \text{Gumbel}(\mu_G, \gamma_G) \) where \( \mu \) corresponds to the location parameter and \( \gamma \) corresponds to the scale parameter. Given that drivers incur an opportunity cost and effort to drive, i.e. \( v \leq 0 \), we truncate the distribution \( G \) so that its support is \((-\infty, 0)\). We think of \( F \) and \( F_0 \) as the distribution of a rider’s per-trip random utility \( \epsilon \), and \( G \) as the distribution of a driver’s opportunity cost (including driving expenses) \( v \leq 0 \). Meanwhile, \( p^b \) and \(-p^s\) correspond to the per-trip fare and drivers’ per-ride gross earnings (fare after deducting platform commission).

To relate the driver per-trip earnings to the distribution of per-hour opportunity cost, we assume a linear relationship between drivers’ per-ride earnings and per-hour earnings. Let the said linear scaling factor be denoted \( \beta \), which can be interpreted as the average number of trips performed by a driver per hour. Finally, we interpret riders’ outside option as a conventional taxi, which is the closest substitute to ride-hailing services, and let the per-trip taxi fare be \( p_0 \).

To fix ideas, suppose there is total demand for trips desired by riders denoted by \( T \), and trips can be done either through ride-hailing services or conventional taxis. For simplicity, the capacity of the conventional taxis is assumed to be fixed and larger than \( T \) so that there are no unfulfilled trips even if ride-hailing platforms are absent. Meanwhile, the capacity of each ride-hailing platform increases proportionally with the number of drivers on the platform. Specifically, given that \( G \) is the distribution of driver’s per-hour opportunity cost of driving for platforms, in the symmetric equilibrium the capacity of each ride-hailing platform is \((1 - G(\beta p^s)) \alpha\), where \( \alpha \) is some factor of proportionality. Then, whether each unit of capacity leads to a realized trip depends on the probability of a rider confirming each ride on the platform, that is, \( B_i^{(N)} \).

Then, we can write the profit function of a platform \( i \) in the symmetric equilibrium as

\[
\Pi_i = \left( \frac{p^b + p^s - c}{\text{per-ride margin}} \right) \times \left( 1 - G(\beta p^s) \right) \alpha T \times \left( B_i^{(N)} \right) \times \left( \text{prob. of rider confirming each ride on } i \right).
\]

Note the capacity of platform \( i \) is the same as the total capacity of all platforms given
that all drivers multihome in our equilibrium. The total rider surplus in equilibrium is

\[ RS = \left[ \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \max \{ \epsilon - p^b, \epsilon_0 - p_0 \} \, dF(n) \, dF_0 (\epsilon_0) \right] (1 - G(\beta p^s)) \alpha T \]

utility from all trips where ride-hailing services are available

\[ + \left[ \int_{-\infty}^{\infty} \{ \epsilon_0 - p_0 \} \, dF_0 (\epsilon_0) \right] (G(\beta p^s)) \alpha T \]

utility from all trips where ride-hailing services are not available

Finally, total driver surplus in equilibrium is

\[ DS = \frac{n B_i^{(N)} \alpha T}{\text{number of rides confirmed through platforms}} \times \int_{\beta p^s}^{\infty} (v - \beta p^s) \, dG(v) \]

Total welfare is then defined as \( W = RS + DS + n \Pi_i \). We note that \( \alpha \) and \( T \) are linear scaling factors and do not affect the equilibrium outcome, and so they play no role in the welfare assessment if we focus on the percentage changes in welfare and surpluses rather than calculating the absolute values of welfare and surpluses. For this reason, we do not attempt to identify \( \alpha \) and \( T \) in our calibration exercise.

6.3.2 Calibration methodology and results

We base our calibration exercise on the documentation of ride-hailing services by Cook et. al. (2018) and Mishel (2018), based on UberX/UberPool data in the United States from January 2015 to March 2017:

<table>
<thead>
<tr>
<th>Variable</th>
<th>Value</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Per-trip fare ( p^b )</td>
<td>$12.63</td>
<td>Cook et al. (2018)</td>
</tr>
<tr>
<td>Per-trip driver earning -( p^s )</td>
<td>$7.92</td>
<td>Mishel (2018), Fare - 25% commission to Uber - $1.55 booking fee</td>
</tr>
<tr>
<td>Trips per hour ( \beta )</td>
<td>1.75</td>
<td>Mishel (2018)</td>
</tr>
<tr>
<td>Driver opportunity cost -( E(v) )</td>
<td>$19.77</td>
<td>Mishel (2018), Driving expenses ($4.78) + mean wage for lowest-paid major service occupation ($14.99)</td>
</tr>
</tbody>
</table>

We let \( n = 2 \) and calibrate model parameters \( \lambda, c, p_0 \), and distribution parameters \((\mu_F, \gamma_F)\) and \((\mu_G, \gamma_G)\) such that the equilibrium fees predicted by the model exactly match the observed values documented in the table above. The resulting parameters from the calibration exercise are summarized as follow:

\[ \sigma_{\lambda} = 0.57, \text{rider price-elasticity of } -2.68 \]

(that is, a 1% increase in fare reduces rider quasi-demand by 2.68%), and driver (per-hour) earning-elasticity of 0.35.

---

\[ \text{These calibrated parameters imply a buyer loyalty index of } \sigma_{\lambda} = 0.57, \text{rider price-elasticity of } -2.68 \]

(that is, a 1% increase in fare reduces rider quasi-demand by 2.68%), and driver (per-hour) earning-elasticity of 0.35.
<table>
<thead>
<tr>
<th>Variable</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fraction of multihoming riders $\lambda$</td>
<td>0.549</td>
</tr>
<tr>
<td>Platform per-trip marginal cost $c$</td>
<td>$0</td>
</tr>
<tr>
<td>Cost of rider outside option $p_o$</td>
<td>$16.03</td>
</tr>
<tr>
<td>Gumbel parameters for riders $(\mu_F, \gamma_F)$</td>
<td>(0, 2.64)</td>
</tr>
<tr>
<td>Gumbel parameters for drivers $(\mu_G, \gamma_G)$</td>
<td>(-4.07, 27.20)</td>
</tr>
</tbody>
</table>

We are interested in the effects of the following two exogenous changes: (i) An increase in the number of platforms from $n = 2$ to $n = 3$; and (ii) A 10% increase in the fraction of multihoming riders. Based on the calibrated parameters, we simulate how these two changes affect prices, surpluses, profit, and welfare. The results are summarized as follows.

<table>
<thead>
<tr>
<th>Percentage change in</th>
<th>(i) Entry</th>
<th>(ii) Increase in multihoming</th>
</tr>
</thead>
<tbody>
<tr>
<td>Per-trip fare</td>
<td>-9.9%</td>
<td>+2.3%</td>
</tr>
<tr>
<td>Per-trip driver earning</td>
<td>-5.1%</td>
<td>+4.2%</td>
</tr>
<tr>
<td>Rider surplus</td>
<td>+3.2%</td>
<td>-0.1%</td>
</tr>
<tr>
<td>Driver surplus</td>
<td>-3.0%</td>
<td>+7.1%</td>
</tr>
<tr>
<td>Individual platform profit</td>
<td>-44.1%</td>
<td>+1.5%</td>
</tr>
<tr>
<td>Welfare</td>
<td>+2.1%</td>
<td>+1.1%</td>
</tr>
</tbody>
</table>

Table 1: Simulated effect of entry and buyer multihoming

These results suggest that under the calibrated parameters, the changes in surpluses appear to be aligned with the changes in fares and earnings. Specifically, a decrease (increase) in fare is associated with an increase (decrease) in rider surplus, and likewise a decrease (increase) in driver earning is associated with a decrease (increase) in driver surplus. Meanwhile, both entry and more multihoming appears to improve welfare, which is perhaps intuitive given that the total fee charged by platforms (fare after deducting driver earning) when there is platform entry or more multihoming.

7 Conclusion

This paper investigated two-sided market pricing by oligopolistic platforms when platforms set transaction fees on both user sides. We addressed the important yet relatively unexplored questions of buyers’ multihoming behaviour and how it interacts with platform competition. The main results are summarized in Table 2.\(^{22}\)

\(^{21}\) For additional robustness, we performed the same exercise using alternative calibration targets of a higher per-ride earning $-p^* = $8.50 and $9.00 (to account for the possibility that drivers earn additional incentive payments from the platform) as well as a lower opportunity cost of drivers $E(-v) = $15 and $10 (to account for the possibility that some drivers drive on a part-time basis hence their opportunity cost is much lower). The signs of changes in prices, surpluses, profit, and welfare obtained in Table 1 continue to hold in each case.

\(^{22}\) Owing to the generalization in Section 5, the results in Table 2 have their counterparts in a continuous version, where the three scenarios are replaced by “A higher fraction of multihoming buyers”, “Platform
Our analysis and discussion have focused on changing either the extent of buyer-multihoming or the extent of platform competition while holding one of these two constant. However, the advancement in multihoming technology that makes multihoming easier for buyers may be a response to the increased number of competing start-up platforms that buyers can use, suggesting these two changes may occur simultaneously. Following this line of thinking, to predict what might happen to prices over time one may want to know the effect of a simultaneous increase in the extent of multihoming and the number of platforms. Our results (e.g. Table 2) suggests that while total fees will decline, initially the effect on the price structure is ambiguous because platform competition and more multihoming have opposite effects when the extent of multihoming is low to begin with. However, once the extent of multihoming gets sufficiently large, more competition and more multihoming will have the same effect on platform price structure, suggesting an overall trend towards higher fees for buyers and lower fees for sellers in the long run.

Our analysis can usefully be extended in several major directions. An obvious direction would be to consider competition between asymmetric platforms. One way to model asymmetric platforms while ensuring tractability is to utilize the technique of aggregative games as in Anderson and Peitz (2019) in their model of media platforms. Their analysis focuses on a competitive bottleneck setup where buyers (consumers) singlehome while sellers (advertisers) multihome. It would be interesting to try to extend their approach to our setting of two-sided multihoming and pricing.

Throughout we have focused on the role of buyer multihoming and that sellers are free to multihome. A natural extension is to consider the possibility of seller singlehoming, which may arise due to exclusive contracts, where an individual platform signs up sellers exclusively. However, in order to avoid a degenerate outcome in which platforms compete away all their profit margin to attract sellers exclusively, this would require an extended model where platforms are differentiated from the perspective of sellers, which, following the canonical framework of Rochet and Tirole (2003), we do not consider. In our current model, each seller is indexed by a real number. So, the characterization of sellers’ (off-equilibrium) quasi-demand conveniently divides sellers into multiple intervals according to the number of platforms they multihome on. The analysis becomes techni-
cally complicated when platforms are differentiated from the sellers’ perspective, whereby
sellers will each be indexed by a $n$-dimensional duplet on $\mathbb{R}^n$. Future work may look into
alternative settings to address the issue of seller singlehoming.
A Appendix

A.1 Demand derivation for the case of multihoming buyer

In this appendix, we complete the demand derivation by considering the seller participation profile under a downward deviation \( p_i^* < \hat{p}^* \). It is obvious that if a seller joins at least one platform, then the seller must also join platform \( i \) given that \( i \) charges the lowest seller fee. A seller will join \( i \) as long as \( v \geq p_i^* \).

Then, in deciding whether to join \( i \) exclusively or join a platform \( j \neq i \) in addition, the seller faces the same trade off between extra access and buyer diversion as we found for the upward deviation. When \( v \) is larger than some threshold, the gain from extra access dominates and the seller will join an additional platform \( j \). However, the fact that all other platforms \( j \neq i \) set \( \hat{p}^* \) does not necessarily imply that the seller will join all these platforms together in a “block”. This is because when \( p_i^* < \hat{p}^* \), any additional platform that a seller joins will divert additional buyers away from the lowest-fee platform \( i \) to the newly joined platform. Therefore, the number of platforms a seller multihomes on will depend on \( v \) in general.

Specifically, consider a seller who chooses to join platform \( i \) together with \( m \) other (symmetric) platforms. We denote this set of platforms as \( N_{i,m} \) (the seller joins \( m + 1 \) platforms in total). Note that \( N_{i,0} = \{i\} \) and \( N_{i,n-1} = N \) so \( m \) is bounded between 0 and \( n - 1 \). The corresponding number of buyers who use \( i \) for transactions is

\[
B_i^{(N_{i,m})} = \Pr\left( \epsilon_i - p_i^b \geq \max_{j \in N_{i,m}} \left\{ \epsilon_j - p_j^b, \alpha_0 \right\} \right).
\]

Clearly a higher \( m \) implies more buyers diverted from platform \( i \) since \( B_i^{(N_{i,m})} \) decreases with \( m \). The following lemma states sellers’ multihoming decision formally:

**Lemma 4** Suppose \( p_i^* < \hat{p}^* \). For \( m = 1, \ldots, n - 1 \), define cutoffs

\[
\hat{v}_m \equiv \frac{p_i^* \left( B_i^{(N_{i,m})} - B_i^{(N_{i,m-1})} \right) + \hat{p}^* \left[ mB_j^{(N_{i,m})} - (m - 1) B_j^{(N_{i,m-1})} \right]}{B_i^{(N_{i,m})} - B_i^{(N_{i,m-1})} + mB_j^{(N_{i,m})} - (m - 1) B_j^{(N_{i,m-1})}},
\]

(14)

A type \( v \) seller joins no platform if \( v \in [v, p_i^*) \), joins only platform \( i \) if \( v \in [p_i^*, \hat{v}_1) \), joins platform \( i \) together with \( m \) randomly chosen symmetric platform(s) from \( j \neq i \) if \( v \in [\hat{v}_m, \hat{v}_{m+1}) \), and joins all platforms if \( v > \hat{v}_{n-1} \).

**Proof.** Consider a type \( v \) seller that has joined platform \( i \) and that is contemplating whether to join one of the platforms \( j \neq i \) in addition. The utility of joining \( i \) alone (so \( m = 0 \)) is \((v - p_i^*) B_i^{(N_{i,0})}\), so this is superior than joining no platforms as long as \( v \geq p_i^* \). Meanwhile the utility from joining another platform \( j \neq i \) (so that \( m = 1 \)) is \((v - p_i^*) B_i^{(N_{i,1})} + (v - \hat{p}^*) B_j^{(N_{i,1})}\). Comparing the two utilities yields the first cutoff

\[
\hat{v}_1 \equiv \frac{p_i^* \left( B_i^{(N_{i,1})} - B_i^{(N_{i,0})} \right) + \hat{p}^* B_j^{(N_{i,1})}}{B_i^{(N_{i,1})} - B_i^{(N_{i,0})} + B_j^{(N_{i,1})}}.
\]

Now suppose a seller has joined platform \( i \) plus \( m - 1 \) other platforms where \( m \leq n - 1 \), i.e. the set of platforms \( N_{i,m-1} \). Owing to the symmetry of all platforms \( j \neq i \), the seller’s utility can be written as \((v - p_i^*) B_i^{(N_{i,m-1})} + (m - 1) (v - \hat{p}^*) B_j^{(N_{i,m-1})}\). The utility of joining one more platform — so that the seller joins platform \( i \) plus \( m \) other platforms, i.e. the set of platforms \( N_{i,m} \), is \((v - p_i^*) B_i^{(N_{i,m})} + m(v - \hat{p}^*) B_j^{(N_{i,m})}\). Comparing the two utilities yields cutoffs (14) for all \( m \leq n - 1 \).

Combining with the case of upward deviation derived in the main text, the complete demand function
faced by platform $i$ is piece-wise defined by

\[ Q_i \left( p_b^i, p_s^i; \hat{p} \right) = \begin{cases} \sum_{m=0}^{n-1} \left[ G \left( \hat{v}_{m+1} \right) - G \left( \hat{v}_m \right) \right] B_i^{(N,m)} & \text{if } p_s^i < \hat{p}^s \\ \left( 1 - G \left( \hat{v} \right) \right) B_i^{(N)} & \text{if } p_s^i \geq \hat{p}^s \end{cases} \]

(15)

where we denote $\hat{v}_0 \equiv p_s^i$ and $\hat{v}_n \equiv \hat{v}$ (so that $G(\hat{v}_n) = 1$). Note that when $p_s^i < \hat{p}^s$, the volume takes into account sellers’ heterogenous multihoming behavior. Figure 6 provides an illustration of function (15) assuming $n = 5$:}

(When $p_s^i \geq \hat{p}^s$)

The left panel of Figure 6 depicts $Q_i \left( p_b^i, p_s^i; \hat{p} \right)$ when $p_s^i \geq \hat{p}^s$. In this case, only sellers with $v \geq \hat{v}$ join platform $i$, and the mass of buyers who use platform $i$ to transact with each of these sellers is $B_i^{(N)}$, that is, those who find $i$ most attractive when all $n$ platforms are available for transactions. The right panel of Figure 6 depicts the case of $p_s^i < \hat{p}^s$, where we recall that $m$ denotes the number of platforms that a seller multihomes on in addition to platform $i$. Sellers with $v \in [p_s^i, \hat{v}_1)$ join platform $i$ exclusively, so that buyers who transact with these sellers can only choose between transacting through $i$ or transacting directly. The mass of buyers who use $i$ to transact with these sellers is $B_i^{(N,0)}$, that is, those who find $i$ more attractive than the outside option. Sellers with $v \in [\hat{v}_1, \hat{v}_2)$ join platform $i$ and a randomly selected platform $j \neq i$, so that buyers who transact with these sellers can choose between transacting through $i$, $j$, or transacting directly. Notably, the mass of buyers who use $i$ to transact with these sellers is $B_i^{(N,1)}$, which is smaller than $B_i^{(N,0)}$ due to the availability of an additional alternative platform for transactions. Extending this idea forward, for sellers who multihome on more platforms, the mass of buyers who use $i$ to transact with these sellers is lower given that there are more alternative platforms available for transactions (as can be seen from $B_i^{(N, m)}$ being decreasing in $m$).

In Section B of the Online Appendix, we examine whether the piece-wise demand function $Q_i \left( p_b^i, p_s^i; \hat{p} \right)$ (15) is continuous, the properties of its derivatives, as well as the conditions under which the corresponding profit function is globally quasi-concave.

A.2 Proofs

**Proof. (Lemma 1).** Let $\epsilon(n)$ denote the highest order statistic (out of $n$ draws of $\epsilon$), and denote

\[
\bar{X} = \frac{1}{n} (1 - F (\epsilon_0 + p)^n - f (\epsilon_0 + p) F (\epsilon_0 + p)^{n-1})
\]
as the buyer inverse semi-elasticity for given non-random outside option $\epsilon_0 + p$. Then, from definition (5) and exploit the alternative expression of

$$\int_\epsilon^\infty \int_\epsilon^\infty 1 - F\left(\max\{\epsilon, \epsilon_0 + p^b\}\right) dF(\epsilon)^{-1} dF_0(\epsilon_0) = \frac{1}{n} \int_\epsilon^\infty [1 - F(\epsilon_0 + p^n)] dF_0(\epsilon_0),$$

we can rewrite $X (p; n)$ as

$$\frac{1}{X (p; n)} = \int_\epsilon^\infty \left[ \int_\epsilon^\infty \frac{f(\epsilon)}{1 - F(\epsilon_0 + p^n)} dF(\epsilon)^{-1} \right] \left[ \int_\epsilon^\infty \frac{(1 - F(\epsilon_0 + p^n))}{f(\epsilon_0 + p^n)} dF_0(\epsilon_0) \right] dF_0(\epsilon_0).$$

Define a new random variable $\tilde{\epsilon}_0 \equiv \epsilon_0 + p$ with support over $[\epsilon + p, \epsilon + p]$, and define the cdf of $\tilde{\epsilon}_0$ conditioned on it being smaller than $\epsilon(n)$:

$$H (x; n, p) \equiv \Pr (\tilde{\epsilon}_0 < x | \tilde{\epsilon}_0 < \epsilon(n)) = \frac{\int_{\epsilon + p}^x (1 - F (\tilde{\epsilon}_0)^n) f_0 (\tilde{\epsilon}_0 - p) \tilde{\epsilon}_0}{\int_{\epsilon + p}^x (1 - F (\tilde{\epsilon}_0)^n) f_0 (\tilde{\epsilon}_0 - p) \tilde{\epsilon}_0}.$$ (16)

Then,

$$\frac{1}{X (p; n)} = \int_\epsilon^\infty \left[ \frac{1}{X(\tilde{\epsilon}; n)} \right] dH (\tilde{\epsilon}_0; n, p).$$

Lemma 4 of Zhou (2017) shows that $1/X (\tilde{\epsilon}_0; n)$ is increasing in $\tilde{\epsilon}_0$ and $n$. Hence, to conclude that $\frac{1}{X (p; n)}$ is increasing in $p$ and $n$, it remains to show that the conditional random variable $\tilde{\epsilon}_0|_{\tilde{\epsilon}_0 < \epsilon(n)}$ is increasing in $n$ and $p$ in the sense of first-order stochastic dominance (FOSD), i.e. $H (x; n, p)$ is decreasing in $p$ and $n$ at each given $x$.

Claim: $\tilde{\epsilon}_0|_{\tilde{\epsilon}_0 < \epsilon(n)}$ is FOSD increasing in $p$. From the cdf function, the relevant derivative $\frac{\partial H(x; n, p)}{\partial p}$ can be shown to be negative if

$$\int_{\epsilon + p}^x [1 - F(\tilde{\epsilon}_0)^n] f_0 (\tilde{\epsilon}_0 - p) d\tilde{\epsilon}_0 \geq \int_{\epsilon + p}^x \left[ 1 - F(\tilde{\epsilon}_0)^n \right] f_0 (\tilde{\epsilon}_0 - p) \tilde{\epsilon}_0.$$

Given $x \leq \epsilon$, establishing (17) is equivalent to showing that the left-hand side of (17) is decreasing in $x$. If we define distribution function

$$\tilde{H} (y; x) = \Pr (\tilde{\epsilon}_0 < y | \tilde{\epsilon}_0 < \max\{\epsilon(n), x\}) = \frac{\int_y^{\epsilon + p} [1 - F(\tilde{\epsilon}_0)^n] f_0 (\tilde{\epsilon}_0 - p) d\tilde{\epsilon}_0}{\int_y^{\epsilon + p} [1 - F(\tilde{\epsilon}_0)^n] f_0 (\tilde{\epsilon}_0 - p) d\tilde{\epsilon}_0} \quad \text{for } y \in [\epsilon + p, x]$$

then we can rewrite the left-hand side of (17) as $\int_y^{\epsilon + p} \left[ \frac{\tilde{H}(y; x)}{\tilde{F}(y; x)} \right] d\tilde{H} (y; x)$. Log-concavity of $f_0$ implies that $\frac{\tilde{H}(y; x)}{\tilde{F}(y; x)}$ is decreasing in $y$. Meanwhile it is easily verified from the definition that $\tilde{H} (y; x)$ is FOSD increasing in $x$. Therefore, we conclude that the left-hand side of (17) is decreasing in $x$, so that inequality (17) indeed holds.

Claim: $\tilde{\epsilon}_0|_{\tilde{\epsilon}_0 < \epsilon(n)}$ is FOSD increasing in $n$. From the cdf function, the relevant derivative $\frac{\partial H(x; n, p)}{\partial n}$ can be shown to be negative if

$$\int_{\epsilon + p}^x [- \ln F(\tilde{\epsilon}_0) F(\tilde{\epsilon}_0)^n] f_0 (\tilde{\epsilon}_0 - p) d\tilde{\epsilon}_0 \geq \int_{\epsilon + p}^x [- \ln F(\tilde{\epsilon}_0) F(\tilde{\epsilon}_0)^n] f_0 (\tilde{\epsilon}_0 - p) \tilde{\epsilon}_0.$$ (18)
so that \( \frac{\partial H(x, n, p)}{\partial n} \leq 0 \) if the left-hand side of (18) is increasing in \( x \). Applying the same technique used in the previous claim, we can write the left-hand side of (18) as

\[
\int_{\xi+p}^{x} \left[ \frac{-\ln F(y) F(y)^{n}}{1 - F(y)^{n}} \right] d\tilde{H}(y; x).
\]

Since \( -\ln F(y) \geq 0 \), we know that \( \frac{-\ln F(y) F(y)^{n}}{1 - F(y)^{n}} \) is increasing in \( y \). This fact, together with the fact that \( \tilde{H}(y; x) \) is FOSD increasing in \( x \), implies that left-hand side of (18) is increasing in \( x \), so that inequality (18) indeed holds. ■

**Proof.** (Proposition 1). The demand derivatives, after imposing symmetry, can be calculated as follows:

\[
\begin{align*}
Q_{i}(\tilde{p}; \tilde{p}) &= (1 - G(\tilde{p}^{s})) B_{i}^{(N)}|_{p_{i}^{s}=\tilde{p}^{b}} \\
&= (1 - G(\tilde{p}^{s})) \int_{\xi}^{\bar{e}} \int_{\xi}^{\bar{e}} 1 - F(\max \{\epsilon, \epsilon_{0} + p^{b}\}) dF(\epsilon)^{n-1} dF_{0}(\epsilon_{0}) \\
\frac{dQ_{i}(\tilde{p}; \tilde{p})}{dp_{i}^{b}} &= (1 - G(\tilde{p}^{s})) \frac{\partial B_{i}^{(N)}}{\partial p_{i}^{b}}|_{p_{i}^{s}=\tilde{p}^{b}} \\
&= - (1 - G(\tilde{p}^{s})) \int_{\xi}^{\bar{e}} \int_{\xi}^{\bar{e}} f(\max \{\epsilon, \epsilon_{0} + p^{b}\}) dF(\epsilon)^{n-1} dF_{0}(\epsilon_{0}) \\
\frac{dQ_{i}(\tilde{p}; \tilde{p})}{dp_{i}^{s}} &= -g(\tilde{p}^{s}) B_{i}^{(N)}|_{p_{i}^{s}=\tilde{p}^{s}}.
\end{align*}
\]

The standard first-order condition yields (6). Denote \( M(\tilde{p}^{s}) \equiv \frac{1 - G(\tilde{p}^{s})}{g(\tilde{p}^{s})} \), and \( \bar{T} \equiv \bar{p}^{b} + \bar{p}^{s} \). To prove the existence and uniqueness of \( \bar{p}^{b} \) and \( \bar{p}^{s} \) defined in (6), from the equilibrium characterization equation we know for \( n \) fixed:

\[
\bar{T} - c = X(\bar{p}^{b}) = M(\bar{p}^{s}).
\]

We know from Lemma 1 that \( X(\cdot) \) is weakly decreasing. If \( X(\cdot) \) is constant, then it is easily seen that \( \bar{T}, \bar{p}^{b}, \) and \( \bar{p}^{s} \) can be uniquely pinned down. When \( X(\cdot) \) is strictly decreasing, we can express \( \bar{p}^{b} = X^{-1}(\bar{T} - c) \), and \( \bar{p}^{s} = M^{-1}(\bar{T} - c) \), where both \( X^{-1} \) and \( M^{-1} \) are strictly decreasing in \( \bar{T} - c \). Therefore, we can rewrite \( \bar{T} \equiv \bar{p}^{b} + \bar{p}^{s} \) as

\[
\bar{T} = X^{-1}(\bar{T} - c) + M^{-1}(\bar{T} - c),
\]

where the right-hand side is strictly decreasing in \( \bar{T} \). This implies that there exist a unique fixed point \( \bar{T} \) that solves (20) and hence solves (19) by construction. Then, \( \bar{p}^{b} \) and \( \bar{p}^{s} \) can be uniquely determined from the one-to-one relations \( \bar{p}^{b} = X^{-1}(\bar{T} - c) \) and \( \bar{p}^{s} = M^{-1}(\bar{T} - c) \). ■

**Proof.** (Lemma 3). Rewrite \( \sigma(p; n) \) as

\[
\sigma(p; n) = \frac{\int_{\xi}^{\bar{e}} [nF(\epsilon_{0} + p)]^{n-1} (1 - F(\epsilon_{0} + p)) \ dF_{0}(\epsilon_{0})}{\int_{\xi}^{\bar{e}} p [1 - F(\epsilon_{0} + p)^{n}] \ dF_{0}(\epsilon_{0})} = \Pr(\epsilon_{(n-1)} < \tilde{\epsilon}_{0} | \epsilon_{(n)} > \tilde{\epsilon}_{0}),
\]

where \( \tilde{\epsilon}_{0} \equiv \epsilon_{0} + p \).
To show $\sigma (p; n) \text{ increases with } p$, we write
\[
\sigma (p; n) = \int_{\xi_p}^{\eta_p} \Pr (\epsilon_{(n-1)} < y | \epsilon_{(n)} > y) \Pr (\tilde{\epsilon}_0 = y | \tilde{\epsilon}_0 < \epsilon_{(n)}) \, dy \\
= \int_{\xi_p}^{\eta_p} \Pr (\epsilon_{(n-1)} < y | \epsilon_{(n)} > y) \, dH (\tilde{\epsilon}_0; n, p),
\] (21)

where $H (\tilde{\epsilon}_0; n, p)$ is the conditional distribution function defined in (16). We first observe that $\Pr (\epsilon_{(n-1)} < y | \epsilon_{(n)} > y)$ is increasing in $y$:
\[
\frac{d}{dy} \Pr (\epsilon_{(n-1)} < y | \epsilon_{(n)} > y) = \frac{nF(y)^{n-1} (1 - F(y))}{(1 - F(y)^n)}
\]
\[
= 1 - F(y) - \frac{1}{n} (1 - F(y)^n) f(y) n^2 \geq 0.
\]

We also know from the proof of Lemma 1 that the conditional random variable $\tilde{\epsilon}_0 | \epsilon_0 < \epsilon_{(n)}$, associated with cdf $H$ is FOSD increasing in $p$. This fact, together with the observation that the integrand of (21) is an increasing function, imply that $\sigma (p; n)$ is increasing in $p$ as required.

To show $\sigma (p; n)$ decreases with $n$, we write
\[
\sigma (p; n) = \int_{\xi}^{\eta} \Pr (\epsilon_{(n-1)} < \tilde{\epsilon}_0 | \epsilon_{(n)} = y) \Pr (\epsilon_{(n)} = y | \epsilon_0 > \tilde{\epsilon}_0) \, dy
\]

Then, we make the following two claims:

Claim: For arbitrary constant $y \in [\xi, \eta]$, $\Pr (\epsilon_{(n-1)} < \tilde{\epsilon}_0 | \epsilon_{(n)} = y)$ is decreasing in $n$ and $y$. By definition,
\[
\Pr (\epsilon_{(n-1)} < \tilde{\epsilon}_0 | \epsilon_{(n)} = y) = \frac{\int_{\xi_p}^{\eta_p} nF (\min \{\tilde{\epsilon}_0, y\})^{n-1} f(y) \, dF_0 (\tilde{\epsilon}_0 - p)}{nF(y)^{n-1} f(y)} = \int_{\xi_p}^{\eta_p} \left( \frac{F (\min \{\tilde{\epsilon}_0, y\})}{F(y)} \right)^{n-1} \, dF_0 (\tilde{\epsilon}_0 - p),
\]
which is clearly decreasing in $n$ and $y$.

Claim: $\epsilon_{(n)} | \epsilon_0 > \tilde{\epsilon}_0$ is FOSD increasing in $n$. By definition, the corresponding CDF is
\[
\Pr (\epsilon_{(n)} < x | \epsilon_{(n)} > \tilde{\epsilon}_0) = \int_{\xi_p}^{\eta_p} \left[ F (x)^n - F (\min \{\tilde{\epsilon}_0, x\})^n \right] \, dF_0 (\tilde{\epsilon}_0 - p) \\
= \int_{\xi_p}^{\eta_p} \left[ 1 - F (\min \{\tilde{\epsilon}_0, x\})^n \right] \, dF_0 (\tilde{\epsilon}_0 - p) \\
= \int_{\xi_p}^{\eta_p} \left( \frac{F (x)^n - F (\min \{\tilde{\epsilon}_0, x\})^n}{1 - F (\min \{\tilde{\epsilon}_0, x\})^n} \right) \, dH (\tilde{\epsilon}_0; n, p),
\]
where $H (\tilde{\epsilon}_0; n, p)$ is the conditional distribution function defined in (16). We first observe that the integrand is decreasing in $n$: showing this is equivalent to showing $\frac{n^a - 1}{b^a - 1}$, $(1 < a < b)$ is decreasing in $n$, which is easily verified from calculating $\frac{d}{dn} \left( \frac{n^a - 1}{b^a - 1} \right) \leq 0$. Likewise, the integrand is decreasing in $\tilde{\epsilon}_0$. These two observations, together with the proven result that the conditional random variable $\tilde{\epsilon}_0 | \epsilon_0 < \epsilon_{(n)}$ associated with cdf $H$ is FOSD increasing in $n$, implies $\Pr (\epsilon_{(n)} < x | \epsilon_{(n)} > \tilde{\epsilon}_0)$ is decreasing in $n$ as required.
Using these two claims, we have for any \( n' \geq n \),
\[
\sigma(p; n) \\
\geq \int_\tilde{c} \Pr(\epsilon(n') < \tilde{\epsilon}_0|\epsilon(n') = y) \Pr(\epsilon(n) = y|\epsilon(n') > \tilde{\epsilon}_0) \, dy \\
\geq \int_\tilde{c} \Pr(\epsilon(n') < \tilde{\epsilon}_0|\epsilon(n') = y) \Pr(\epsilon(n') = y|\epsilon(n') > \tilde{\epsilon}_0) \, dy \\
= \Pr(\epsilon(n') < \tilde{\epsilon}_0|\epsilon(n') > \tilde{\epsilon}_0) = \sigma(p; n').
\]
So \( \sigma(p; n) \) is indeed decreasing in \( n \). □

**Proof. (Proposition 2).** The first-order conditions for buyer and seller fees, after applying symmetry, are given by (12). To prove the existence and uniqueness of \( \hat{p}^b \) and \( \hat{p}^s \) defined in (12), denote \( \hat{T} \equiv \hat{p}^b + \hat{p}^s \) and \( M(\hat{p}^s) \equiv \frac{1 - G(\hat{p}^s)}{g(\hat{p}^s)} \). From equilibrium characterization equation, we know for \( n \) fixed:
\[
\hat{T} - c = X(\hat{p}^b) = M(\hat{p}^s) \sigma(\hat{p}^b).
\]
We know from Lemma 1 that \( X(.) \) is weakly decreasing. If either \( X(.) \) is constant, then it is easily seen that \( \hat{T}, \hat{p}^b \), and \( \hat{p}^s \) can be uniquely pinned down. When \( X(.) \) is strictly decreasing, we can express \( \hat{p}^b = X^{-1}(\hat{T} - c) \) and \( \hat{p}^s = M^{-1} \left( \frac{\hat{T} - c}{\sigma(X^{-1}(\hat{T} - c))} \right) \), so that
\[
\hat{T} = X^{-1}(\hat{T} - c) + M^{-1} \left( \frac{\hat{T} - c}{\sigma(X^{-1}(\hat{T} - c))} \right),
\]
where the right-hand side is strictly decreasing in \( \hat{T} \) because \( X^{-1} \) and \( M^{-1} \) are both strictly decreasing, while \( \sigma \) is an increasing function (Lemma 3). This implies that there exist a unique fixed point \( \hat{T} \) that solves (23) and hence solves (22) by construction. Then, \( \hat{p}^b \) and \( \hat{p}^s \) can be uniquely determined from the one-to-one relations \( \hat{p}^b = X^{-1}(\hat{T} - c) \) and \( \hat{p}^s = M^{-1} \left( \frac{\hat{T} - c}{\sigma(X^{-1}(\hat{T} - c))} \right) \). □

**Proof. (Proposition 3).** The proposition is equivalent to a comparative static statement with respect to an exogenously given \( \sigma \) on an equilibrium pinned down by (12). This is because, if we treat \( \sigma \) as exogenous, then (6) is simply a special case of (12) with \( \sigma = 1 \), so it suffices to show \( \frac{d\hat{p}^b}{d\sigma} < 0 \) and \( \frac{d\hat{p}^s}{d\sigma} > 0 \). Applying total differentiation on (6) and writing in matrix form, we have
\[
\begin{bmatrix}
1 - \frac{\partial X}{\partial p^b} \\
1 - \frac{\partial X}{\partial p^s} \\
1 - \sigma \frac{\partial^2 G(\hat{p}^s)}{g(\hat{p}^s)} \\
1 - \sigma \frac{\partial^2 G(\hat{p}^s)}{g(\hat{p}^s)}
\end{bmatrix} \begin{bmatrix}
\frac{d\hat{p}^b}{d\sigma} \\
\frac{d\hat{p}^s}{d\sigma} \\
\frac{d\hat{p}^b}{d\sigma} \\
\frac{d\hat{p}^s}{d\sigma}
\end{bmatrix} = \begin{bmatrix}
0 \\
0 \\
0 \\
0
\end{bmatrix}.
\]
Given \( \frac{\partial X}{\partial p^b} \leq 0 \) (Lemma 1) and \( \frac{\partial^2 G(\hat{p}^s)}{g(\hat{p}^s)} > 0 \), the matrix in (24) has determinant
\[
\text{Det} \equiv \left( 1 - \frac{\partial X}{\partial p^b} \right) \left( 1 - \sigma \frac{\partial^2 G(\hat{p}^s)}{g(\hat{p}^s)} \right) > 1 > 0.
\]
Cramer’s rule gives
\[
\frac{d\hat{p}^s}{d\sigma} = \frac{1}{\text{Det}} \begin{bmatrix}
1 - \frac{\partial X}{\partial p^b} \\
0 \\
1 - \sigma \frac{\partial^2 G(\hat{p}^s)}{g(\hat{p}^s)} \\
1 - \sigma \frac{\partial^2 G(\hat{p}^s)}{g(\hat{p}^s)}
\end{bmatrix} > 0, \quad \text{and} \quad \frac{d\hat{p}^b}{d\sigma} = \frac{1}{\text{Det}} \begin{bmatrix}
0 \\
1 - \sigma \frac{\partial^2 G(\hat{p}^s)}{g(\hat{p}^s)} \\
1 - \frac{\partial^2 G(\hat{p}^s)}{g(\hat{p}^s)} \\
1 - \frac{\partial^2 G(\hat{p}^s)}{g(\hat{p}^s)}
\end{bmatrix} \sigma < 0.
\]
Finally,
\[
\frac{dp^s}{d\sigma} + \frac{dp^b}{d\sigma} = -\frac{1}{\Det} \left( \frac{\partial X}{\partial \vec{p}} \right) \left( 1 - G(\hat{p}^s) \right) \geq 0.
\]

\[\blacksquare\]

**Proof. (Proposition 4).** Total differentiation on (6) and write in matrix form, we have
\[
\begin{bmatrix}
1 - \frac{\partial X}{\partial \vec{p}} & 1 \\
1 - M \frac{\partial \sigma}{\partial \vec{p}} & 1 - \sigma \frac{\partial M}{\partial \vec{p}}
\end{bmatrix}
\begin{bmatrix}
\frac{dp^b}{dn} \\
\frac{dp^s}{dn}
\end{bmatrix}
= \begin{bmatrix}
\frac{\partial X}{\partial n} \\
\frac{\partial M}{\partial n}
\end{bmatrix}.
\]

By Lemma 1, \(\frac{\partial X}{\partial \vec{p}} < 0\). A direct application of Cramer’s rule then shows \(\frac{dp^s}{dn} > 0\), \(\frac{dp^b}{dn} < 0\), and \(\frac{dp^s}{dn} + \frac{dp^b}{dn} \leq 0\) as required. \[\blacksquare\]

**Proof. (Proposition 5).** Denote \(M(\hat{p}^s) \equiv \frac{1 - G(\hat{p}^s)}{\sigma(\hat{p}^s)}\). Total differentiation on (12), in matrix form, gives
\[
\begin{bmatrix}
1 - \frac{\partial X}{\partial \vec{p}} & 1 \\
1 - M \frac{\partial \sigma}{\partial \vec{p}} & 1 - \sigma \frac{\partial M}{\partial \vec{p}}
\end{bmatrix}
\begin{bmatrix}
\frac{dp^b}{dn} \\
\frac{dp^s}{dn}
\end{bmatrix}
= \begin{bmatrix}
\frac{\partial X}{\partial n} \\
\frac{\partial M}{\partial n}
\end{bmatrix}.
\]

Since \(\frac{\partial X}{\partial \vec{p}} \leq 0\), \(\frac{\partial M}{\partial \vec{p}} < 0\), and \(\frac{\partial \sigma}{\partial \vec{p}} > 0\) (Lemma 1 and Lemma 3), so accordingly the matrix in (25) has determinant
\[
\Det \equiv \left( 1 - \frac{\partial X}{\partial \vec{p}} \right) \left( 1 - \sigma \frac{\partial M}{\partial \vec{p}} \right) - 1 + M \frac{\partial \sigma}{\partial \vec{p}} > 0.
\]

By Cramer’s rule,
\[
\frac{dp^s}{dn} = \frac{1}{\Det} \det \left( \begin{array}{cc}
1 - \frac{\partial X}{\partial \vec{p}} & \frac{\partial X}{\partial m} \\
1 - M \frac{\partial \sigma}{\partial \vec{p}} & M \frac{\partial M}{\partial m}
\end{array} \right) + \frac{\sigma}{\Det} \Det \left( \frac{\partial \sigma}{\partial \vec{p}} \frac{\partial X}{\partial \vec{p}} - \frac{\partial \sigma}{\partial \vec{p}} \frac{\partial X}{\partial \vec{p}} \frac{\partial \sigma}{\partial \vec{p}} \frac{\partial X}{\partial \vec{p}} \right) \geq 0.
\]
\[
\frac{dp^b}{dn} = \frac{1}{\Det} \det \left( \begin{array}{cc}
\frac{\partial X}{\partial m} & 1 - \frac{\partial X}{\partial \vec{p}} \\
M \frac{\partial \sigma}{\partial \vec{p}} & 1 - \sigma \frac{\partial M}{\partial \vec{p}}
\end{array} \right) - \frac{\partial \sigma}{\partial \vec{p}} \Det \left( \frac{\partial \sigma}{\partial \vec{p}} \frac{\partial X}{\partial \vec{p}} - \frac{\partial \sigma}{\partial \vec{p}} \frac{\partial X}{\partial \vec{p}} \frac{\partial \sigma}{\partial \vec{p}} \frac{\partial X}{\partial \vec{p}} \right) \leq 0.
\]

We know \(\frac{\partial X}{\partial m} \leq 0\) and \(\frac{\partial \sigma}{\partial \vec{p}} \leq 0\) (Lemma 1 and Lemma 3), therefore
\[
\frac{dp^s}{dn} + \frac{dp^b}{dn} = \frac{1}{\Det} \Det \left( \begin{array}{cc}
\frac{\partial \sigma}{\partial \vec{p}} \frac{\partial X}{\partial \vec{p}} & \frac{\partial \sigma}{\partial \vec{p}} \frac{\partial X}{\partial \vec{p}} \\
\frac{\partial \sigma}{\partial \vec{p}} \frac{\partial X}{\partial \vec{p}} & 1 - \sigma \frac{\partial M}{\partial \vec{p}}
\end{array} \right) \leq 0.
\]

Meanwhile, to show \(\frac{dp^s}{dn} \leq 0\), from (27) it suffices to show that \(M \frac{\partial \sigma}{\partial \vec{p}} - \frac{\partial X}{\partial m} \leq 0\). To show that, we denote
\[
\Lambda \equiv \int_{\hat{p}^s}^{\hat{p}^b} \int_{0}^{t} f \left( \max \{\epsilon, \epsilon_0 + \hat{p}^b \} \right) dF(\epsilon)^{n-1} dF_0(\epsilon_0),
\]
and let \(\Lambda'\) be its partial derivative wrt \(\vec{n}\). We note that if \(f\) is decreasing then the fact that \(F(\epsilon)^{n-1}\) being FOSD increasing in \(\vec{n}\) means that \(\Lambda' \leq 0\). Computing the relevant derivatives and utilizing the
equilibrium condition \( M = \frac{X}{\sigma} \), we get

\[
M \frac{\partial\sigma}{\partial n} - \frac{\partial X}{\partial n} = \frac{\int_{\bar{\epsilon}}^{\epsilon} [1 - F (\epsilon_0 + \bar{p}^b)] dF_0 (\epsilon_0)}{n\Lambda} \times \left( \frac{\int_{\bar{\epsilon}}^{\epsilon} \left[ \ln (F (\epsilon_0 + \bar{p}^b)) F (\epsilon_0 + \bar{p}^b)^{n-1} (1 - F (\epsilon_0 + \bar{p}^b)) \right] dF_0 (\epsilon_0)}{\int_{\bar{\epsilon}}^{\epsilon} [F (\epsilon_0 + \bar{p}^b)^{n-1} (1 - F (\epsilon_0 + \bar{p}^b))] dF_0 (\epsilon_0)} + \frac{\Lambda'}{\Lambda} \right)
\]

\[
< 0
\]

where the inequality is due to \( \ln (F (\epsilon_0 + \bar{p}^b)) < 0 \) and \( \Lambda' \leq 0 \).

Finally, to show \( \frac{\partial \bar{p}^b}{\partial n} \geq 0 \), we first note that \( g \) being decreasing implies that \( \frac{\partial M}{\partial \bar{p}^b} \geq -1 \). Therefore,

\[
\frac{\partial \bar{p}^b}{\partial n} \geq \frac{1}{Det} \left[ (1 + \sigma) \frac{\partial X}{\partial n} - M \frac{\partial \sigma}{\partial n} \right].
\]

Substitute for the relevant terms and again utilizing the equilibrium condition \( M = \frac{X}{\sigma} \), then \( (1 + \sigma) \frac{\partial X}{\partial n} - M \frac{\partial \sigma}{\partial n} \geq 0 \) if and only if

\[
0 \geq \frac{\int_{\bar{\epsilon}}^{\epsilon} \left[ \ln (F (\epsilon_0 + \bar{p}^b)) F (\epsilon_0 + \bar{p}^b)^{n-1} (1 - F (\epsilon_0 + \bar{p}^b)) \right] dF_0 (\epsilon_0)}{\int_{\bar{\epsilon}}^{\epsilon-p} [F (\epsilon_0 + \bar{p}^b)^{n-1} (1 - F (\epsilon_0 + \bar{p}^b))] dF_0 (\epsilon_0)}
\]

\[
+ \sigma \left( \frac{1}{n} + \frac{\int_{\bar{\epsilon}}^{\epsilon} [1 - F (\epsilon_0 + \bar{p}^b)^n] dF_0 (\epsilon_0)}{\int_{\bar{\epsilon}}^{\epsilon} [1 - F (\epsilon_0 + \bar{p}^b)^n] dF_0 (\epsilon_0)} \right) + \frac{\Lambda'}{\Lambda} (1 + \sigma).
\]

We know \( \Lambda' \leq 0 \). Meanwhile, applying L’Hopital rule twice shows that the first two components converges to zero when \( \bar{p}^b \to (\bar{\epsilon} - \epsilon) \). Moreover, calculating the first-order derivative shows that the sum of the first two components to be increasing in \( \bar{p}^b \), hence the sum is non-positive for all \( \bar{p}^b \leq \bar{\epsilon} - \epsilon \). ■
References


This online appendix contains proofs of omitted results and details from the main paper, and provides a more detailed analysis of model extensions described in Section 5.

B Further details: multihoming buyers

B.1 Properties of the demand function

We examine the continuity and the property of demand derivative of demand function $Q_i(p_i^b, p_i^s; \hat{p})$ in (15).

Claim 1 For any $\hat{p}$, $Q_i(p_i^b, p_i^s; \hat{p})$ is continuous in $p_i^b$ and $p_i^s$.

Proof. Continuity with respect to $p_i^b$ is obvious. To show continuity with respect to $p_i^s$, note from (14) that $\lim_{p_i^s \to p_i^s^-} \hat{v}_m = \hat{p}^s$ for $m = 1, ..., n - 1$. Similarly, note from Lemma 2 that $\lim_{p_i^s \to p_i^s^+} \hat{v} = \hat{p}^s$. Thus,

$$\lim_{p_i^s \to p_i^s^-} Q_i(p_i^b, p_i^s; \hat{p}) = [1 - G(\hat{p}^s)] B_i^{(N)} + \sum_{m=0}^{n-2} [G(\hat{p}^s) - G(\hat{p}^s)] B_i^{(N,m)}$$

and

$$\lim_{p_i^s \to p_i^s^+} Q_i(p_i^b, p_i^s; \hat{p}) = [1 - G(\hat{p}^s)] B_i^{(N)}$$

so $Q_i(p_i^b, p_i^s; \hat{p})$ is continuous for all $p_i^b$ and $p_i^s$, which includes $(p_i^b, p_i^s) = \hat{p}$. □

Claim 2 For any $\hat{p}$,

$$\lim_{p_i^s \to p_i^s^-} \frac{dQ_i}{dp_i^s} (p_i^b, p_i^s; \hat{p}) \geq \lim_{p_i^s \to p_i^s^+} \frac{dQ_i}{dp_i^s} (p_i^b, p_i^s; \hat{p})$$

Equality holds if in addition (i) $n = 2$, or (ii) $F, F_0 \sim \text{Gumbel}(\mu, \gamma)$.

Proof. Consider first $p_i^s \geq \hat{p}^s$. Then the right-hand side derivative is

$$\lim_{p_i^s \to p_i^s^-} \frac{dQ_i}{dp_i^s} (p_i^b, p_i^s; \hat{p}) = \lim_{p_i^s \to p_i^s^-} \frac{d\hat{v}}{dp_i^s} B_i^{(N)} g(p_i^s)$$

and

$$\lim_{p_i^s \to p_i^s^+} \frac{dQ_i}{dp_i^s} (p_i^b, p_i^s; \hat{p}) = \lim_{p_i^s \to p_i^s^+} \sum_{j \in N, j \neq i} \left( B_j^{(N)} - B_j^{(N-1)} \right) + B_j^{(N)} B_i^{(N)} g(p_i^s)$$

so

$$\lim_{p_i^s \to p_i^s^-} \frac{dQ_i}{dp_i^s} (p_i^b, p_i^s; \hat{p}) = \lim_{p_i^s \to p_i^s^+} \sum_{j \in N, j \neq i} \left( B_j^{(N)} - B_j^{(N-1)} \right) + B_j^{(N)} B_i^{(N)} g(p_i^s).$$
Evaluating the above at $p_i^b = \hat{p}^b$, all platforms become symmetry so that functions $B_{i}^{(\Theta)}$ are the same for any set $\Theta$ and any given $j \in \Theta$. So, for simplicity we denote any such generic term as $B^{(\Theta)}$, hence we have

$$
\lim_{n_i \to \hat{p}_{i}^{+}} \frac{dQ_i}{dp_i} (p_i^b; \hat{p}) = \frac{-B^{(N)}B^{(N)}}{nB^{(N)} - (n - 1)B^{(N-i)}} g(\hat{p})
$$

(B.1)

When $p_i^* > \hat{p}$, the left hand side derivative is

$$
\lim_{p_i^* \to \hat{p}_{i}^{+}} \frac{dQ_i}{dp_i} (p_i^b; p_i^*; \hat{p})
$$

$$
= \lim_{p_i^* \to \hat{p}_{i}^{+}} \sum_{m=0}^{n-1} \left[ \frac{d\hat{v}_{m+1}}{dp_i} - \frac{d\hat{v}_{m}}{dp_i} \right] g(\hat{v}_m) B^{i}_{1}(N_{i,m})
$$

$$
= g(\hat{p}) \left[ \sum_{m=0}^{n-1} \left[ \frac{d\hat{v}_{m+1}}{dp_i} - \frac{d\hat{v}_{m}}{dp_i} \right] B^{(N_{i,m})} \right]
$$

(B.2)

where $\frac{d\hat{v}_{m}}{dp_i} = 0$ because $\hat{v}_n \equiv \hat{v}$, $\frac{d\hat{v}_{n-1}}{dp_i} = 1$ since $\hat{v}_0 \equiv p_i^b$, while

$$
\frac{d\hat{v}_m}{dp_i} \bigg|_{p_i^b = \hat{p}} = \frac{B^{(N_{i,m})} - B^{(N_{i,m-1})}}{(m+1)B^{(N_{i,m})} - mB^{(N_{i,m-1})}}
$$

in which $B^{(i)}$ is as denoted earlier due to symmetry. Hence, evaluating at $p_i^b = \hat{p}^b$, (B.2) can be expanded

$$
\frac{1}{g(\hat{p})} \lim_{p_i^* \to \hat{p}_{i}^{+}} \frac{dQ_i}{dp_i} (p_i^b; p_i^*; \hat{p})
$$

$$
= -\frac{d\hat{v}_{n-1}}{dp_i} B^{(N)} + \sum_{m=1}^{n-2} \left[ \frac{d\hat{v}_{m+1}}{dp_i} - \frac{d\hat{v}_m}{dp_i} \right] B^{(N_{i,m})} + \left( \frac{d\hat{v}_1}{dp_i} - 1 \right) B^{(N_{i,o})}
$$

$$
= \frac{-B^{(N_{i,n-1})}B^{(N_{i,n-1})}}{nB^{(N_{i,n-1})} - (n - 1)B^{(N_{i,n-2})}}
$$

$$
+ \frac{B^{(N_{i,n+1})} - B^{(N_{i,m})}}{(m+1)B^{(N_{i,m})} - mB^{(N_{i,m-1})}} B^{(N_{i,m})} \text{(B.3)}
$$

By definition, proving differentiability at $(p_i^b, p_i^*) = \hat{p}$ requires us to show

$$
\lim_{p_i^* \to \hat{p}_{i}^{+}} \frac{dQ_i}{dp_i} (p_i^b; p_i^*; \hat{p}) = \lim_{p_i^* \to \hat{p}_{i}^{+}} \frac{dQ_i}{dp_i} (\hat{p}^b, p_i^*; \hat{p})
$$

(B.4)

To prove this, we note that $N_{i,n-1} = N$ and that when all platforms are symmetry we have $N_{i,n-2} = N_{i-n}$ (because both sets denote a set of $n-1$ symmetry platforms). Then, substituting for (B.1) we can rewrite (B.3) as

$$
\lim_{p_i^* \to \hat{p}_{i}^{+}} \frac{dQ_i}{dp_i} (p_i^b; p_i^*; \hat{p}) = \lim_{p_i^* \to \hat{p}_{i}^{+}} \frac{dQ_i}{dp_i} (\hat{p}^b, p_i^*; \hat{p}) + g(\hat{p}) \Phi(n)
$$
where \( \Phi(n) \) is defined as the last three lines of (B.3), i.e.

\[
\Phi(n) = \frac{B^{(N_i,n-2)}B^{(N_i,n-1)}}{nB^{(N_i,n-1)} - (n-1)B^{(N_i,n-2)}}
\]

\[
+ \sum_{m=1}^{n-2} \left[ \frac{B^{(N_i,m+1)} - B^{(N_i,m)}}{(m+2)B^{(N_i,m+1)} - (m+1)B^{(N_i,m)}} - \frac{B^{(N_i,m)} - B^{(N_i,m-1)}}{(m+1)B^{(N_i,m)} - mB^{(N_i,m-1)}} \right]B^{(N_i,m)}
\]

\[
+ \left( \frac{B^{(N_i,1)} - B^{(N_i,0)}}{2B^{(N_i,1)} - B^{(N_i,0)}} - 1 \right)B^{(N_i,0)}
\]

To conclude (B.4), it suffices to prove by induction that \( \Phi(n) \geq 0 \) for all \( n \geq 2 \). First, when \( n = 2 \) we have \( N_{i,1} = N \) so

\[
\Phi(2) = \frac{B^{(N_i,0)}B^{(N_i,1)}}{2B^{(N_i,1)} - B^{(N_i,0)}} + \left( \frac{B^{(N_i,1)} - B^{(N_i,0)}}{2B^{(N_i,1)} - B^{(N_i,0)}} - 1 \right)B^{(N_i,0)}
\]

\[
= \left[ \frac{B^{(N_i,0)}B^{(N_i,1)}}{2B^{(N_i,1)} - B^{(N_i,0)}} - \frac{B^{(N_i,0)}B^{(N_i,1)}}{2B^{(N_i,1)} - B^{(N_i,0)}} + B^{(N_i,0)} \right] = 0
\]

Note that this also proves the first part of the claim, that is, the case of \( n = 2 \).

By inductive hypothesis, suppose \( \Phi(n-1) \geq 0 \). For \( n \geq 3 \), if we expand one more term from the summation in \( \Phi(n) \) and rearrange terms we get

\[
\Phi(n) = \frac{B^{(N_i,n-2)}B^{(N_i,n-1)}}{nB^{(N_i,n-1)} - (n-1)B^{(N_i,n-2)}}
\]

\[
+ \frac{B^{(N_i,n-1)} - B^{(N_i,n-2)}}{nB^{(N_i,n-1)} - (n-1)B^{(N_i,n-2)}} - \frac{B^{(N_i,n-2)} - B^{(N_i,n-3)}}{(n-1)B^{(N_i,n-2)} - (n-2)B^{(N_i,n-3)}}
\]

\[
+ \sum_{m=1}^{n-3} \left[ \frac{B^{(N_i,m+1)} - B^{(N_i,m)}}{(m+2)B^{(N_i,m+1)} - (m+1)B^{(N_i,m)}} - \frac{B^{(N_i,m)} - B^{(N_i,m-1)}}{(m+1)B^{(N_i,m)} - mB^{(N_i,m-1)}} \right]B^{(N_i,m)}
\]

\[
+ \frac{B^{(N_i,n-2)}B^{(N_i,n-1)}}{nB^{(N_i,n-1)} - (n-1)B^{(N_i,n-2)}} - \frac{B^{(N_i,n-2)}B^{(N_i,n-1)}}{nB^{(N_i,n-1)} - (n-1)B^{(N_i,n-2)}} - \frac{B^{(N_i,n-2)}B^{(N_i,n-1)}}{nB^{(N_i,n-1)} - (n-1)B^{(N_i,n-2)}} + \Phi(n-1)
\]

By inductive hypothesis \( \Phi(n-1) \geq 0 \). Therefore, to prove \( \Phi(n) \geq 0 \) it remains to show

\[
\frac{2B^{(N_i,n-1)}B^{(N_i,n-2)}}{nB^{(N_i,n-1)} - (n-1)B^{(N_i,n-2)}} \geq \frac{B^{(N_i,n-2)}}{(n-1)B^{(N_i,n-2)} - (n-2)B^{(N_i,n-3)}}.
\]

Rearranging the terms and cancel out common coefficients, the inequality above is equivalent to

\[
0 \leq \frac{(B^{(N_i,n-2)} - B^{(N_i,n-3)})}{B^{(N_i,n-1)}} - \frac{(B^{(N_i,n-3)} - B^{(N_i,n-4)})}{B^{(N_i,n-3)}}.
\]

(B.6)

We know \( \frac{\partial}{\partial k} B^{(N_i,k)} \leq 0 \), so we simply need to show that \( B \) is decreasing in \( k \) with a decreasing magnitude, i.e. \( B \) is convex in \( k \). Recall from definition that for \( k \in \{0, \ldots, n-1\} \), we have

\[
B^{(N_i,k)} = \int_{\bar{z}}^{\bar{z} - \rho^k} \left[ \frac{1}{k+1} \right] dF_0(\epsilon_0).
\]
Convexity of $B$ in $k$ then follows from the observation that $\frac{1-F(\cdot;\gamma)}{\gamma^{k+1}}$ is convex in $k$, and that convexity is preserved by integration when the integrand is always positive over the entire region of integration. So, $\Phi(n) \geq 0$ for all $n \geq 2$ as required. Finally, in the special case of $F$ following the Gumbel distribution whereby

$$B^{(N)} = \frac{\exp \{-p_i^b/\gamma\}}{1 + (k + 1) \exp \{-p_i^b/\gamma\}},$$

it can be directly verified that (B.6) holds in equality, so that $\Phi(n) = 0$ for all $n \geq 2$. \qed

\section*{B.2 Quasi-concavity of profit function}

A difficulty to establish equilibrium existence in the case with multihoming buyers is that the profit function may not be globally quasi-concave. Providing sufficient conditions under which it is quasi-concave is very difficult because the bivariate demand function is piece-wise defined by (10) and (15), the latter of which is a sum of $n$ integrals $B^{(N+1)}$. The difficulty is that even if each of these integrals is log-concave, it is not guaranteed that the sum will be quasi-concave. The only exception we found is when $F$ and $F_0$ correspond Gumbel distribution and $G$ correspond to uniform distribution, whereby the demand function $Q_i(p_i^b, p_i^s; \hat{p})$ turns out to be log-concave in $(p_i^b, p_i^s)$ so that the profit function is quasi-concave, as claimed in the main text:

\begin{claim}
If $F, F_0 \sim \text{Gumbel}(\mu, \gamma)$, and $G \sim \text{Uniform over} \ [\bar{v}, \bar{v}]$, then for all $(p_i^b, p_i^s)$,

$$Q_i(p_i^b, p_i^s; \hat{p}) = \left(\frac{\hat{v} + \hat{p}^s (n-1) \exp \{-\hat{p}^b/\gamma\}}{\hat{v} - \hat{v}} \times \frac{\exp \{-p_i^b/\gamma\}}{1 + \exp \{-p_i^b/\gamma\} + (n-1) \exp \{-\hat{p}^b/\gamma\}}\right).$$

Moreover, $Q_i(p_i^b, p_i^s; \hat{p})$ is jointly log-concave in $(p_i^b, p_i^s)$.
\end{claim}

\begin{proof}
We first consider $p_i^s \geq \hat{p}^s$. When $F$ and $F_0$ correspond Gumbel distribution,

$$B_i^{(N)} = \frac{\exp \{-p_i^b/\gamma\}}{1 + \exp \{-p_i^b/\gamma\} + (n-1) \exp \{-\hat{p}^b/\gamma\}},$$

$$B_j^{(N-1)} = \frac{\exp \{-\hat{p}^b/\gamma\}}{1 + \exp \{-p_i^b/\gamma\} + (n-1) \exp \{-\hat{p}^b/\gamma\}},$$

$$B_j^{(N)} = \frac{\exp \{-\hat{p}^b/\gamma\}}{1 + \exp \{-p_i^b/\gamma\} + (n-1) \exp \{-\hat{p}^b/\gamma\}}.$$\end{proof}

Substituting for these terms and simplifying, we get

$$\hat{v} = p_i^s \hat{v} \frac{B_i^{(N)} - p_i^s \sum_{j \in N-i} B_j^{(N-1)} - B_j^{(N)}}{B_i^{(N)} - \sum_{j \in N-i} (B_j^{(N-1)} - B_j^{(N)})} \geq \hat{p}^b,$$

$$= p_i^s (1 + (n-1) \exp \{-\hat{p}^b/\gamma\}) - \hat{p}^b (n-1) \exp \{-\hat{p}^b/\gamma\}.$$
Therefore

\[
Q_i(p_i^b, p_i^s; \hat{p}) \mid p_i^b \geq \bar{p}^* = (1 - G(\bar{v})) B_1^{(N)}
\]

\[
= \left( \frac{\hat{v} + \bar{p}^* (n - 1) \exp \{-\bar{p}^b/\gamma\} - \left[ 1 + (n - 1) \exp \{-\bar{p}^b/\gamma\} \right] p_i^s}{\hat{v} - \bar{v}} \right) \frac{\exp \{-p_i^b/\gamma\}}{1 + \exp \{-p_i^b/\gamma\} + (n - 1) \exp \{-\bar{p}^b/\gamma\}}
\]

Likewise, when \( p_i^s < \bar{p}^* \). The Gumbel assumption implies

\[
B_i^{(N, m)} = \frac{\exp \{-p_i^b/\gamma\}}{1 + \exp \{-p_i^b/\gamma\} + m \exp \{-\bar{p}^b/\gamma\}} \quad \text{and} \quad B_j^{(N, m)} = \frac{\exp \{-\bar{p}^b/\gamma\}}{1 + \exp \{-\bar{p}^b/\gamma\} + m \exp \{-\bar{p}^b/\gamma\}}.
\]

Substituting for these terms and simplifying, we get for \( m = 1, \ldots, n - 1 \):

\[
\hat{v}_m = \frac{p_i^s \left( B_i^{(N, m)} - B_i^{(N, m - 1)} \right) + \bar{p}^* \left( m B_j^{(N, m)} - (m - 1) B_j^{(N, m - 1)} \right)}{B_i^{(N, m)} - B_i^{(N, m - 1)} + m B_j^{(N, m)} - (m - 1) B_j^{(N, m - 1)}}
\]

\[
= -p_i^s \exp \{-p_i^b/\gamma\} + \bar{p}^* (1 + \exp \{-\bar{p}^b/\gamma\})
\]

Notice that \( \hat{v}_m \) is independent of \( m \), implying that \( \hat{v}_1 = \hat{v}_2 = \ldots = \hat{v}_{n-1} \). Therefore, we can write

\[
Q_i(p_i^b, p_i^s; \hat{p}) \mid p_i^b < \bar{p}^*
\]

\[
= \left[ (1 - G(\hat{v}_{n-1})) B_1^{(N, n-1)} + [G(\hat{v}_1) - G(p_i^b^s)] B_i^{(N, n)} \right]
\]

\[
= \left( \frac{\hat{v} + \bar{p}^* (n - 1) \exp \{-\bar{p}^b/\gamma\} - \left[ 1 + (n - 1) \exp \{-\bar{p}^b/\gamma\} \right] p_i^s}{\hat{v} - \bar{v}} \right) \frac{\exp \{-p_i^b/\gamma\}}{1 + \exp \{-p_i^b/\gamma\} + (n - 1) \exp \{-\bar{p}^b/\gamma\}}
\]

\[
+ \left( \frac{-p_i^s \exp \{-p_i^b/\gamma\} + \bar{p}^* \left[ 1 + \exp \{-\bar{p}^b/\gamma\} \right] - p_i^s}{\hat{v} - \bar{v}} \right) \frac{\exp \{-p_i^b/\gamma\}}{1 + \exp \{-p_i^b/\gamma\}}
\]

\[
\quad = \left( (\hat{v} + \bar{p}^* (n - 1) \exp \{-\bar{p}^b/\gamma\} - \left[ 1 + (n - 1) \exp \{-\bar{p}^b/\gamma\} \right] p_i^s) \right) \frac{\exp \{-p_i^b/\gamma\}}{1 + \exp \{-p_i^b/\gamma\} + (n - 1) \exp \{-\bar{p}^b/\gamma\}}
\]

\[
= Q_i(p_i^b, p_i^s; \hat{p}) \mid p_i^b \geq \bar{p}^*.
\]

Therefore, under the imposed distributional assumptions the function form of \( Q_i(p_i^b, p_i^s; \hat{p}) \) is the same regardless of \( p_i^b \). Moreover, \( Q_i(p_i^b, p_i^s; \hat{p}) \) is multiplicatively separable in \( p_i^b \) and \( p_i^s \), whereby each multiplicative component is obviously log-concave in \( p_i^b \) and \( p_i^s \) respectively (recall that a logit-demand form is necessarily log-concave). Given that log-concavity is preserved by multiplication, we conclude that \( Q_i(p_i^b, p_i^s; \hat{p}) \) is log-concave in \( (p_i^b, p_i^s) \). ■

In order to determine the global quasi-concavity of the profit function in Section 4.2 for other distribution functions, we rely on numerical calculations. Specifically, we verified that quasi-concavity is satisfied in the two numerical examples presented in Section 6. In addition, we considered the case of \( F \) and \( F_0 \sim \text{Gumbel}(\mu, \gamma) \) (location parameter \( \gamma \) can be normalized to 1 without loss of generality) and \( G \sim \text{Normal}(\mu_G, \sigma^2) \) for all combinations of the following parameter values: \( n \in \{2, 3, 4\} \), \( c \in \{0.1, 1\} \), \( \mu \in \{0.5, 1, 2\} \), \( \mu_G \in \{-4, 0, 3\} \), and \( \sigma^2 \in \{1, 2\} \). We also considered alternative distribution assumptions for \( F \), \( F_0 \), and \( G \); specifically, \( \text{Uniform}(0, 1) \) and \( \text{Exponential} \) (1). In all the cases considered, the quasi-concavity assumption was satisfied, suggesting it does not require very special conditions to hold. Details and codes of the numerical calculations are available from the authors upon request.
C Multihoming buyers with logit buyer quasi-demand

In this section, we analyze in detail the special case when $F$ and $F_0 \sim \text{Gumbel}(\mu, \gamma)$ with location parameter $\gamma$ normalized to 1. In this case, it can be shown that the buyer quasi-demand as defined in (1) follows the standard multinomial logit form widely used in the industrial organization literature:

$$B_i^{(\Theta)} = \frac{\exp \{-p_i^b/\gamma\}}{1 + \sum_{j \in \Theta} \exp \{-p_j^b/\gamma\}}.$$

We can then explicitly compute the inverse semi-elasticity function (5) and the loyalty index function (11) as:

$$X(p^b; n) = \mu \left( \frac{1 + n \exp \{-p^b/\gamma\}}{1 + (n - 1) \exp \{-p^b/\gamma\}} \right) \text{ and } \sigma(p^b; n) = \frac{1}{1 + (n - 1) \exp \{-p^b/\gamma\}}. \quad \text{(C.1)}$$

Then the following proposition is analogous to the second part of Proposition 5 in the main text. Note that the effect entry on the total fee has already been proven in the first part of Proposition 5, so we do not repeat it here.

**Proposition 9** (Platform entry with buyer-multihoming) In the equilibrium characterized by Proposition 2, platform entry (increasing $n$) decreases seller fee $\hat{p}^s$. Platform entry increases buyer fee $\hat{p}^b$ if in addition

$$4(n - 1) > \epsilon_s(p) \text{ for all } p \in [\underline{v}, \overline{v}], \quad \text{(C.2)}$$

where $\epsilon_s(p) \equiv -\frac{d}{dp} \left( \frac{1 - G(p)}{g(p)} \right)$ is the log-curvature index of seller quasi-demand.

**Proof.** We proceed by total differentiation and Cramer rule as in the proof of Proposition 5, and we arrive at

$$\frac{d\hat{p}^s}{dn} = \frac{1}{\text{Det}} \begin{vmatrix} 1 - \frac{\partial X}{\partial \hat{p}^b} & \frac{\partial X}{\partial \hat{p}^b} \\ 1 - M \frac{\partial \sigma}{\partial \hat{p}^b} & M \frac{\partial \sigma}{\partial \hat{p}^b} \end{vmatrix} = \frac{1}{\text{Det}} \begin{pmatrix} M \frac{\partial \sigma}{\partial \hat{p}^b} - \frac{\partial X}{\partial \hat{p}^b} \\ \frac{\partial X}{\partial \hat{p}^b} - M \frac{\partial \sigma}{\partial \hat{p}^b} \end{pmatrix} < 0; \quad \text{(C.3)}$$

$$\frac{d\hat{p}^b}{dn} = \frac{1}{\text{Det}} \begin{vmatrix} M \frac{\partial \sigma}{\partial \hat{p}^s} & M \frac{\partial \sigma}{\partial \hat{p}^s} \\ 1 - \sigma \frac{\partial M}{\partial \hat{p}^s} & 1 - \sigma \frac{\partial M}{\partial \hat{p}^s} \end{vmatrix} = \frac{1}{\text{Det}} \begin{pmatrix} \frac{\partial X}{\partial \hat{p}^s} - M \frac{\partial \sigma}{\partial \hat{p}^s} \\ M \frac{\partial \sigma}{\partial \hat{p}^s} - \frac{\partial M}{\partial \hat{p}^s} \frac{\partial X}{\partial \hat{p}^s} \end{pmatrix}. \quad \text{(C.4)}$$

Utilizing from (12) that $M = \frac{1 - G(p)}{g(p)} = X/\sigma = \mu \left( 1 + n \exp \{-\hat{p}^b/\gamma\} \right)$, we can verify

$$M \frac{\partial \sigma}{\partial \hat{p}^b} - \frac{\partial X}{\partial \hat{p}^b} = \frac{-\mu \exp \{-\hat{p}^b/\gamma\}}{1 + (n - 1) \exp \{-\hat{p}^b/\gamma\}} < 0.$$

It remains to show that (C.3) is positive under the regularity condition (C.2). After substituting for the corresponding expressions, one can simplify (C.3) as

$$\frac{d\hat{p}^b}{dn} = \frac{1}{\text{Det}} \left( \frac{\mu \exp \{-\hat{p}^b/\gamma\}^2}{[1 + (n - 1) \exp \{-\hat{p}^b/\gamma\}]^3} \left( \frac{[1 + (n - 1) \exp \{-\hat{p}^b/\gamma\}]^2}{\exp \{-\hat{p}^b/\gamma\}} \right) + \frac{\partial M}{\partial \hat{p}^s} \right). \quad \text{(C.4)}$$
Hence, $\frac{dp^b}{dp} > 0$ if and only if

$$\frac{[1 + (n - 1) \exp \{-p^b/\gamma\}]^2}{\exp \{-p^b/\gamma\}} > -\frac{\partial M}{\partial p^s}. \tag{C.5}$$

We can bound the left-hand side (LHS) of (C.5) from below as follows:

$$LHS = \frac{1}{\exp \{-p^b/\gamma\}} + (n - 1)^2 \exp \{-p^b/\gamma\} + 2(n - 1) \geq 4(n - 1)$$

where the last inequality is due to the inequality of arithmetic and geometric means. Therefore, condition (C.2) implies (C.5) as required.

The regularity condition (C.2) requires that seller quasi-demand $1 - G$ is not too log-concave, that is, $\epsilon_s$ is not too high relative to $n$. We first note that $\epsilon_s > 0$ if $1 - G$ is strictly log-concave, $\epsilon_s \geq 1$ if $1 - G$ is concave, and $\epsilon_s \leq 1$ if $1 - G$ is convex. The latter implies that (C.2) is immediately satisfied by all distributions with weakly decreasing densities (whereby $1 - G$ is convex), e.g. a normal distribution right-truncated at the mean. Demand log-curvature index $\epsilon_s$ has been featured prominently in the classical price theory and public finance (see, e.g. Bulow and Pfleiderer, 1983; Bagnoli and Bergstrom, 2005; Weyl and Farbinger, 2013). These works adopt the following equivalent definition for log-curvature index:

$$\epsilon_s = -\frac{d^2}{dp^2} \ln [1 - G(p)] = -\frac{d}{dp} \ln [1 - G(p)]^2.$$

In addition, a statistical way of viewing $\epsilon_s$, as discussed extensively by Weyl and Farbinger (2013), is to notice that if we let $G$ be the generalized Pareto distribution (GPD) with tail index $\alpha$ then $\epsilon_s = \alpha$ is a constant. Formally, a GPD can be described with cdf

$$G(p) = 1 - (1 - \lambda \alpha (p - 1))^{1/\alpha},$$

where $\lambda > 0$ is a scale parameter and $\alpha > -1$ is the tail index (or shape parameter). Our assumption on log-concavity requires $\alpha > 0$, whereby the support of $G$ is $[1, 1 + 1/\lambda \alpha]$. GPD is also known as the class of constant pass-through demand as proposed by Bulow and Pfeiderer (1983), and it includes commonly used distributions such as uniform ($\epsilon_s = 1$) and exponential ($\epsilon_s \to 0$) as special cases. Under GPD, (C.2) is satisfied if $n$ is large relative to constant $\alpha$.

Intuitively, condition (C.2) governs the relative magnitude between the shift in $P^s (p^b; n)$ curve (reduced loyalty effect) and the shift in $P^b (p^s; n)$ curve (reduced markup effect). Under the current demand specification, the reduced loyalty effect dominates so that the shift in $P^s (p^b; n)$ curve is always greater in magnitude than $P^b (p^s; n)$ curve, and the equilibrium seller fee always decreases. However, how the equilibrium buyer fee changes will generally depend on the relative magnitude of shift between these two curves. To illustrate this point, we consider a numerical example in Figure 7 assuming $G \sim \text{Normal}(\mu_G, \sigma^2)$. In the first panel, condition (C.2) holds so that the shift in $P^s (p^b; n)$ is much greater than $P^b (p^s; n)$ curve, and consequently the equilibrium buyer fee increases with entry. In contrast, in the second panel where the condition (C.2) fails, the two curves shifts in approximately the same magnitude, so that entry decreases buyer and seller fees.
Figure 7: Platform entry \((n = 2 \text{ to } n = 3)\) with buyer-multihoming when \(c = 0, \mu = 1,\) and: (A) \(G \sim \text{Normal}(2, 1);\) and (B) \(G \sim \text{Normal}(2, 0.5).\)

D Extension: partial-multihoming buyers

In this extended model, an exogenous fraction \(\lambda > 0\) of buyers are allowed to multihome while the remaining fraction \(1 - \lambda\) of buyers are restricted to singlehome. We continue to assume that buyers do not observe seller-side fees, and they hold passive beliefs on that. With slight abuse of notation, we continue to denote the equilibrium fees in this setting as \((\hat{p}_b, \hat{p}_s)\).

We first derive the demand functions facing each platform as in Section 4.2. Again, we can focus on the participation equilibrium in which all multihoming buyers join all platforms, while singlehoming buyers each selects one of the platforms to join. Consider a deviating platform \(i\) that charges \((p^*_b, p^*_s) \neq (\hat{p}_b, \hat{p}_s)\). Note that the decisions of singlehoming buyers and multihoming buyers can be similarly derived as in the main text hence omitted here.

To analyze seller decisions, suppose \(p^*_i \geq \hat{p}_s\). For a seller with type \(v\), we write her total surplus from joining all platforms \(j \neq i\) as

\[
(v - \hat{p}_s) \sum_{j \in \mathbb{N} - i} \left( (1 - \lambda) B_j^{(\mathbb{N})} + \lambda B_j^{(\mathbb{N} - i)} \right)
= (v - \hat{p}_s) \sum_{j \in \mathbb{N} - i} \hat{B}_j^{(\mathbb{N} - i)},
\]

where the function \(B_j^{(\Theta)} = \Pr (\epsilon_i - p^*_i \geq \max_{j \in \Theta} \{ \epsilon_j - p^*_j, \epsilon_0 \})\) as in (7), while

\[
\hat{B}_j^{(\Theta)} \equiv (1 - \lambda) B_j^{(\mathbb{N})} + \lambda B_j^{(\Theta)}
\]

which can be thought of as a “composite” buyer quasi-demand that consists of (i) the mass of singlehoming buyers who join and use platform \(i \in \Theta\) for transactions, and (ii) the mass of multihoming buyers who join all platforms and choose to use platform \(i \in \Theta\) for transactions. Likewise, if the seller joins all
platforms including $i$, then her total surplus is
\[
(v - \tilde{p}^s) \left[ (1 - \lambda) \sum_{j \in \mathbb{N}_{-i}} \tilde{B}_j^{(N)} + \lambda \sum_{j \in \mathbb{N}_{-i}} B_j^{(N)} \right] + (v - p_i^s) \left[ (1 - \lambda) B_i^{(N)} + \lambda \tilde{B}_i^{(N)} \right]
\]
\[
= (v - \tilde{p}^s) \sum_{j \in \mathbb{N}_{-i}} \tilde{B}_j^{(N)} + (v - p_i^s) \tilde{B}_i^{(N)}. 
\]  \hfill (D.2)

Comparing (D.1) and (D.2), we can pin down the threshold $\dot{v}$ and profit function $B_i$ can be characterized by the usual first-order condition. We numerically verified that $\Pi_i$ while the derivation for $\dot{v}$

We note that $\dot{B}_i^{(N)} - \dot{B}_i^{(N-1)} \leq 0$, and it can be shown that $\dot{v} \geq \hat{p}^s$ using $p_i^s \geq \tilde{p}^s$. Likewise, when $p_i^s < \hat{p}^s$, with the similar calculations we can pin down thresholds $\dot{v}_m$ for $m = 1, ..., n - 1$ as in Lemma 4:

\[
\dot{v}_m \equiv \frac{p_i^s [\tilde{B}_i^{(N,m)} - \tilde{B}_i^{(N,m-1)}] + \hat{p}^s [mB_j^{(N,m)} - (m - 1) \dot{B}_j^{(N,m-1)}]}{\tilde{B}_i^{(N,m)} - \tilde{B}_i^{(N,m-1)} + m\dot{B}_j^{(N,m)} - (m - 1) \dot{B}_j^{(N,m-1)}},
\]

We can further define $\dot{v}_n \equiv \dot{v}$ and $\dot{v}_0 \equiv p_i^s$. Then the following lemma formalize seller participation decisions

**Lemma 5** (Seller participation with partial-multihoming buyers)

1. Suppose $p_i^s > \hat{p}^s$. A type $v$ seller joins all platforms $j \neq i$ if $v \geq \hat{p}^s$. The seller joins platform $i$ in addition if $v \geq \dot{v}$.
2. Suppose $p_i^s = \hat{p}^s$. A type $v$ seller joins all platforms if $v \geq \hat{p}^s$, otherwise she joins no platform.
3. Suppose $p_i^s < \hat{p}^s$. A type $v$ seller joins no platform if $v \in [\hat{v}_n, p_i^s)$, joins only platform $i$ if $v \in [p_i^s, \dot{v}_1)$, joins platform $i$ together with $m$ randomly chosen symmetric platform(s) from $j \neq i$ if $v \in [\dot{v}_m, \dot{v}_{m+1})$, and joins all platforms if $v > \dot{v}_{n-1}$.

**Proof.** It is easy to see that If a seller joins at least one platform, then the seller must also join all platform(s) that set(s) the (common) lowest seller fee. The derivation for $\dot{v}$ follows from the main text, while the derivation for $\dot{v}_m$ simply follows from the proof of Lemma 4 after replacing $B_i^{(1)}$ with $\dot{B}_i^{(1)}$ and $B_j^{(1)}$ with $\dot{B}_j^{(1)}$.

Given the user decisions characterized above, we can write down platform $i$’s demand as

\[
Q_i (p_i^b, p_i^s; \hat{p}) = \left\{ \begin{array}{ll}
\sum_{m=0}^{n-1} \left[ 1 - G(\dot{v}_m) \right] \tilde{B}_i^{(N,m)} & \text{if } p_i^s \geq \hat{p}^s \\
\sum_{m=0}^{n-1} \left[ G(\dot{v}_{m+1}) - G(\dot{v}_m) \right] \tilde{B}_i^{(N,m)} & \text{if } p_i^s < \hat{p}^s
\end{array} \right.
\]

and profit function

\[
\Pi_i = (p_i^b + p_i^s - c) Q_i (p_i^b, p_i^s; \hat{p}).
\]

It is easily verified that $Q_i (p_i^b, p_i^s; \hat{p})$ is indeed continuous, and that

\[
\lim_{p_i^s \to \hat{p}^-} \frac{dQ_i}{dp_i^s} (\hat{p}, p_i^s; \hat{p}) \geq \lim_{p_i^s \to \hat{p}^+} \frac{dQ_i}{dp_i^s} (\hat{p}, p_i^s; \hat{p}),
\]

as in the baseline model. We assume that $\Pi_i$ is quasi-concave in $(p_i^b, p_i^s)$ so that the equilibrium can be characterized by the usual first-order condition. We numerically verified that $\Pi_i$ is quasi-concave for
\[ \lambda \in \{0.1, 0.5, 0.9\} \] over all the distributional and parameter configurations considered in Section B.2, so that quasi-concavity is indeed a reasonable assumption. The details and codes of the simulations are available from the authors upon request.

Then, we can focus on the equilibrium characterized by the following first-order condition:

\[ \frac{d\Pi_i}{dp_i} \bigg|_{p_i^l = \hat{p}, p_i^r = \hat{p}^r} = \frac{d\Pi_i}{dp_i} \bigg|_{p_i^l = \hat{p}, p_i^r = \hat{p}^r} = 0. \] (D.3)

Equation (D.3) can be rewritten explicitly as

\[ \hat{p}_b + \hat{p}_s - c = -\frac{Q_i(\hat{p}, \hat{p}^r, \hat{p}^s)}{dQ_i(\hat{p}, \hat{p}^r, \hat{p}^s)/dp_i} = -\lim_{p_i^r \to \hat{p}^r} \frac{dQ_i(p_i^l, p_i^r, \hat{p}^s)}{dp_i}. \]

Deriving and substituting for the relevant terms, we have:

**Proposition 10** *(Equilibrium with partial-multihoming buyers)* A pure symmetric pricing equilibrium can be characterized by all platforms choosing \( \hat{p} = (\hat{p}^b, \hat{p}^s) \) that uniquely solves

\[ \hat{p}_b + \hat{p}_s - c = X(\hat{p}^b; n) = \frac{1 - G(\hat{p}^s)}{g(\hat{p}^s)} \sigma_\Lambda(\hat{p}^b; n), \] (D.4)

where

\[ X(\hat{p}^b; n) \equiv \frac{\int_{\epsilon}^\epsilon \int_{\epsilon}^\epsilon 1 - F(\max \{\epsilon, \epsilon_0 + p^b\})dF(\epsilon)^{n-1}dF_0(\epsilon_0)}{\int_{\epsilon}^\epsilon \int_{\epsilon}^\epsilon f(\max \{\epsilon, \epsilon_0 + p^b\})dF(\epsilon)^{n-1}dF_0(\epsilon_0)}, \]

\[ \sigma_\Lambda(\hat{p}^b; n) \equiv \lambda \frac{\int_{\epsilon}^\epsilon \frac{F(\epsilon_0 + p)}{n-\gamma} - F^n(\epsilon_0 + p)}{\int_{\epsilon}^\epsilon 1 - F^n(\epsilon_0 + p)dF_0(\epsilon_0)} + 1 - \lambda. \]

**Proof.** *(Proposition 10).* The demand derivatives, after imposing symmetry, can be calculated as follows:

\[ Q_i(\hat{p}; \hat{p}) = (1 - G(\hat{p}^s)) \hat{B}_i^{(N)} \bigg|_{\hat{p}_i^l = \hat{p}} = (1 - G(\hat{p}^s)) \frac{\exp \left\{-\frac{\hat{p}^s}{\gamma} \right\}}{1 + n \exp \left\{-\frac{\hat{p}^s}{\gamma} \right\}}, \]

\[ \frac{dQ_i(\hat{p}; \hat{p})}{dp_i} = (1 - G(\hat{p}^s)) \frac{\partial \hat{B}_i^{(N)}}{\partial p_i} \bigg|_{\hat{p}_i^l = \hat{p}} = -\frac{1}{X} Q_i(\hat{p}; \hat{p}), \]

and

\[ \lim_{p_i^r \to \hat{p}^r} \frac{dQ_i(\hat{p}; \hat{p})}{dp_i} = -\frac{d\tilde{\sigma}}{dp_i} g(\hat{p}^s) \hat{B}_i^{(N)} \bigg|_{\hat{p}_i^l = \hat{p}} \]

\[ = \left( \frac{\hat{B}_i^{(N)}}{(n-1)^2 \left( \hat{B}_i^{(N)} - \hat{B}_j^{(N-1)} \right) + \hat{B}_i^{(N)} \right) \left( 1 - G(\hat{p}^s) \right) Q_i(\hat{p}; \hat{p}) \]

\[ = \frac{1}{\sigma_\Lambda} \frac{g(\hat{p}^s)}{1 - G(\hat{p}^s)} Q_i(\hat{p}; \hat{p}). \]

Substituting for the relevant terms yield the equation for equilibrium fees. To prove the existence and uniqueness of \( \hat{p}^b \) and \( \hat{p}^s \) defined in the proposition, we can follow the exact same steps as in the proof of Proposition 2. \( \blacksquare \)

Given the equilibrium characterization, we are now ready to prove the formal results stated in Section 5 of the main text.
Proof. (Proposition 7). Denote $M \equiv 1 - G(\hat{p}^*)/\hat{p}$. Applying total differentiation with respect to $\lambda$ on (D.4) and writing in matrix form, we have

$$
\begin{bmatrix}
1 - \frac{\partial X}{\partial \hat{p}^*} & 1 \\
1 - M \frac{\partial \sigma}{\partial \hat{p}^*} & 1 - \sigma \frac{\partial M}{\partial \hat{p}^*}
\end{bmatrix}
\begin{bmatrix}
\frac{dp^b}{d\lambda} \\
\frac{dp^s}{d\lambda}
\end{bmatrix} =
\begin{bmatrix}
0 \\
M \frac{\partial \sigma}{\partial X}
\end{bmatrix}.
$$

Given that $\sigma = \lambda \sigma + (1 - \lambda)$ where $\sigma \in [0, 1]$ is defined in (11), we know immediately from Lemma 3 that $\frac{\partial \lambda}{\partial X} \leq 0$, and $\frac{\partial \sigma}{\partial \hat{p}^*} \geq 0$. Then

$$
Det \equiv \begin{vmatrix}
1 - \frac{\partial X}{\partial \hat{p}^*} & 1 \\
1 - M \frac{\partial \sigma}{\partial \hat{p}^*} & 1 - \sigma \frac{\partial M}{\partial \hat{p}^*}
\end{vmatrix} = \left(1 - \frac{\partial X}{\partial \hat{p}^*}\right) \left(1 - \sigma \frac{\partial M}{\partial \hat{p}^*}\right) - 1 + M \frac{\partial \sigma}{\partial X} > 0.
$$

By Cramer’s rule,

$$
dp^b = \frac{1}{Det} \begin{vmatrix} 0 & 1 - \sigma \frac{\partial M}{\partial \hat{p}^*} \end{vmatrix} > 0, \quad \text{and} \quad dp^s = \frac{1}{Det} \begin{vmatrix} 1 - \frac{\partial X}{\partial \hat{p}^*} & 0 \end{vmatrix} < 0.
$$

as required. ■

Proof. (Proposition 8). Given the decomposition of $\sigma = \lambda \sigma + (1 - \lambda)$, after a total differentiation on (D.4) and apply Cramer rule, we can write

$$
dp^s = \frac{\partial X}{\partial n} = \frac{1}{Det} \left[ M \frac{\partial \lambda}{\partial n} \frac{\partial X}{\partial \hat{p}^*} + M \left( \frac{\partial \lambda}{\partial \hat{p}^*} \frac{\partial X}{\partial n} - \frac{\partial \lambda}{\partial \hat{p}^*} \frac{\partial X}{\partial \hat{p}^*} \right) \right]$$

$$
= \frac{1}{Det} \left[ \frac{\partial X}{\partial n} + \lambda M \left( \frac{\partial \sigma}{\partial n} + \frac{\partial X}{\partial \hat{p}^*} \frac{\partial \sigma}{\partial n} - \frac{\partial \sigma}{\partial \hat{p}^*} \frac{\partial X}{\partial \hat{p}^*} \right) \right].
$$

We know that $\frac{dp^s}{dn} > 0$ when $\lambda = 0$ and $\frac{dp^s}{dn} < 0$ when $\lambda = 1$ and $f$ is decreasing. By continuity we have $\frac{dp^s}{dn} \geq 0$ if $\lambda \to 0$ and $\frac{dp^s}{dn} \leq 0$ if $\lambda \to 1$. Likewise,

$$
dp^b = \frac{\partial X}{\partial n} = \frac{1}{Det} \left[ \frac{\partial X}{\partial n} - \left( \lambda \sigma + (1 - \lambda) \right) \frac{\partial M}{\partial \hat{p}^*} \frac{\partial X}{\partial \hat{p}^*} - \lambda M \frac{\partial \sigma}{\partial \hat{p}^*} \right].
$$

We know that $\frac{dp^b}{dn} < 0$ when $\lambda = 0$ and $\frac{dp^b}{dn} > 0$ when $\lambda = 1$ and $f$ and $g$ are decreasing. By continuity we have $\frac{dp^b}{dn} \geq 0$ if $\lambda \to 0$ and $\frac{dp^b}{dn} \leq 0$ if $\lambda \to 1$. ■

Proof. (Remark 1). Denote $M \equiv \frac{1 - G(\hat{p}^*)}{\hat{p}^*}$, and recall that log-concavity of $1 - G$ implies $\frac{\partial M}{\partial \hat{p}^*} \leq 0$. Applying total differentiation with respect to $n$ on (D.4) and writing in matrix form, we have

$$
\begin{bmatrix}
1 - \frac{\partial X}{\partial \hat{p}^*} & 1 \\
1 - M \frac{\partial \sigma}{\partial \hat{p}^*} & 1 - \sigma \frac{\partial M}{\partial \hat{p}^*}
\end{bmatrix}
\begin{bmatrix}
\frac{dp^b}{dn} \\
\frac{dp^s}{dn}
\end{bmatrix} =
\begin{bmatrix}
\frac{\partial X}{dn} \\
M \frac{\partial \sigma}{dn}
\end{bmatrix}.
$$

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By Cramer’s rule,

\[
\frac{dp^b}{dn} = \frac{1}{Det} \begin{vmatrix}
\frac{\partial X}{\partial n} & \frac{1}{\sigma} \frac{\partial M}{\partial p^b} \\
M \frac{\partial \sigma}{\partial n} & 1 - \sigma \frac{\partial M}{\partial p^b} 
\end{vmatrix},
\]

and \(\frac{dp^s}{dn} = \frac{1}{Det} \begin{vmatrix}
1 - \frac{\partial X}{\partial p^b} & \frac{\partial M}{\partial p^s} \\
\frac{\partial \sigma}{\partial n} & M \frac{\partial \sigma}{\partial n} 
\end{vmatrix},
\]

where \(Det\) is defined as in (D.5). Sum up these two expressions and rearrange, we get

\[
d\hat{p} + \frac{dp^s}{dn} = \frac{1}{Det} \begin{vmatrix}
M \left( \frac{\partial \sigma}{\partial n} \frac{\partial X}{\partial n} - \frac{\partial \sigma}{\partial p^b} \frac{\partial X}{\partial n} \right) - \frac{\partial M}{\partial n} \frac{\partial X}{\partial n}
\end{vmatrix}.
\]

Likewise,

\[
d\hat{p} - \frac{dp^s}{dn} = \frac{1}{Det} \begin{vmatrix}
2 \left( \frac{\partial X}{\partial n} - M \frac{\partial \sigma}{\partial n} \right) - \frac{\partial M}{\partial n} \frac{\partial X}{\partial n} - M \left( \frac{\partial \sigma}{\partial n} \frac{\partial X}{\partial n} - \frac{\partial M}{\partial p^s} \frac{\partial X}{\partial n} \right)
\end{vmatrix}.
\]

Substituting for the terms and simplifying, it can be shown that \(\frac{dp^b}{dn} - \frac{dp^s}{dn} > 0\) if and only if

\[
\left( \frac{\lambda}{1 + (1 - \lambda)(n - 1) \exp \{-\hat{p}^b/\gamma\}} \right) \frac{1 + n \exp \{-\hat{p}^b/\gamma\}}{\exp \{-\hat{p}^b/\gamma\}} \left( 2 + \frac{\exp \{-\hat{p}^b/\gamma\}}{1 + (n - 1) \exp \{-\hat{p}^b/\gamma\}} \right) - 2 > -\sigma \frac{\partial M}{\partial p^b}.
\]

Denote LHS of (D.6) as \(\eta (\lambda, n, \hat{p}^b (\lambda))\). We claim that there exists a unique threshold \(\bar{\lambda}\) such that (D.6) — that is, \(\frac{1}{\sigma} \eta (\lambda, n, \hat{p}^b (\lambda)) > -\frac{\partial M}{\partial p^b}\) — holds if and only if \(\lambda > \bar{\lambda}\). We have

\[
\frac{d\eta (\lambda, n, \hat{p}^b (\lambda))}{d\lambda} = \frac{\partial \psi}{\partial \lambda} + \frac{\partial \psi}{\partial \hat{p}^b} \frac{dp^b}{d\lambda} > 0,
\]

and

\[
\frac{d\sigma}{d\lambda} = \frac{\partial \sigma}{\partial \lambda} + \frac{\partial \sigma}{\partial \hat{p}^b} \frac{dp^b}{d\lambda} < 0,
\]

so that \(\frac{1}{\sigma} \eta (\lambda, n, \hat{p}^b (\lambda))\) is increasing in \(\lambda\). Hence, \(\frac{1}{\sigma} \eta (\lambda, n, \hat{p}^b (\lambda))\) is minimized at \(\lambda = 0\) and maximized at \(\lambda = 1\), in which

\[
\frac{1}{\sigma} \eta (\lambda, n, \hat{p}^b (\lambda)) \bigg|_{\lambda=0} = -2 < -\frac{\partial M}{\partial \hat{p}^b},
\]

while it can be verified that

\[
\frac{1}{\sigma} \eta (\lambda, n, \hat{p}^b (\lambda)) \bigg|_{\lambda=1} = \left( 1 + (n - 1) \exp \{-\hat{p}^b/\gamma\} \right) \left[ 1 + n \exp \{-\hat{p}^b/\gamma\} \right] \left( 2 + \frac{\exp \{-\hat{p}^b/\gamma\}}{1 + (n - 1) \exp \{-\hat{p}^b/\gamma\}} \right) - 2
\]

\[> 4n - 1 > 4(n - 1),\]

so that condition (C.2) implies that \(\frac{1}{\sigma} \eta (\lambda, n, \hat{p}^b (\lambda)) \bigg|_{\lambda=1} > -\frac{\partial M}{\partial \hat{p}^b}\). Hence, by the intermediate value theorem, there exists a unique threshold \(\bar{\lambda} \in [0, 1]\) such that \(\frac{1}{\sigma} \eta (\lambda, n, \hat{p}^b (\lambda)) > -\frac{\partial M}{\partial \hat{p}^b}\) if and only if \(\lambda > \bar{\lambda}\), as required.