A Continuous consumer types

Suppose that we have a continuum of consumer types, and each consumer is indexed by the convenience benefit \( b \) obtained from performing transactions through \( M \). We assume \( b \in [b_L, b_H] \) is distributed with cdf \( G(\cdot) \) and corresponding log-concave density \( g(\cdot) \), where \( b_L \geq -\infty \) and \( b_H \leq \infty \). Everything else is like in the baseline model. We assume the gap \( b_H - b_L \) is large enough so that equilibrium prices are always interior.

A.1 Pure marketplace

Recall fringe suppliers always set inside and outside prices at \( p_i = c + \tau + \Delta \) and \( p_o = c + \Delta \). Consider \( S \)'s pricing problem after it joins the marketplace. It chooses \( p_i \) and \( p_o \) to maximize its profit

\[
(p_o - c) G(p_i - p_o) + (p_I - \tau - c) (1 - G(p_i - p_o))
\]

subject to \( p_i \leq c + \tau + \Delta \) and \( p_o \leq c + \Delta \).

Since \( S \) makes sales on both channels, it can always increase profit by raising its prices in both channels (by the same amount) whenever both pricing constraints do not bind. Therefore, at least one pricing constraint must bind. If \( p_i^* = c + \tau + \Delta \), then \( S \) solves

\[
\max_{p_o \leq c + \Delta} \{ (p_o - c) G(p_i^* - p_o) + \Delta (1 - G(p_i^* - p_o)) \}.
\]

For all \( p_o \leq c + \Delta \), an increase in \( p_o \): (i) raises the margin in the outside channel; (ii) shifts demand from the lower-margin outside channel to the higher-margin inside channel (due to the constraint on \( p_o \)), and so we must have \( p_o^* = c + \Delta \). If we start with \( p_o^* = c + \Delta \) instead, then \( S \) solves

\[
\max_{p_i \leq c + \tau + \Delta} \{ \Delta G(p_i - p_o^*) + (p_i - \tau - c) (1 - G(p_i - p_o^*)) \},
\]

and the same logic as above implies \( p_i^* = c + \tau + \Delta \). Therefore, we conclude that \( p_i^* = c + \tau + \Delta \) and \( p_o^* = c + \Delta \), and \( S \) earns profit \( \pi^{market} = \Delta \).

On the other hand, if \( S \) does not participate, its profit is \( \max_{p_o \leq c + \Delta} (p_o - c) G(c + \tau + \Delta - p_o) < \pi^{market} \). Therefore, \( S \) always participates.

Since the cross-channel utility difference is \( p_i^* - p_o^* = \tau \), the number of consumers buying through the intermediary is \( 1 - G(\tau) \). Thus, \( M \)'s profit as a pure marketplace is

\[
\Pi^{market} = \max_{\tau} \{ \tau (1 - G(\tau)) \},
\]

so the optimal commission follows the usual monopoly price formula:

\[
\tau^m = \frac{1 - G(\tau^m)}{g(\tau^m)}.
\]

As in the baseline model with discrete consumer types, the marketplace’s profit is independent of \( \Delta \).
The reason is that the innovative supplier can fully extract the value of its innovation (inside and outside the platform).

A.2 Pure reseller

$S$ sets its outside price to maximize $(p_o - c) G (p_m - p_o + \Delta)$ subject to $p_o \leq c + \Delta$. Meanwhile, $M$’s cost is zero so it maximizes $p_m (1 - G (p_m - p_o + \Delta))$. If the constraint on $p_o$ is non-binding, the equilibrium prices are jointly pinned down by:

$$p_o^* = c + \frac{G (p_m^* - p_o^* + \Delta)}{g (p_m^* - p_o^* + \Delta)} \quad \text{and} \quad p_m^* = \frac{1 - G (p_m^* - p_o^* + \Delta)}{g (p_m^* - p_o^* + \Delta)}.$$  

It is useful to denote the equilibrium cross-channel utility difference by $A$, where $A$ is the unique solution to

$$A = \Delta - c + \frac{1 - 2G (A)}{g (A)}. \quad (A.1)$$

Then we have $p_o^* = c + \frac{G (A)}{g (A)}$ and $p_m^* = \frac{1 - G (A)}{g (A)}$. The constraint $p_o \leq c + \Delta$ is non-binding if and only if $\frac{G (A)}{g (A)} < \Delta$. If instead $\frac{G (A)}{g (A)} \geq \Delta$, then the equilibrium prices are $p_o^* = c + \Delta$ and $p_m^* = \frac{1 - G (B)}{g (B)}$, where $B$ is the unique solution to

$$B = -c + \frac{1 - G (B)}{g (B)}. \quad (A.2)$$

It is useful to note that the log-concavity of $g$ implies $\frac{1-G}{g}$ and $\frac{1-2G}{g}$ are decreasing functions, so

$$A \leq B \iff \Delta \leq \frac{G (A)}{g (A)},$$

where equality holds when $\Delta = \frac{G (A)}{g (A)}$, so that the condition in the right-hand side can be equivalently written as $\Delta \leq \frac{G (B)}{g (B)}$ whenever convenient. Then, the equilibrium profits can be summarized as $\Pi^\text{resell} = \min \left\{ \frac{(1-G (B)^2}{g (B)} , \frac{(1-G (A)^2}{g (A)} \right\}$ and $\pi^\text{resell} = \min \left\{ \Delta G (B), \frac{G (A)^2}{g (A)} \right\}$.

We relegate all cross-mode comparisons to Section A.4.

A.3 Dual mode

There are, in general, two possible types of equilibrium in the pricing subgame: (i) $M$ makes all the inside sales (reseller equilibrium), and (ii) $S$ makes all the inside sales (intermediation equilibrium). As opposed to the baseline model with discrete consumer types, the heterogeneity in consumer types implies there is no direct sales equilibrium (i.e. in which no consumers buy through $M$) because the assumptions on $G(.)$ mean there are always some consumers who buy through $M$.

A.3.1 Dual mode - reseller equilibrium

Consider first the extreme case where $\tau$ is sufficiently high so that $M$ always wins the on-platform competition without being constrained by within-channel competition. Suppose $S$ participates on $M$, and sets its outside price to maximize $(p_o - c) G (p_m - p_o + \Delta)$ subject to $p_o \leq c + \Delta$. Then $M$ solves

$$\max_{p_m} \left( p_m (1 - G (p_m - p_o + \Delta)) \right) \text{ subject to } p_m \leq c - \Delta + \tau.$$  

We first rule out any reseller equilibrium in which the constraint on $p_m$ is binding. Suppose by contradiction such a reseller equilibrium exists. Then in equilibrium we must have $p_m^* = p_o^* - \Delta = c + \tau - \Delta$ and $p_o^* = c + \min \left\{ \Delta, \frac{G (c + \tau - p_o^*)}{g (c + \tau - p_o^*)} \right\}$. However, $M$ can profitably deviate from the candidate equilibrium
by setting a very high \( p_m \) to let \( S \) win the inside competition, earning deviation profit
\[
\Pi^{dev} = \tau \left(1 - G(c + \tau - p_m^*)\right) > \Pi^{eqm} = (c + \tau - \Delta) \left(1 - G(c + \tau - p_o^*)\right).
\]

Next, suppose in equilibrium the constraint on \( p_m \) is non-binding. We can obtain the equilibrium of the simultaneous pricing game:
\[
\begin{align*}
p_o^* &= c + \min \left\{ \frac{G(p_m^* - p_o^* + \Delta)}{g(p_m^* - p_o^* + \Delta)}, \Delta \right\} \\
p_m^* &= 1 - G(p_m^* - p_o^* + \Delta) \quad \frac{1}{g(p_m^* - p_o^* + \Delta)}.
\end{align*}
\]
This is the same pricing equilibrium as in the reseller mode, so the subgame equilibrium can be concisely described as
\[
(p_o^*, p_m^*) = \begin{cases} 
(c + \Delta, \frac{1 - G(B)}{g(B)}) & \text{if } \Delta \leq \frac{G(A)}{g(A)} \\
(c + \frac{G(A)}{g(A)}, \frac{1 - G(A)}{g(A)}) & \text{if } \Delta > \frac{G(A)}{g(A)}
\end{cases}
\tag{A.3}
\]
and \( p_i^* \geq c + \tau \), where \( A \) and \( B \) are defined in (A.1) and (A.2).

The equilibrium (A.3) is sustainable provided that (i) \( S \) has no incentive to undercut, and (ii) \( M \) has no incentive to let \( S \) win. Condition (i) is equivalent to
\[
c + \tau + \Delta \geq \frac{1 - G(\max \{B, A\})}{g(\max \{B, A\})}.
\tag{A.4}
\]
If \( \tau \) does not satisfy this condition, then \( p_m^* > c - \Delta + \tau \) and so \( S \) has an incentive to undercut from (A.3).

Now consider condition (ii) required for the equilibrium (A.3) to exist. If \( \Delta > \frac{G(A)}{g(A)} \), then \( M \) has no incentive to let \( S \) win if and only if \( \tau > \tilde{\tau}_A \), where \( \tilde{\tau}_A \) is the largest solution to the following indifference equation that equates \( M \)'s deviation profit (by letting \( S \) win inside with its price \( c + \tau \)) with \( M \)'s equilibrium profit:
\[
\tilde{\tau}_A \left(1 - G\left(\tilde{\tau}_A - \frac{G(A)}{g(A)}\right)\right) = \frac{(1 - G(A))^2}{g(A)}.
\tag{A.5}
\]
Note that \( \tilde{\tau}_A \geq \Delta - c + \frac{1 - G(B)}{g(B)} \), so that \( \tau \geq \tilde{\tau}_A \) implies (A.4). If \( \Delta \leq \frac{G(A)}{g(A)} \), then \( M \) has no incentive to let \( S \) win if and only if \( \tau > \tilde{\tau}_B \), where \( \tilde{\tau}_B \) is the largest solution to the following indifference equation that equates \( M \)'s deviation profit (by letting \( S \) wins inside) with \( M \)'s equilibrium profit:
\[
\tilde{\tau}_B \left(1 - G(\tilde{\tau}_B - \Delta)\right) = \frac{(1 - G(B))^2}{g(B)}.
\tag{A.6}
\]
Again, note that \( \tilde{\tau}_B \geq \Delta - c + \frac{1 - G(B)}{g(B)} \), so that \( \tau \geq \tilde{\tau}_B \) implies (A.4). Meanwhile, it can be easily verified that \( \tilde{\tau}_B > \tilde{\tau}_A \) if and only if \( \Delta > \frac{G(A)}{g(A)} \).

To summarize the construction of the reseller equilibrium in dual mode:
- If \( \Delta > \frac{G(A)}{g(A)} \), the reseller equilibrium is sustainable if and only if \( \tau > \tilde{\tau}_A \). In this case \( \Pi^{eqm} = \frac{(1 - G(A))^2}{g(A)g(A)} \), \( \tau^{eqm} = \frac{G(A)^2}{g(A)} \), \( (p_o^*, p_m^*) = \left(c + \frac{G(A)}{g(A)}, \frac{1 - G(A)}{g(A)}\right) \) and \( p_i^* \geq c + \tau \).

\footnote{As opposed to the baseline model, any \( p_i < c + \tau \) is dominated by \( p_i = c + \tau \) because regardless of \( p_o \) there is always some consumers buying through the marketplace.}

\footnote{This follows from the observation that if we substitute \( \Delta - c + \frac{1 - G(A)}{g(A)} \) for \( \tilde{\tau}_A \) in the left-hand side of (A.5), then the left-hand side becomes greater than the right-hand side (recall that by definition \( A = \Delta - c + \frac{1 - 2G(A)}{g(A)} \)).}

\footnote{This follows from the observation that if we substitute \( \Delta - c + \frac{1 - G(B)}{g(B)} \) for \( \tilde{\tau}_B \) in the left-hand side of (A.6), then the left-hand side becomes greater than the right-hand side.}
If $\Delta \leq \frac{G(A)}{g(A)}$, the reseller equilibrium is sustainable if $\tau > \bar{\tau}_B$. In this case $\Pi^{eqm} = \frac{(1-G(B))^2}{g(B)}$, $\pi^{eqm} = \Delta G(B)$, $(p^*_o, p^*_m) = \left( c + \Delta, \frac{1-G(B)}{\pi(B)} \right)$ and $p^*_i \geq c + \tau$.

A.3.2 Dual mode - intermediation equilibrium

Given that in any intermediation equilibrium $S$ makes all sales in both channels, it can always profitably increase both $p_i$ and $p_o$ until one of the following constraints binds: $p_i \leq \min \{p^*_m + \Delta, c + \Delta + \tau \}$ and $p_o \leq c + \Delta$, where $p^*_m$ is some arbitrarily given price set by $M$. If only the constraint on the outside price binds, then $p^*_o = c + \Delta$ while $p_i$ is interior and solves

$$\max_{p_i \leq \min \{p^*_m + \Delta, c + \Delta + \tau \}} \left\{ (p^*_o - c) G(p_i - p^*_o) + (p_i - \tau - c) (1 - G(p_i - p^*_o)) \right\} = \max_{p_i \leq \min \{p^*_m + \Delta, c + \Delta + \tau \}} \left\{ \Delta G(p_i - c - \Delta) + (p_i - \tau - c) (1 - G(p_i - c - \Delta)) \right\}.$$  

The first-order condition implies $p_i = c + \Delta + \tau + \frac{1-G(p_i-c-\Delta)}{g(p_i-c-\Delta)} > c + \Delta + \tau$, violating the constraint on $p_i$. Therefore the constraint on $p_i$ must bind. For any given $p^*_o$, $p^*_m$ solves

$$\max_{p_o \leq c+\Delta} \left\{ (p_o - c) G(p^*_i - p_o) + (p^*_i - \tau - c) (1 - G(p^*_i - p_o)) \right\}.$$  

It is useful to define

$$\phi_\tau \equiv \tau - \frac{G(\phi_\tau)}{g(\phi_\tau)} \quad (A.7)$$

so that the first-order condition implies

$$p^*_o = \min \left\{ c + \Delta, \frac{G(\phi_\tau)}{g(\phi_\tau)} + p^*_i - \tau \right\}. \quad (A.8)$$

Then, asymmetric Bertrand competition on the marketplace implies $p^*_i = p^*_m + \Delta$ and $p^*_m \in [\max \{0, c + \tau - \Delta \}, \tau]$. Note $p^*_m$ is indeterminate because $M$ makes no sales in equilibrium. We cannot have $p^*_m > \tau$ because in any such equilibrium $M$ would have an incentive to undercut $S$ and make the inside sales, earning a margin strictly greater than $\tau$. Likewise, any $p^*_m < \max \{0, c + \tau - \Delta \}$ means either $M$ or $S$ is playing a dominated strategy in equilibrium.

To confirm this is an equilibrium, we need to make sure $M$ does not have an incentive to deviate. $M$’s equilibrium profit is

$$\Pi^{eqm} = \tau (1 - G(p^*_i - p^*_o)) = \tau (1 - G(\max \{p^*_m - c, \phi_\tau\})),$$

which is decreasing in $p^*_m$. Given we are looking for equilibrium that maximizes $M$’s profit, we must have $p^*_m = \max \{0, c + \tau - \Delta \}$, and so $p^*_o = \min \{c + \Delta, p^*_m + \Delta - \phi_\tau\}$, while $p^*_i = p^*_m + \Delta$. There are four possible equilibrium configurations (ignoring firms’ incentive to deviate):

- **Configuration 1**: $p^*_m = 0$, $p^*_o = c + \Delta$. This requires $\tau \leq \min \{\Delta - c, \frac{G(-c)}{g(-c)} - c\}$. $\Pi^{eqm} = \tau (1 - G(-c))$ and $\pi = \Delta - (c + \tau) G(-c)$.

- **Configuration 2**: $p^*_m = 0$, $p^*_o = \Delta - \phi_\tau$. This requires $\tau \in (\frac{G(-c)}{g(-c)} - c, \Delta - c)$. $\Pi^{eqm} = \tau (1 - G(\phi_\tau))$ and $\pi = \Delta - (c + \tau) G(\phi_\tau)$.

- **Configuration 3**: $p^*_m = c + \tau - \Delta$, $p^*_o = c + \tau - \phi_\tau$. This requires $\tau \in [\Delta - c, \bar{\tau}_1)$, where

$$\bar{\tau}_1 \equiv \phi_\tau + \Delta \quad (A.9)$$

is such that $\tau < \bar{\tau}_1 \Leftrightarrow \tau - \phi_\tau < \Delta$. $\Pi^{eqm} = \tau (1 - G(\phi_\tau))$ and $\pi = \Delta G(\phi_\tau)$. 


Ignoring the upperbound constraint, the deviation profit is maximized at
\[ p^* = c + \tau - \Delta, \quad p^*_n = c + \Delta. \]
This requires \( \tau \geq \max\{\Delta - c, \bar{\tau}_1\} \). \( \Pi^{eqm} = \tau(1 - G(\tau - \Delta)) \) and \( \pi = \Delta G(\tau - \Delta) \).

Bertrand competition means \( S \) has no incentive to deviate in any of these equilibria. So we simply need to make sure \( M \) has no incentive to deviate (by undercutting \( S \)) for equilibria with \( p^*_m = c + \tau - \Delta > 0 \). To do so, we will use the following technical lemma:

**Lemma A.1** \( \Delta > \frac{G(-c)}{g(-c)} \) if and only if \( \bar{\tau}_1 > \Delta - c \)

**Proof.** Given \( \frac{dp}{d\tau} \in (0, 1) \), we know \( \bar{\tau}_1 > \Delta - c \) if and only if \( \Delta - c < \phi_{\Delta-c} + \Delta \), or \( \phi_{\Delta-c} > c \). Using (A.7), the last condition is equivalent to \( \Delta > \frac{G(-c)}{g(-c)} \). ■

Suppose \( \Delta \leq \frac{G(-c)}{g(-c)} \), or equivalently, \( \bar{\tau}_1 \leq \Delta - c \). This rules out configurations 2 and 3. For all \( \tau \leq \Delta - c \), configuration 1 applies, and clearly \( M \) cannot profitably undercut \( S \). For \( \tau > \Delta - c \), configuration 4 applies, and \( M \)'s deviation profit is

\[
\Pi^{dev} = \max_{p^*_m = c + \tau - \Delta} p^*_m \left(1 - G(p^*_m - c)\right).
\]

Ignoring the upperbound constraint, the deviation profit is maximized at \( p^*_m = \frac{1 - G(B)}{g(B)} \). For all \( \tau \leq \Delta - c + \frac{1 - G(B)}{g(B)} = B + \Delta \), the upperbound constraint on \( p^*_m \) binds so \( \Pi^{dev} = (\tau - \Delta + c)(1 - G(\tau - \Delta)) \) if and only if \( \tau > \Delta - c + \frac{1 - G(B)}{g(B)} \), or equivalently, \( \tau > \bar{\tau}_B \), where

\[
\bar{\tau}_B = (1 - G(B)) \cdot \frac{g(B)}{B - \tau}.
\]

By definition \( \bar{\tau}_B < B + \Delta \) because \( B + \Delta = \Delta - c + \frac{1 - G(B)}{g(B)} > \frac{1 - G(B)}{g(B)} \). By transitivity, \( \tau^*_B < \Delta - c + \frac{1 - G(B)}{g(B)} < \bar{\tau}_B \).

Suppose \( \Delta > \frac{G(-c)}{g(-c)} \). For \( \tau \leq \Delta - c \), configurations 1 and 2 apply, and clearly \( M \) cannot profitably undercut \( S \). Consider \( \tau \in (\Delta - c, \bar{\tau}_1) \). From configuration 3, the best deviation profit that \( M \) can achieve is

\[
\Pi^{dev} = \max_{p^*_m = c + \tau - \Delta} \left\{ p^*_m \left(1 - G(p^*_m - c + \phi_\tau - \tau + \Delta)\right) \right\}.
\]

Ignoring the upperbound constraint, the deviation profit is maximized at \( p^*_m = \frac{1 - G(X) + \phi_\tau}{g(X)} \), where

\[
X_\tau \equiv \Delta - c - \tau + \phi_\tau + \frac{1 - G(X)}{g(X)}.
\]

For all \( \tau \leq \Delta - c + \frac{1 - G(X)}{g(X)} \) (or equivalently, \( \tau \leq A + \frac{G(A)}{g(A)} \)),\(^7\) the upperbound constraint on \( p^*_m \) binds so \( \Pi^{dev} = (\tau - \Delta + c)(1 - G(\phi_\tau)) \) if and only if \( \tau > \Delta - c + \frac{1 - G(X)}{g(X)} \) (or equivalently, \( \tau > A + \frac{G(A)}{g(A)} \)).

\[
\Pi^{dev} = \frac{(1 - G(X))g(X)}{g(X)g(X)}, \quad M \text{ has no incentive to deviate if and only if } \tau \leq \bar{\tau}_X, \text{ where}
\]

\[
\bar{\tau}_X = (1 - G(\phi_\tau)) \cdot \frac{g(X)}{g(X)}.
\]

\(^7\)Specifically, \( \tau \leq A + \frac{G(A)}{g(A)} \Leftrightarrow \phi_\tau \leq A \Rightarrow X_\tau \leq A \). Therefore, \( \tau \leq A + \frac{G(A)}{g(A)} = \Delta - c + \frac{1 - G(A)}{g(A)} \) implies \( \tau \leq \Delta - c + \frac{1 - G(X)}{g(X)} \). Likewise, \( \tau > A + \frac{G(A)}{g(A)} = \Delta - c + \frac{1 - G(X)}{g(X)} \) implies \( \tau > \Delta - c + \frac{1 - G(X)}{g(X)} \). Therefore the two conditions are equivalent.

then RHS is \( \frac{(1 - G(X))^2}{g(X)} \), LHS is \( (\Delta - c + \frac{1 - G(A)}{g(A)})(1 - G(A)) \). Therefore, for general \( \tau > A + \frac{G(A)}{g(A)} \), we have \( X_\tau < A \).

---

5
The existence of $\tau_X$ follows from the intermediate value theorem. In what follows, we assume $\tau(1 - G(\phi_\tau))$ is quasiconcave.\(^8\) Let

$$\tau^*_X \equiv \arg \max_\tau \tau(1 - G(\phi_\tau)),$$

or $\tau^*_X = \frac{1 - G(\phi_{\tau^*_X})}{g(\tau^*_X) / d\phi_{\tau^*_X}/d\tau}$. Finally, for $\tau \geq \bar{\tau}_1$, configuration 4 applies and the analysis follows from the previous paragraph. In particular, the configuration is an equilibrium if and only if $\tau \leq \tau_B$.

The following two technical lemmas identify the relative ordering of these cutoffs.

**Lemma A.2** (i) $\bar{\tau}_X \leq \bar{\tau}_1 \iff \bar{\tau}_B \leq \bar{\tau}_1$; (ii) $\tau_X \leq \bar{\tau}_1 \iff \tau_X \geq \tau_B$; (iii) $\tau_B \geq B + \Delta$, and $\bar{\tau}_X \geq A + \frac{G(A)}{g(A)}$; (iv) $\tau^*_X \leq \bar{\tau}_1 \iff \tau^*_B \leq \bar{\tau}_1$.

**Proof.** (i) From definitions, $\bar{\tau}_X \leq \bar{\tau}_1 \iff \bar{\tau}_1 (1 - G(\phi_{\tau_1})) \leq \frac{(1 - G(X_{\tau_1}))^2}{g(X_{\tau_1})} = \frac{(1 - G(B))^2}{g(B)}$, where the last equality used $X_{\tau_1} = B$, while $\tau_X \leq \bar{\tau}_1 \iff \bar{\tau}_1 (1 - G(\phi_{\tau_1})) \leq \frac{(1 - G(B))^2}{g(B)}$. So $\tau_X \leq \bar{\tau}_1 \iff \bar{\tau}_B \leq \bar{\tau}_1$. (ii) To show $\tau_X > \tau_B$, consider

$$\Gamma(\tau) \equiv \tau(1 - G(c + \tau - p^*_m)) - \max_{p^*_m} (1 - G(p^*_m - p^*_o + \Delta)).$$

If we denote $p^{dev} = \max_{p^*_m} (1 - G(p^*_m - p^*_o + \Delta))$, then by envelope theorem:

$$\frac{d\Gamma(\tau)}{dp^*_m} = \left(\tau - (p^{dev} - c) \frac{g(p^{dev} - p^*_o + \Delta)}{g(c + \tau - p^*_o)}\right) g(c + \tau - p^*_o) \geq 0,$$

where we used log-concavity of $g$ and $p^{dev} \leq c + \tau - \Delta$. Given the supposition $\tau_X \leq \bar{\tau}_1$, we have $p^*_o = c + \tau_X - \phi_{\tau_X} \leq c + \Delta$, and so

$$\Gamma(\tau_{p^*_o = c + \tau_X - \phi_{\tau_X}}) = 0 \leq \Gamma(\tau_{p^*_o = c + \Delta}).$$

Suppose by contradiction $\tau_X > \tau_B$, then given $\Gamma(\tau_{p^*_o = c + \Delta})$ is decreasing for $\tau \geq \tau_B$ and $\Gamma(\tau_B)_{p^*_o = c + \Delta} = 0$, we have $\Gamma(\tau_X)_{p^*_o = c + \Delta} < 0$, a contradiction. Therefore, $\tau_X \leq \tau_B$ must hold. (iii) From the definitions, we know $\tau_B \geq \Delta - c + \frac{1 - G(B)}{g(B)}$, and $\tau_X \geq \Delta - c + \frac{1 - G(X_{\tau_X})}{g(X_{\tau_X})}$. (iv) From definitions $\tau^*_X \leq \bar{\tau}_1 \iff \tau^*_1 > \frac{1 - G(\phi_{\tau^*_1})}{g(\phi_{\tau^*_1})}$, and $\tau^*_B \leq \bar{\tau}_1 \iff \bar{\tau}_1 > \frac{1 - G(\tau^*_B - \Delta)}{g(\tau^*_B - \Delta)} = \frac{1 - G(\phi_{\tau^*_1})}{g(\phi_{\tau^*_1})}$. So $\tau^*_X \leq \bar{\tau}_1 \iff \tau^*_B \leq \bar{\tau}_1$, given that $d\phi_{\tau^*_1}/d\tau \in (0, 1)$.

**A.3.3 Dual mode - overall equilibrium**

We can now combine both types of equilibrium to pin down $S$’s participation decision and $M$’s optimization problem in setting $\tau$. Recall that if $S$ does not participate, then the pricing subgame unfolds as if $M$ operated as a pure reseller.

Suppose $\Delta \leq \frac{G(-c)}{g(-c)} < \frac{G(A)}{g(A)}$ and $S$ participates. In the pricing subgame, the reseller equilibrium exists if and only if $\tau \geq \bar{\tau}_B$. Meanwhile, from the previous subsection, we know the intermediation equilibrium exists if and only if $\tau \leq \tau_B$. To summarize the outcome of the post-participation subgame:

<table>
<thead>
<tr>
<th>Range of $\tau$</th>
<th>$\Pi^{eqm}(\tau)$</th>
<th>$\pi^{eqm}(\tau)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tau \leq \Delta - c$</td>
<td>$\tau(1 - G(-c))$</td>
<td>$\Delta - (c + \tau)(1 - G(-c))$</td>
</tr>
<tr>
<td>$\tau \in (\Delta - c, \bar{\tau}_B]$</td>
<td>$\tau(1 - G(\tau - \Delta))$</td>
<td>$\Delta G(\tau - \Delta)$</td>
</tr>
<tr>
<td>$\tau \geq \bar{\tau}_B$</td>
<td>$(\frac{1 - G(B)^2}{g(B)})$</td>
<td>$\Delta G(B)$</td>
</tr>
</tbody>
</table>

\(^8\)A sufficient condition is $\phi_{\tau}$ being convex, which is satisfied if $G$ is uniform.
If $S$ does not participate, its profit is $\pi^{np} = \Delta G(B)$. For $\tau \leq \Delta - c$, we have $\pi^{eqm} \geq \pi^{np}$ if and only if $\tau(1 - G(-c)) \leq \Delta (1 - G(B)) - c(1 - G(-c))$, implying $\tau(1 - G(-c)) \leq (\Delta - c)(1 - G(B))$ due to $-c < B$. For $\tau \geq \Delta - c$, we have $\pi^{eqm} \geq \pi^{np}$ if and only if $\tau \geq B + \Delta$. Note that $B + \Delta \leq \bar{\tau}_B$ by Lemma A.2 point (iii), so $\tau \geq B + \Delta$ is feasible. By setting $\tau = B + \Delta$, $M$ achieves the profit $\Pi^{eqm} = (B + \Delta)(1 - G(B))$, which is higher than the profit from setting $\tau > \bar{\tau}_B$ or $\tau \leq \Delta - c$ that still ensures participation. Moreover, $\bar{\tau}_B < B + \Delta$ and so $M$ cannot achieve higher profit by setting $\tau \in (B + \Delta, \bar{\tau}_B]$. We conclude $\tau^{dual} = B + \Delta$.

Next, suppose $\frac{G(-c)}{g(-c)} < \Delta \leq \frac{G(A)}{g(A)}$ and $S$ participates. The reseller equilibrium exists if and only if $\tau \geq \bar{\tau}_B$. Meanwhile, $A + \frac{G(A)}{g(A)} \geq \bar{\tau}_1 \iff \phi_{\bar{\tau}_1} \leq A \iff \bar{\tau}_1 = \frac{G(A)}{g(A)}$.

From Lemma A.2 point (iii), we know that $\bar{\tau}_X \geq A + \frac{G(A)}{g(A)}$, and so $\Delta \leq \frac{G(A)}{g(A)} \implies \bar{\tau}_X \geq \bar{\tau}_1 \iff \bar{\tau}_B \leq \bar{\tau}_1$ by Lemma A.2 point (i). Therefore, from the analysis of the intermediation equilibrium for the case $\Delta > \frac{G(-c)}{g(-c)}$, we know that the intermediation equilibrium exists if and only if $\tau \leq \bar{\tau}_B$. To summarize the outcome of the subgame that starts after $S$'s decision to participate:

<table>
<thead>
<tr>
<th>Range of $\tau$</th>
<th>$\Pi^{eqm}(\tau)$</th>
<th>$\pi^{eqm}(\tau)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tau \leq \frac{G(-c)}{g(-c)} - c$</td>
<td>$\tau(1 - G(-c))$</td>
<td>$\Delta - (c + \tau)(1 - G(-c))$</td>
</tr>
<tr>
<td>$\tau \in \left[\frac{G(-c)}{g(-c)} - c, \Delta - c\right]$</td>
<td>$\tau(1 - G(\phi_{\tau}))$</td>
<td>$\Delta - (c + \tau)(1 - G(\phi_{\tau}))$</td>
</tr>
<tr>
<td>$\tau \in [\bar{\tau}_1, \bar{\tau}_B]$</td>
<td>$\tau(1 - G(\tau - \Delta))$</td>
<td>$\Delta G(\tau - \Delta)$</td>
</tr>
<tr>
<td>$\tau \geq \bar{\tau}_B$ (reseller eqm)</td>
<td>$\frac{(1 - \bar{\tau}_B)\gamma}{g(B)}$</td>
<td>$\Delta G(B)$</td>
</tr>
</tbody>
</table>

If $S$ does not participate, its profit is $\pi^{np} = \Delta G(B)$. To proceed, first note that the definitions of $A$, $B$ and $\bar{\tau}_1$ imply $\Delta \leq \frac{G(A)}{g(A)} \implies \Delta \leq \frac{G(B)}{g(B)} \implies B + \Delta \geq \bar{\tau}_1$.

This means that for all $\tau \leq \bar{\tau}_1$, we have $\phi_{\tau} < \phi_{\bar{\tau}_1} = \bar{\tau}_1 - \Delta \leq B$. For $\tau \leq \Delta - c$, similar to the previous paragraph, $\pi^{eqm} \geq \pi^{np}$ only if $\Pi^{eqm}(\tau) \leq (\Delta - c)(1 - G(B))$. For $\tau \in [\Delta - c, \bar{\tau}_1]$, we have $G(\phi_{\tau})^2 / g(\phi_{\tau}) < \Delta G(\phi_{\tau}) \leq \Delta G(B)$, where the first inequality used $G(\phi_{\tau}) / g(\phi_{\tau}) < \Delta$ for all $\tau < \bar{\tau}_1$ given how $\bar{\tau}_1$ is defined. Therefore $S$ does not participate for $\tau$ within this region. For $\tau \geq \Delta - c$, we have $\pi^{eqm} \geq \pi^{np}$ if and only if $\tau \geq B + \Delta$, and note $B + \Delta \leq \bar{\tau}_B$ and so $\tau \geq B + \Delta$ is feasible. By setting $\tau = B + \Delta$, $M$ achieves the profit $\Pi^{eqm} = (B + \Delta)(1 - G(B))$, higher than the profit from setting $\tau \notin [\bar{\tau}_1, \bar{\tau}_B]$ (while still ensuring participation). Moreover, we know $\bar{\tau}_B < B + \Delta$ and so $M$ cannot achieve higher profit by setting $\tau \in (B + \Delta, \bar{\tau}_B]$. We conclude $\tau^{dual} = B + \Delta$.

Suppose $\Delta > \frac{G(A)}{g(A)}$. Note that $\Delta > \frac{G(A)}{g(A)} \implies A > -c + \frac{1 - G(A)}{g(A)} \implies A > -c$, implying $\Delta > \frac{G(-c)}{g(-c)}$. The reseller equilibrium exists if and only if $\tau \geq \bar{\tau}_A$. If $\bar{\tau}_B > \bar{\tau}_1$, then the reseller equilibrium exists if and only if $\tau \leq \bar{\tau}_B$, and $\bar{\tau}_B > \bar{\tau}_A$ given $\Delta > \frac{G(A)}{g(A)}$. If $\bar{\tau}_B \leq \bar{\tau}_1$, then the intermediation equilibrium exists if and only if $\tau \leq \bar{\tau}_X$, and note $\bar{\tau}_X > A + \frac{G(A)}{g(A)} > \bar{\tau}_A$ by Lemma A.2 point (iii). There is therefore a parameter region in which both the reseller equilibrium and the intermediation equilibrium coexist.
Consider first the subcase of $\Delta \geq \frac{G(A)}{g(A)}$ with $\bar{\tau}_1 > \bar{\tau}_1$. We have

<table>
<thead>
<tr>
<th>Range of $\tau$</th>
<th>$\Pi^{eqm}(\tau)$</th>
<th>$\pi^{eqm}(\tau)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tau \leq \frac{G(-c)}{g(-c)} - c$</td>
<td>$\tau(1 - G(-c))$</td>
<td>$\Delta - (c + \tau)(1 - G(-c))$</td>
</tr>
<tr>
<td>$\tau \in \left( \frac{G(-c)}{g(-c)} - c, \Delta - c \right)$</td>
<td>$\tau(1 - G(\phi_{\tau}))$</td>
<td>$\Delta - (c + \tau)(1 - G(\phi_{\tau}))$</td>
</tr>
<tr>
<td>$\tau \in [\Delta - c, \bar{\tau}_1)$</td>
<td>$\tau(1 - G(\phi_{\tau}))$</td>
<td>$G(\phi_{\tau})^2 / g(\phi_{\tau})$</td>
</tr>
<tr>
<td>$\tau \in [\bar{\tau}_1, \bar{\tau}_B)$</td>
<td>$\tau(1 - G(\tau - \Delta))$</td>
<td>$\Delta G(\tau - \Delta)$</td>
</tr>
<tr>
<td>$\tau \geq \bar{\tau}_A$ (reseller eqm)</td>
<td>$\frac{(1 - G(A))^2}{g(A)}$</td>
<td>$G(A)^2 / g(A)$</td>
</tr>
</tbody>
</table>

If $S$ does not participate, its profit is $\pi^{np} = G(A)^2 / g(A)$. For $\tau \leq \Delta - c$, we have $\Delta - c < A + \frac{G(A)}{g(A)}$ implies $\phi_{\tau} < A$ (due to the definition of $A$), and so $\pi^{eqm} \geq \pi^{np}$ if and only if

$$\Pi^{eqm}(\tau) \leq \Delta - c(1 - G(\phi_{\tau})) - \frac{G(A)^2}{g(A)} = (\Delta - c)(1 - G(\phi_{\tau})) - \left( \Delta G(\phi_{\tau}) - \frac{G(A)^2}{g(A)} \right).$$

For $\tau \in [\Delta - c, \bar{\tau}_1)$, we have $\pi^{eqm} = G(\phi_{\tau})^2 / g(\phi_{\tau})$. Given $\Delta > \frac{G(A)}{g(A)}$, we know $A + \frac{G(A)}{g(A)} \in [\Delta - c, \bar{\tau}_1]$ and so $\tau \geq A + \frac{G(A)}{g(A)}$ is feasible. Notice that for $\tau \geq A + \frac{G(A)}{g(A)}$, we have $\pi^{eqm} = G(\phi_{\tau})^2 / g(\phi_{\tau})$ increasing and continuous in $\tau$ and $\pi^{eqm} = G(\phi_{\tau})^2 / g(\phi_{\tau})$ when $\tau = A + \frac{G(A)}{g(A)}$. Therefore, $\pi^{eqm} \geq G(A)^2 / g(A) = \pi^{np}$. Moreover, by setting $\tau = A + \frac{G(A)}{g(A)}$, $M$ achieves the profit $\Pi^{eqm} = (\Delta - c + \frac{G(A)}{g(A)}(1 - G(A)))$. For $\tau \in [\bar{\tau}_1, \bar{\tau}_B)$, given $\Delta > \frac{G(A)}{g(A)} \implies \bar{\tau}_1 > B + \Delta > \bar{\tau}_B$, we know that $\Pi^{eqm}(\tau)$ is decreasing for all $\tau$ in this range. Therefore, all $\tau \in [\bar{\tau}_1, \bar{\tau}_B]$ is dominated by $\bar{\tau}_1$. We conclude that $M$ either sets $\tau \in \left[ A + \frac{G(A)}{g(A)}, \bar{\tau}_1 \right]$, or sets $\tau \leq \Delta - c$.

Next consider the subcase of $\Delta > \frac{G(A)}{g(A)}$ with $\bar{\tau}_X \leq \bar{\tau}_1$. We have

<table>
<thead>
<tr>
<th>Range of $\tau$</th>
<th>$\Pi^{eqm}(\tau)$</th>
<th>$\pi^{eqm}(\tau)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tau \leq \frac{G(-c)}{g(-c)} - c$</td>
<td>$\tau(1 - G(-c))$</td>
<td>$\Delta - (c + \tau)(1 - G(-c))$</td>
</tr>
<tr>
<td>$\tau \in \left( \frac{G(-c)}{g(-c)} - c, \Delta - c \right)$</td>
<td>$\tau(1 - G(\phi_{\tau}))$</td>
<td>$\Delta - (c + \tau)(1 - G(\phi_{\tau}))$</td>
</tr>
<tr>
<td>$\tau \in [\Delta - c, \bar{\tau}_X)$</td>
<td>$\tau(1 - G(\phi_{\tau}))$</td>
<td>$G(\phi_{\tau})^2 / g(\phi_{\tau})$</td>
</tr>
<tr>
<td>$\tau \geq \bar{\tau}_A$</td>
<td>$\frac{(1 - G(A))^2}{g(A)}$</td>
<td>$G(A)^2 / g(A)$</td>
</tr>
</tbody>
</table>

By a very similar analysis to that in the previous paragraph, we can establish that $M$ optimally sets $\tau \in \left[ A + \frac{G(A)}{g(A)}, \bar{\tau}_X \right]$, or sets some $\tau \leq \Delta - c$.

To summarize:

**Proposition A.1** In the overall equilibrium, $M$ always sets $\tau$ to induce the intermediation equilibrium.

- If $\Delta \leq \frac{G(A)}{g(A)}$, $M$ sets $\tau^{\text{dual}} = B + \Delta$ and $S$ participates. In equilibrium, $p_m^* = c + \tau^{\text{dual}} - \Delta$, $p_s^* = c + \tau^{\text{dual}}$ and $p_r^* = c + \Delta$.

- If $\Delta > \frac{G(A)}{g(A)}$, either (i) $M$ sets $\tau^{\text{dual}} \in \left[ A + \frac{G(A)}{g(A)}, \min \{ \bar{\tau}_1, \bar{\tau}_X \} \right]$ and $S$ participates, with equilibrium prices $p_m^* = c + \tau^{\text{dual}} - \Delta$, $p_s^* = c + \tau^{\text{dual}}$ and $p_r^* = c + \tau^{\text{dual}} - \phi_{\tau^{\text{dual}}}$; or (ii) $M$ sets $\tau^{\text{dual}} \leq \Delta - c$, and $S$ participates, with equilibrium prices $p_m^* = 0$, $p_s^* = \Delta$ and $p_r^* = \Delta + \min \{ c, -\phi_{\tau^{\text{dual}}} \}$.

To complete the equilibrium characterization, we explore the case $\Delta > \frac{G(A)}{g(A)}$ with numerical simulations. Based on Proposition A.1, $M$’s optimization problem is:

$$\max_{\tau} \Pi^{eqm}(\tau) \text{ subject to } \pi^{eqm}(\tau) \geq \pi^{np},$$
where

<table>
<thead>
<tr>
<th>Range of $\tau$</th>
<th>$\Pi^{eqm}(\tau)$</th>
<th>$\pi^{eqm}(\tau) - \pi^{np}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tau \leq \Delta - c$</td>
<td>$\tau(1 - G(\max{\phi, -c}))$</td>
<td>$\Delta - (c + \tau)(1 - G(\max{\phi, -c})) - \frac{G(A)^2}{g(A)}$</td>
</tr>
<tr>
<td>$\tau \in \max{\bar{\tau}_1, \bar{\tau}_X}$</td>
<td>$\tau(1 - G(\phi))$</td>
<td>$\frac{G(\phi)^2}{g(\phi)} - \frac{G(A)^2}{g(A)}$</td>
</tr>
</tbody>
</table>

Intuitively, there are two possible regimes for $M$’s commission: (i) a high $\tau$ that makes $S$ earns zero profit from inside sales, but $\tau$ is high enough so that sufficiently many consumers purchase from $S$ outside to ensure $S$ participates, and (ii) a sufficiently low commission that makes $S$ earns sufficient profit from inside sales, thereby ensuring its participation.

**Example 1 (Numerical example)** Let $G \sim U[-1, 1]$, $c = 0.5$, and $\Delta \in [1.25, 3]$. Here, $\Delta > \frac{G(A)}{g(A)}$ is equivalent to $\Delta \geq 1.25$.

Figure 1 below plots $M$’s optimal commission $\tau$ for any given $\Delta$, and the induced inside and outside prices $p_i^*$ and $p_o^*$. For $\Delta$ that is low enough, we have a “high commission regime” where $M$ optimally sets a high commission at $\tau_{dual} \in \left[A + \frac{G(A)}{g(A)}, \min\{\bar{\tau}_1, \bar{\tau}_X\}\right]$. For this set of parameters, the participation constraint binds in the high commission regime, so $\tau_{dual} = A + \frac{G(A)}{g(A)} = \frac{2(\Delta + 1)}{3}$. As $\Delta$ becomes higher, this participation constraint becomes tight, and $M$ optimally switches to the “low commission regime” of $\tau_{dual} \leq \Delta - c$. Therefore, for intermediate value of $\Delta$, the participation constraint binds $M$ commission below $\tau_{dual} \leq \arg \max \tau(1 - G(\phi)) = 1.5$. In the low commission regime, the participation constraint relaxes when $\Delta$ increases, and so $\tau_{dual} \to 1.5$ as $\Delta$ increases.

![Dual mode - fee and prices](image)

Figure 1: Equilibrium characterization of dual mode when $\Delta > G(A)/g(A)$, assuming $G \sim U[-1, 1]$ and $c = 0.5$.

### A.4 Comparisons of the different modes

We first compare between the two pure modes:

---

9Generally, if $G$ follows $U[b_L, b_H]$, then $B = \frac{1}{2}(b_H - c)$, $A = \frac{\Delta - \epsilon + b_H + b_L}{\frac{g(A)}{2}}$, $\tau^m = \frac{b_H}{2}$, $\phi = \frac{\epsilon + b_H}{2}$, $\bar{\tau}_1 \equiv b_L + 2\Delta$, $\bar{\tau}_X = \frac{b_H}{2} - b_L$, and we have $\Delta > \frac{G(A)}{g(A)}$ if and only if $\Delta > \frac{b_H - c}{2} - b_L$. 

---

Proposition A.2

- M’s profit: $\Pi^\text{market} > \Pi^\text{resell}$ if and only if $\Delta > c + \frac{G(r^m)}{g(r^m)}$.
- S’s profit: $\pi^\text{market} > \pi^\text{resell}$.
- Total consumer surplus: $CS^\text{market} \leq CS^\text{resell}$, where the inequality is strict if $c > 0$.
- Welfare: $W^\text{market} > W^\text{resell}$ if and only if $\int_{-\infty}^{\max\{A,B\}} (\Delta - c) dG(b) - \int_{\max\{A,B\}}^{r^m} b dG(b) > 0$.

Proof. We know $\Pi^\text{market} = \frac{1-G(r^m)}{g(r^m)}$ and $\Pi^\text{resell} = \frac{(1-G(\max\{A,B\}))^2}{g(\max\{A,B\})}$, so that $\Pi^\text{market} > \Pi^\text{resell}$ if and only if $r^m < \max\{A, B\}$. The stated condition in the proposition, $\Delta > c + \frac{G(r^m)}{g(r^m)}$, is equivalent to $\Delta - c + \frac{1-2G(r^m)}{g(r^m)} > \frac{1-G(r^m)}{g(r^m)} = r^m$, which is then equivalent to $A > r^m$ from definition (A.1), implying $\tau^m < \max\{A, B\}$ as required. Next, suppose $\Delta \leq c + \frac{G(r^m)}{g(r^m)}$, which is equivalent to $A \leq r^m$. Moreover, by definition (A.2), we have $B \leq r^m$. Thus, $r^m \geq \max\{A, B\}$, implying $\Pi^\text{market} \leq \Pi^\text{resell}$. Turning to S’s profit, we have $\pi^\text{resell} = \Delta G(b) < c = \pi^\text{market}$. Next, we can write down consumer surplus in each mode as (after doing some substitutions):

$$CS^\text{market} = v + \int_{r^m}^{\infty} (b - c - r^m) dG(b) + \int_{-\infty}^{r^m} -c dG(b),$$

$$CS^\text{resell} = \begin{cases} v + \int_{b}^{\infty} (b - c - B) dG(b) + \int_{-\infty}^{B} -c dG(b) & \text{if } \Delta \leq \frac{G(A)}{g(A)} \frac{G(B)}{g(B)} \\ v + \int_{A}^{\infty} \left(b - \frac{1-G(A)}{g(A)}\right) dG(b) + \int_{-\infty}^{A} \left(\Delta - \frac{G(A)}{g(A)} - c\right) dG(b) & \text{if } \Delta > \frac{G(A)}{g(A)} \frac{G(B)}{g(B)} \end{cases}.$$ 

If $\Delta \leq \frac{G(A)}{g(A)}$, then $CS^\text{resell} \geq CS^\text{market}$ follows from $B \leq r^m$. If $\Delta > \frac{G(A)}{g(A)}$, we note $\frac{1-G(A)}{g(A)}$ is decreasing in $\Delta$, and approaches $\frac{1-G(B)}{g(B)} = B + c < c + r^m$ when $\Delta \rightarrow \frac{G(A)}{g(A)}$. Therefore, $\frac{1-G(A)}{g(A)} < c + r^m$ for all $\Delta > \frac{G(A)}{g(A)}$, implying $CS^\text{resell} > CS^\text{market}$. Finally, we have

$$W^\text{market} = v + \int_{r^m}^{\infty} (\Delta - c + b) dG(b) + \int_{-\infty}^{r^m} -(\Delta - c) dG(b)$$

$$W^\text{resell} = v + \int_{\max\{A,B\}}^{\infty} bdG(b) + \int_{-\infty}^{\max\{A,B\}} -(\Delta - c) dG(b)$$

Rearranging, we get

$$W^\text{market} - W^\text{resell} = \int_{-\infty}^{\max\{A,B\}} (\Delta - c) dG(b) - \int_{\max\{A,B\}}^{r^m} bdG(b).$$

The condition $\Delta > c + \frac{G(r^m)}{g(r^m)}$ for $M$ to prefer the marketplace mode over the reseller mode obtained here is analogous to the condition $\Delta \geq \frac{c}{r^m}$ in the baseline model with discrete consumer types. Thus, $M$ prefers the marketplace mode when $\Delta$ is large relative to $M$’s cost efficiency and the mass of consumers preferring to transact through the direct channel (i.e. those with low $b$), and the reseller mode otherwise. The results for S’s profit and total consumer surplus are consistent with the baseline model in the main text. The new result is the additional condition for $W^\text{market} > W^\text{resell}$. We interpret this condition below, together with the next result.

Compare now the dual mode with the two pure modes. For tractability, we first focus on the case with $\Delta \leq \frac{G(A)}{g(A)}$, in which we have a closed-form solution for $M$’s optimal commission in dual mode. If $G$ follows $U[0_L, b_H]$, the assumption is equivalent to $\Delta \leq \frac{b_H - 2b_L - c}{2}$.

Proposition A.3 Suppose $\Delta \leq \frac{G(A)}{g(A)}$:

- M’s profit: $\Pi^\text{dual} > \Pi^\text{resell} \geq \Pi^\text{market}$. 

10
• S’s profit: $\pi^{\text{market}} > \pi^{\text{dual}} = \pi^{\text{resell}}$.

• Consumer surplus: $CS^{\text{dual}} = CS^{\text{resell}} \geq CS^{\text{market}}$, where the inequality is strict if $c > 0$.

• Welfare: $W^{\text{dual}} > W^{\text{resell}}$; $W^{\text{dual}} > W^{\text{market}}$ if and only if

$$\int_B^m bdG(b) > 0.$$ (A.12)

**Proof.** The supposition $\Delta \leq \frac{G(A)}{g(A)}$ implies $\Pi^{\text{resell}} = \frac{(1-G(B))^2}{g(B)} \geq \frac{(1-G(\tau^m))^2}{g(\tau^m)} = \Pi^{\text{market}}$. Meanwhile, the fact that in dual mode $M$ strictly prefers choosing a $\tau$ that induces the intermediation equilibrium implies that $\Pi^{\text{dual}} = \left(\Delta - c + \frac{1-G(B)}{g(B)}\right)(1-G(B)) > \frac{(1-G(B))^2}{g(B)} = \Pi^{\text{resell}}$. As for S’s profit, we have $\pi^{\text{dual}} = \Delta G(B) < \Delta = \pi^{\text{market}}$. It is straightforward to verify that $CS^{\text{dual}} = CS^{\text{resell}}$ given $\Delta \leq \frac{G(A)}{g(A)}$.

As for welfare:

$$W^{\text{dual}} = v + \int_B^\infty (\Delta - c + b) dG(b) + \int_{-\infty}^B (\Delta - c) dG(b)$$

$$> v + \int_B^\infty bdG(b) + \int_{-\infty}^B (\Delta - c) dG(b) = W^{\text{resell}}.$$  

Finally, $W^{\text{dual}} - W^{\text{market}} = \int_B^m bdG.$

Notably, with a continuum of consumer types we have $W^{\text{dual}} \neq W^{\text{market}}$ in general, in contrast to the baseline model. Given that all consumers purchase one unit of S’s product regardless of which channel they use, the only welfare difference across these two modes is due to a possible distortion arising from cross-channel price differences. To the extent that S’s price is lower in one channel than another, this will induce too many consumers to buy in the channel they do not prefer, potentially forgoing the transaction “benefit” $b$. Both modes potentially involve distortions. The marketplace involves the inside price being set $\tau^m$ higher than the outside price, whereas the dual mode may involve the inside price being set higher or lower than the outside price. If in dual mode the inside price is higher than the outside price,$^{10}$ the distortion is lower under the dual mode. However, if in dual mode the inside price is lower than the outside price, then the comparison is ambiguous. Condition (A.12) can then be understood as requiring that the distortion of inducing excessive usage of the marketplace channel in dual mode is more than offset by the under-utilization of the marketplace in the marketplace mode.

To numerically evaluate the case with $\Delta > \frac{G(A)}{g(A)}$, we consider two distinct sets of parameters: (i) $G \sim U[-1,1]$, $c = 0.5$, as in Example 1; and (ii) $G \sim U[-1,0.2]$, $c = 0.5.$\textsuperscript{11} Figures 2 - 3 plot M’s profit, S’s profit, total consumer surplus, and welfare for each set of parameters.

The following observations are in order. First, $\Pi^{\text{dual}} > \max \{\Pi^{\text{resell}}, \Pi^{\text{market}}\}$, and a ban on the dual mode results in M choosing the reseller mode if $\Delta$ is small, and choosing the marketplace mode if $\Delta$ is high.

Second, $\pi^{\text{market}} > \pi^{\text{dual}} \geq \pi^{\text{resell}}$. The last inequality reflects that S’s participation constraint does not necessarily pin down M’s commission in the dual mode. For $\Delta$ sufficiently large, S achieves a strictly higher profit under the dual mode than under the reseller mode, because the benefit from accessing extra consumers (through being hosted) strictly outweighs the loss from having to pay commissions.

Third, the welfare comparisons are generally ambiguous due to the distortions in channel usage as discussed above. For this reason, the welfare effect of banning the dual mode can be ambiguous, in contrast to the baseline model with discrete consumer types.

\textsuperscript{10}A sufficient condition is $G(0)$ must be sufficiently small relative to $c$, i.e. $c < \frac{1-G(0)}{g(0)}$.

\textsuperscript{11}We also considered a range of other parameter values, obtaining similar qualitative insights. The details and the MATLAB code are available from the authors upon request.
Finally, $CS_{\text{dual}} > CS_{\text{market}}$, while $CS_{\text{dual}} \geq CS_{\text{resell}}$ if and only if $\Delta$ is not too large. Note that in Figure 2, a ban on the dual mode always weakly decreases consumer surplus. This is because in the range of $\Delta$ with $M$ switching to the reseller mode post-ban, we have $CS_{\text{dual}} \geq CS_{\text{resell}}$. In contrast, in Figure 3, a ban on the dual mode increases consumer surplus for some intermediate range of $\Delta$, i.e. the range where $\Pi_{\text{resell}} > \Pi_{\text{market}}$ and $CS_{\text{dual}} < CS_{\text{resell}}$. This insight is consistent with the baseline model with discrete consumer types: whenever $\Delta$ is large enough, the reseller mode leads to higher total consumer surplus as it allows for wider dissemination of the innovation surplus. However, if $\Delta$ is too high or is too low, $M$ prefers to operate as a marketplace, resulting in lower consumer surplus.

B Innovation and product imitation

B.1 Imperfect imitation

In this section, we allow product imitation to be imperfect to explore how this affects the results derived in Section 4.1. Suppose the imitation is imperfect so that it raises the value of $M$’s product to $v + \beta \Delta$, where $\beta \leq 1$ measures the strength of imitation. Section 4.1 is equivalent to setting $\beta = 1$, while Section 3.3 is equivalent to setting $\beta = 0$.

We first restate the equilibrium of the post-participation subgame.
Figure 3: Cross-mode comparisons when $\Delta > G(A)/g(A)$, assuming $G \sim U[-1,0.2]$ and $c = 0.5$.

Lemma B.1 (Dual mode, intermediation) Define $\hat{p} = c + \Delta + \frac{b - \tau (1 - \mu)}{\mu}$.

- If $\Delta < \frac{c}{1 - \beta}$, then there is no intermediation equilibrium.
- If $\Delta \geq \frac{c}{1 - \beta}$, then any price profile satisfying $p^*_i \in \left[ \max \{ c + \tau, (1 - \beta)\Delta \} , \min \{ \hat{p} , \tau + (1 - \beta)\Delta \} \right]$, $p^*_m = p^*_i - (1 - \beta)\Delta$, $p^*_o = c + \Delta$ is an intermediation equilibrium. The intermediation equilibria exist if and only if $\tau \leq b + \mu \min \left\{ \frac{b + c + \beta \Delta}{1 - \mu}, \Delta \right\}$.

Equilibrium profits are $\Pi = \tau (1 - \mu)$ and $\pi = \mu \Delta + (p^*_i - c - \tau) (1 - \mu)$.

Proof. The proof follows from Lemma 1, after adjusting the value of $M$’s product to $v + \beta \Delta + b$. In particular, the definition of $\hat{p}$ (the price that makes $S$ indifferent between selling on the marketplace and selling exclusively through its direct channel) remains the same given $S$ makes sales in both channels. The only difference is that the set $\Phi_i$ defined in (7.6) has to be replaced by

$$\Phi_i = \left[ \max \{ c + \tau, (1 - \beta)\Delta \} , \min \{ \hat{p} , \tau + (1 - \beta)\Delta \} \right].$$

This set is non-empty if and only if (i) $\Delta \geq \frac{c}{1 - \beta}$ and (ii) $\hat{p} \geq \max \{ c + \tau, (1 - \beta)\Delta \}$. We have $\hat{p} \geq (1 - \beta)\Delta \Leftrightarrow \tau \leq b + \mu \left( \frac{b + c + \beta \Delta}{1 - \mu} \right)$, while $\hat{p} \geq c + \tau \Leftrightarrow \tau \leq b + \mu \Delta$. ■
Lemma B.2 (Dual mode, direct sales)

- If \( \Delta < \frac{c}{\beta} \) or \( \Delta(1 - \mu - \beta) < b + c \), then there is no direct sales equilibrium.
- If \( \Delta \geq \frac{c}{\beta} \) and \( \Delta(1 - \mu - \beta) \geq b + c \), then any price profile satisfying \( \hat{p}_i > \Delta(1 - \beta) \), \( p^*_m = 0 \), \( p^*_o = \Delta(1 - \beta) - b \) is a direct sales equilibrium. Direct sales equilibria exist if and only if \( \tau \geq \frac{b + (c + \beta \Delta) \mu}{1 - \mu} \).

Equilibrium profits are \( \Pi = 0 \) and \( \pi = \Delta(1 - \beta) - b - c \).

Proof. The proof follows from Lemma 2, after adjusting the value of \( M \)'s product to \( v + \beta \Delta + b \). ■

Lemma B.3 (Dual mode, reseller) Suppose \( \Delta < \frac{c}{\beta} \).

- If \( \tau > b + \mu \beta (1 - \beta) \Delta \), then in the mixed-strategy equilibrium \( p^*_m \) is distributed according to c.d.f \( F^*_m(p^*_m) = F_m(p^*_m + (1 - \beta) \Delta) \) and \( p^*_o \) is distributed according to c.d.f \( F_o(p^*_o) \), where \( F_m \) and \( F_o \) are defined in Lemma 9. Equilibrium profits are \( \Pi = (c + b + \mu \beta \Delta)(1 - \mu) \) and \( \pi = \mu \Delta \).
- If \( \tau \leq b + \mu \beta (1 - \beta) \Delta \), then in the equilibrium, \( p^*_m = c + \Delta \), \( p^*_i = c + \tau \) and \( p^*_o = c + \tau - (1 - \beta) \Delta \). All regular consumers purchase from \( M \) if \( c > 0 \) and purchase from \( S \) if \( c = 0 \). Equilibrium profits are \( \Pi = (c + \tau - (1 - \beta) \Delta)(1 - \mu) \) and \( \pi = \mu \Delta \).

Suppose \( \Delta \geq \frac{c}{\beta} \).

- If \( \Delta(1 - \mu - \beta) > b + c \) or \( \tau < b + \mu \Delta \), then there is no reseller equilibrium.
- If \( \Delta(1 - \mu - \beta) \leq b + c \) and \( \tau \geq b + \Delta \), then any price profile satisfying \( \hat{p}_i > c + b + \Delta \), with \( p^*_m \) being distributed according to c.d.f \( F^*_m(p^*_m) = F_m(p^*_m(1 - \beta) \Delta) \) and \( p^*_o \) being distributed according to c.d.f \( F_o(p^*_o) \), where \( F_m \) and \( F_o \) are defined in Proposition 2, is a mixed-strategy reseller equilibrium. Equilibrium profits are \( \Pi = (c + b - (1 - \mu - \beta) \Delta)(1 - \mu) \) and \( \pi = \mu \Delta \).
- If \( \Delta(1 - \mu - \beta) \leq b + c \) and \( \tau \in [b + \mu \Delta, b + \Delta] \), then in the mixed-strategy equilibrium, \( p^*_m = c + \tau \), \( p^*_m \) is distributed according to c.d.f \( F^*_m(p^*_m) = F_m(p^*_m - \beta \Delta) \) and \( p^*_o \) is distributed according to c.d.f \( F_o(p^*_o) \), where \( F_m \) and \( F_o \) are defined in Lemma 3. Equilibrium profits are \( \Pi = (b + c - (1 - \mu - \beta) \Delta)(1 - \mu) \) and \( \pi = \mu \Delta \).

Proof. The proof follows from Lemma 9, Proposition 2, and Lemma 3 after adjusting the value of \( M \)'s product to \( v + \beta \Delta + b \). ■

We can now summarize the equilibrium of the post-participation subgame of stage 3:

- If \( \Delta < \frac{c}{\beta} \) then for all \( \tau \) the pricing subgame leads to the reseller equilibrium, with \( S \) earning \( \mu \Delta \).
- Suppose \( \Delta \geq \frac{c}{\beta} \) and \( \Delta(1 - \mu - \beta) \leq b + c \). If \( \tau \leq b + \mu \Delta \), the pricing subgame leads to the intermediation equilibrium, with \( S \) earning

\[
\mu \Delta + \min \left\{ \Delta \frac{b - \tau}{\mu}, (1 - \beta) \Delta - c \right\} (1 - \mu)
\]

Otherwise the pricing subgame leads to the reseller equilibrium, with \( S \) earning \( \mu \Delta \).

---

\(^{12}\)This follows from the tie-breaking rules stated in Section 2. Nonetheless, the equilibrium profit is independent of the tie-breaking rule.
• Suppose \( \Delta \geq \frac{c}{1 - \beta} \) and \( \Delta(1 - \mu - \beta) > b + c \). If \( \tau \leq \frac{b + (c + \beta \Delta) \mu}{1 - \mu} \), the pricing subgame leads to the intermediation equilibrium, with \( S \) earning

\[
\mu \Delta + \min \left\{ \Delta + \frac{b - \tau}{\mu}, (1 - \beta) \Delta - c \right\} (1 - \mu).
\]

Otherwise the pricing subgame leads to the direct sales equilibrium, with \( S \) earning \( \Delta(1 - \beta) - b - c \).

To analyze \( S \)'s decision in stage 2, it is useful and without loss of generality to suppose that \( S \) first chooses \( \Delta \) and then decides whether or not to participate. For each \( \Delta \), if \( S \) does not participate, then there is no imitation and its profit is \( \max \{ \mu \Delta, \Delta - b - c \} \), as described in Lemma 8. Let \( \text{NP} \) denote the equilibrium with \( S \) not participating. Accounting for the possibility of \( \text{NP} \), below we summarize the equilibrium outcomes for any given \( \Delta \) and \( \tau \).

• If \( 1 - \beta < \frac{c(1 - \mu)}{b + c} \), then the resulting equilibria are

\[
\begin{array}{c|cc}
\tau \leq \frac{b + \mu c}{1 - \mu} & \tau > \frac{b + \mu c}{1 - \mu} \\
\hline
\Delta \leq \frac{b + c}{1 - \mu} & \text{RE} & \text{RE} \\
\Delta \in \left( \frac{b + c}{1 - \mu}, \frac{c}{1 - \beta} \right] & \text{NP} & \text{NP} \\
\Delta > \frac{c}{1 - \beta} & \text{IE} & \text{NP} \\
\end{array}
\]

\[
(\text{B.1})
\]

• If \( 1 - \beta \geq \frac{c(1 - \mu)}{b + c} \), then the resulting equilibria are

\[
\begin{array}{c|cc}
\tau \leq \min \left\{ b + \mu \Delta, \frac{b + \mu c}{1 - \mu} \right\} & \tau > \min \left\{ b + \mu \Delta, \frac{b + \mu c}{1 - \mu} \right\} \\
\hline
\Delta < \frac{c}{1 - \beta} & \text{RE} & \text{RE} \\
\Delta \in \left[ \frac{c}{1 - \beta}, \frac{b + c}{1 - \mu} \right) & \text{IE} & \text{RE} \\
\Delta > \frac{b + c}{1 - \mu} & \text{IE} & \text{NP} \\
\end{array}
\]

\[
(\text{B.2})
\]

### B.2 Innovation decision

To simplify the problem, the following lemma narrows down the possible range of profit-maximizing \( \Delta \) chosen by \( S \):

**Lemma B.4** If \( 1 - \beta \leq \frac{c}{b + c} \), then \( S \) never sets \( \Delta \geq \frac{c}{1 - \beta} \).

**Proof.** We can derive \( S \)'s profit function in stage 2, as a function of \( \Delta \). Suppose \( 1 - \beta \leq \frac{c(1 - \mu)}{b + c} \). If \( \tau \leq \frac{b + \mu c}{1 - \mu} \), then

\[
\hat{\pi}^{\text{dual}}(\Delta) = \begin{cases} 
\mu \Delta - K(\Delta) \ (\text{RE}) & \text{if } \Delta \leq \frac{b + c}{1 - \mu} \\
\Delta - b - c - K(\Delta) \ (\text{NP}) & \text{if } \Delta \in \left( \frac{b + c}{1 - \mu}, \frac{c}{1 - \beta} \right] \\
\Delta - K(\Delta) - (1 - \mu)(c + \beta \Delta) \ (\text{IE}) & \text{if } \Delta \geq \frac{c}{1 - \beta}
\end{cases}
\]

\[
(\text{B.3})
\]

If \( \tau > \frac{b + \mu c}{1 - \mu} \), then

\[
\hat{\pi}^{\text{dual}}(\Delta) = \begin{cases} 
\mu \Delta - K(\Delta) \ (\text{RE}) & \text{if } \Delta \leq \frac{b + c}{1 - \mu} \\
\Delta - b - c - K(\Delta) \ (\text{NP}) & \text{if } \Delta \geq \frac{c}{1 - \beta}
\end{cases}
\]

\[
(\text{B.4})
\]

where we have used \( \Delta \geq \frac{b + c}{1 - \mu} \) and \( 1 - \beta < \frac{c(1 - \mu)}{b + c} \) to show in \( \text{IE} \) that \( p_\beta^* = \min \{ \hat{p} - c - \tau, (1 - \beta) \Delta - c \} = \min \left\{ \Delta + \frac{b - \tau}{\mu}, \Delta - c - \beta \Delta \right\} = \Delta - c - \beta \Delta \) when \( \tau \leq \frac{b + \mu c}{1 - \mu} \). Let \( \Delta^M \) denote the solution of

\[
1 - (1 - \mu) \beta \equiv K'(\Delta^M),
\]

15
where \( \Delta^L \leq \Delta^M \leq \Delta^H \). The peak point \( \Delta^M \) of the third expression in (B.3) is below \( \Delta^H \leq \frac{c}{1-\beta} \), so \( \bar{\pi}^{\text{dual}} \) is decreasing for all \( \Delta \geq \frac{c}{1-\beta} \). Notice \( \lim_{\Delta \to \frac{c}{1-\beta}} \bar{\pi}^{\text{dual}}(\Delta) = \lim_{\Delta \to \frac{c}{1-\beta}} \bar{\pi}^{\text{dual}}(\Delta) = \frac{c(1-\mu)}{1-\beta} - b - c > 0 \), hence \( \Delta \geq \frac{c}{1-\beta} \) is never optimal. For (B.4), both possible peak points \( \Delta^L \) and \( \Delta^H \) are below \( \frac{c}{1-\beta} \) by assumption, and the function is clearly continuous in \( \Delta \).

Suppose instead \( 1 - \beta \geq \frac{c(1-\mu)}{1-\beta} \). If \( \tau < \frac{c}{1-\beta} + b \) then

\[
\bar{\pi}^{\text{dual}}(\Delta) = \begin{cases} 
\mu \Delta - K(\Delta) (RE) & \text{if } \Delta \leq \frac{c}{1-\beta} \\
\Delta - K(\Delta) - (1 - \mu) \left( \frac{c}{1-\beta} \right) (IE) & \text{if } \Delta \in \left[ \frac{c}{1-\beta} + b, \frac{c}{1-\beta} \right] \\
\Delta - K(\Delta) - (1 - \mu) (c + \beta \Delta) (IE) & \text{if } \Delta \geq \frac{c}{1-\beta} 
\end{cases}
\]

(B.5)

If \( \tau \in \left[ \frac{c}{1-\beta} + b, \frac{b+c}{1-\beta} \right] \), then

\[
\bar{\pi}^{\text{dual}}(\Delta) = \begin{cases} 
\mu \Delta - K(\Delta) (RE) & \text{if } \Delta \leq \frac{b+c}{1-\beta} \\
\Delta - K(\Delta) - (1 - \mu) \left( \frac{c}{1-\beta} \right) (IE) & \text{if } \Delta \in \left[ \frac{b+c}{1-\beta}, \frac{c}{1-\beta} \right] \\
\Delta - K(\Delta) - (1 - \mu) (c + \beta \Delta) (IE) & \text{if } \Delta \geq \frac{b+c}{1-\beta} 
\end{cases}
\]

(B.6)

If \( \tau > \frac{b+c}{1-\beta} \), then

\[
\bar{\pi}^{\text{dual}}(\Delta) = \begin{cases} 
\mu \Delta - K(\Delta) (RE) & \text{if } \Delta \leq \frac{b+c}{1-\beta} \\
\Delta - b - c - K(\Delta) (NP) & \text{if } \Delta > \frac{b+c}{1-\beta} 
\end{cases}
\]

(B.7)

The peak points of the second and third expressions in (B.5), i.e. \( \Delta^L \) and \( \Delta^H \), are below \( \frac{c}{1-\beta} \), so \( \bar{\pi}^{\text{dual}} \) is decreasing for all \( \Delta \geq \frac{c}{1-\beta} \). Note that \( \lim_{\Delta \to \frac{c}{1-\beta}} \bar{\pi}^{\text{dual}}(\Delta) - \lim_{\Delta \to \frac{c}{1-\beta}} \bar{\pi}^{\text{dual}}(\Delta) = (1 - \mu) \left( \frac{c}{1-\beta} - \frac{c}{1-\beta} \right) > 0 \), so \( \Delta \geq \frac{c}{1-\beta} \) is never optimal. For (B.6) and (B.7), we note the functions are continuous in \( \Delta \) and all peak points are below \( \frac{c}{1-\beta} \leq \min \left\{ \frac{c}{1-\beta}, \frac{b+c}{1-\beta} \right\} \), so \( \Delta \geq \frac{c}{1-\beta} \) is never optimal.

Next, we distinguish two cases, depending on how good M’s imitation is.

**B.3 Case:** \( 1 - \beta \leq \frac{c}{1-\beta} \)

Suppose first imitation by \( M \) is sufficiently good. By Lemma B.4, S always sets \( \Delta < \frac{c}{1-\beta} \). From Tables B.1 - B.2, we get

\[
\bar{\pi}^{\text{dual}}(\Delta) = \begin{cases} 
\mu \Delta - K(\Delta) (RE) & \text{if } \Delta \leq \frac{b+c}{1-\beta} \\
\Delta - b - c - K(\Delta) (NP) & \text{if } \Delta > \frac{b+c}{1-\beta} 
\end{cases}
\]

Therefore, the problem of choosing \( \Delta \) is the same as in Section 4.1. The existing equilibrium characterization from Section 4.1 thus applies.

**Proposition B.1** (Dual mode equilibrium with imperfect product imitation) Assume \( 1 - \beta \leq \frac{c}{1-\beta} \). Then \( M \) sets \( \tau^{\text{dual}} = b + \mu \beta \Delta^L \).

- If \( \frac{c}{1-\beta} \leq \frac{b+c}{1-\beta} \), then \( S \) sets \( \Delta = \Delta^H \) and does not participate in stage 2. In stage 3, \( p_0^* = \Delta^H - b \) and \( p_m^* = 0 \), all regular consumers buy from \( S \) directly, while \( \Pi^{\text{dual}} = 0 \) and \( \pi^{\text{dual}} = \Delta^H - b - c - K(\Delta^H) \).

- If \( \frac{b+c}{1-\beta} > \Delta^L \), then \( S \) sets \( \Delta = \Delta^L \) and participates in stage 2. In stage 3, \( p_0^* = c + \Delta \) and \( p_m^* = p_i^* = c + \tau^{\text{dual}} \), all regular consumers buy from \( M \), while \( \Pi^{\text{dual}} = (1 - \mu) (b + c + \mu \beta \Delta^L) \) and \( \pi^{\text{dual}} = \mu \Delta^L - K(\Delta^L) \).

Then, Proposition 8 becomes (recall \( \Psi \equiv \Delta^H - K(\Delta^H) - (\Delta^L - K(\Delta^L)) \)):

**Proposition B.2** (Ban on dual mode):
• If $\bar{\Delta} > \frac{b + c}{1 - \mu}$, a ban on the dual mode has no effect.

• If $\bar{\Delta} \leq \frac{b + c}{1 - \mu}$ and $\Delta^L \geq \frac{c}{1 - \mu}$, a ban on the dual mode results in $M$ choosing the marketplace mode, with $\Pi$, $CS_{regular}$, and $CS$ decreasing; $CS_{direct}$ not changing; $\Delta$ and $\pi$ increasing; and $W$ decreasing if $\Psi < c(1 - \mu) - (1 - \beta) \Delta^L$ and increasing if $\Psi \geq c(1 - \mu) - (1 - \beta) \Delta^L$.

• If $\bar{\Delta} \leq \frac{b + c}{1 - \mu}$ and $\Delta^L \leq \frac{c}{1 - \mu}$, a ban on the dual mode results in $M$ choosing the reseller mode, with $\Pi$, $CS_{regular}$, and $W$ decreasing; $\pi$ and $\Delta$ not changing; $CS_{direct}$ increasing; and $CS$ increasing if $\Delta^L \geq b + c$.

**Proof.** For welfare, $W_{dual}^{market}$ becomes $v + \beta \Delta^L + (1 - \mu)b - \mu c - K(\Delta^L)$. Therefore, $W_{market}^{dual} > W_{dual}^{market}$ if and only if $\Delta^H - K(\Delta^H) - (\beta \Delta^L - K(\Delta^L)) \geq c(1 - \mu)$, or $\Psi \geq c(1 - \mu) - (1 - \beta) \Delta^L$. Meanwhile $W_{dual}^{market} > W_{resell}^{market}$ follows from the baseline model (since these two modes have the same $\Delta$). In the dual mode, $p_{\Pi}^* = c + \tau_{dual} = c + \mu \beta \Delta^L + b$. When $\Delta^L \geq \frac{c}{1 - \mu}$, we have $CS_{regular}^{dual} = v - c + (1 - \mu) \beta \Delta^L > CS_{regular}^{market} = v - c$. When instead $\Delta^L \leq \frac{c}{1 - \mu}$, the distribution support $p_{\Pi}^* \in [c + \mu \Delta^L, c + \Delta^L]$ in reseller mode implies $CS_{regular}^{resell} < v - c + (1 - \mu) \beta \Delta^L$, so it follows that $CS_{regular}^{dual} > CS_{regular}^{resell}$. Meanwhile $p_{\Pi}^* = c + \Delta$ in both the marketplace mode and the dual mode, so $CS_{direct}^{dual} = CS_{market}^{dual} = v - c < CS_{direct}^{resell}$. It follows that $CS_{dual} > CS_{market}^{dual}$. Next,

$$CS_{resell} = W_{resell} - \Pi_{resell} - \pi_{resell} = v - c + (1 - \mu)^2 \Delta^L + (1 - \mu)(1 - \eta)(\Delta^L - b - c),$$

while $CS_{dual} = CS_{regular}^{dual} + CS_{direct}^{dual} = v - c + (1 - \mu)^2 \beta \Delta^L$. So, $CS_{dual} \leq CS_{resell}$ if $\Delta^L - b - c \geq 0$. ■

There are two key differences relative to the case with perfect imitation. First, whenever a ban on the dual mode results in $M$ choosing the marketplace mode, the range of parameters under which the ban improves total consumer surplus becomes larger. Intuitively, when imitation is imperfect, the overall welfare generated by $M$’s product (which is sold to all regular consumers in dual mode) is lower. Second, whenever a ban on the dual mode results in $M$ choosing the reseller mode, the range of parameters under which the ban improves total consumer surplus becomes larger. This reflects that the net surplus left by $M$ to regular consumers is smaller when imitation is imperfect.

Likewise, the effect of banning product imitation largely follows from Proposition 9. The only difference occurs when $\bar{\Delta} \leq \frac{b + c}{1 - \mu}$, in which case a ban on imitation results in $M$ continuing to operate in dual mode. The range of parameters for the ban to result in higher welfare and profit becomes unambiguously larger when imitation is imperfect.

**Proposition B.3** (Ban on imperfect imitation):

• If $\bar{\Delta} > \frac{b + c}{1 - \mu}$, a ban on imitation results in $M$ switching from the marketplace to the dual mode (i.e. the ban makes the dual mode viable), with $CS_{regular}$, $CS$, and $\Pi$ increasing; $\pi$ decreasing; and $CS_{direct}$, $\Delta$, and $W$ not changing.

• If $\bar{\Delta} \leq \frac{b + c}{1 - \mu}$, a ban on imitation results in $M$ continuing the dual mode, with $\pi$, $\Delta$, $CS_{regular}$, and $CS$ increasing; $CS_{direct}$ not changing; $W$ decreasing if $\Psi < c(1 - \mu) - (1 - \beta) \Delta^L$ and increasing if $\Psi \geq c(1 - \mu) - (1 - \beta) \Delta^L$; and $\Pi$ decreasing if $\Psi < (c - (1 - \beta) \Delta^L) (\frac{1 - \mu}{\mu})$ and increasing if $\Psi \geq (c - (1 - \beta) \Delta^L) (\frac{1 - \mu}{\mu})$.

**Proof.** The only new comparison concerns $M$’s profit. Its dual mode profit with imitation $(1 - \mu)(b + c + \mu \beta \Delta^L)$ is higher than its dual mode profit without imitation $(b + c \bar{\Delta})(1 - \mu)$ if and only if

$$\Psi \equiv \Delta^H - \Delta^L - (K(\Delta^H) - K(\Delta^L)) \leq (c - (1 - \beta) \Delta^L) (\frac{1 - \mu}{\mu}).$$
B.4 Case: $1 - \beta > \frac{c}{\Delta \pi}$

Now suppose imitation by $M$ is not very good. In this case, the analysis of $S$’s innovation decision is not generally tractable. For this reason, we focus on the special case of $\beta \to 0$, i.e. imitation is nearly impossible. From Table B.2, if $\tau \leq \frac{b+c}{\mu}$, then

$$\bar{\pi}^\text{dual} (\Delta) = \begin{cases} 
\mu \Delta - K(\Delta) (RE) & \text{if } \Delta < \frac{\tau - b}{\mu} \\
\Delta + (1 - \mu) \min \left\{ \frac{b+\tau}{\mu} - c \right\} - K(\Delta) (IE) & \text{if } \Delta \geq \frac{\tau - b}{\mu} 
\end{cases}$$

If $\tau > \frac{b+c}{\mu}$, then

$$\bar{\pi}^\text{dual} (\Delta) = \begin{cases} 
\mu \Delta - K(\Delta) (RE) & \text{if } \Delta \leq \frac{b+c}{\mu} \\
\Delta - b - c - K(\Delta) (NP) & \text{if } \Delta > \frac{b+c}{\mu} 
\end{cases}$$

We first derive $S$’s optimal choice of innovation in stage 2.

**Lemma B.5 (Innovation level in dual mode)** In stage 2:

- Suppose $\bar{\Delta} \geq \frac{b+c}{\mu}$. If $\tau \leq \frac{b+c}{\mu}$, then $S$ chooses $\Delta^H$, resulting in the intermediation equilibrium; if $\tau > \frac{b+c}{\mu}$, then $S$ chooses $\Delta^H$, resulting in the non-participation equilibrium.

- Suppose $\bar{\Delta} \leq \frac{b+c}{\mu}$. If $\tau \leq b+\mu \bar{\Delta}$, then $S$ chooses $\Delta^H$, resulting in the intermediation equilibrium; if $\tau > b+\mu \bar{\Delta}$, then $S$ chooses $\Delta^L$, resulting in the reseller equilibrium.

**Proof. (Lemma B.5).** Denote the piece-wise components of $\bar{\pi}^\text{dual} (\Delta)$ separately as:

$$X (\Delta) \equiv \begin{cases} 
\mu \Delta - K(\Delta) & \text{if } \Delta < \frac{\tau - b}{\mu} \\
\Delta + (1 - \mu) \min \left\{ \frac{b+\tau}{\mu} - c \right\} - K(\Delta) & \text{if } \Delta \geq \frac{\tau - b}{\mu} 
\end{cases}$$

and

$$\bar{\pi}^\text{dual} (\Delta) = \begin{cases} 
\mu \Delta - K(\Delta) & \text{if } \Delta \leq \frac{b+c}{\mu} \\
\Delta - b - c - K(\Delta) & \text{if } \Delta > \frac{b+c}{\mu} 
\end{cases}$$

where $\bar{\pi}^\text{dual} (\Delta) = X (\Delta)$ if $\tau \leq \frac{b+c}{\mu}$ and $\bar{\pi}^\text{dual} (\Delta) = Y (\Delta)$ if $\tau > \frac{b+c}{\mu}$. We establish the following two claims.

**Claim 1:** arg $\max X (\Delta) = \Delta^L$ if $\tau > b + \mu \bar{\Delta}$ and arg $\max X (\Delta) = \Delta^H$ if $\tau \leq b + \mu \bar{\Delta}$, where $\bar{\Delta}$ is defined in Lemma 4.

**Proof:** For all $\tau \leq b + \mu \Delta^L$, $X (\Delta)$ has exactly one interior peak point at $\Delta = \Delta^H > \Delta^L \geq \frac{\tau - b}{\mu}$, so $S$ optimally chooses $\Delta^H$. For all $\tau > b + \mu \Delta^H$, $X (\Delta)$ has exactly one interior peak point at $\Delta = \Delta^L < \Delta^H \leq \frac{\tau - b}{\mu}$, so $S$ optimally chooses $\Delta^L$. For $\tau \in (b + \mu \Delta^L, b + \mu \Delta^H)$, $X (\Delta)$ has two interior peak points:

$$\max_{\Delta < \frac{\tau - b}{\mu}} X (\Delta) = \mu \Delta^L - K(\Delta^L) \quad \text{and} \quad \max_{\Delta \geq \frac{\tau - b}{\mu}} X (\Delta) = \Delta^H + (1 - \mu) \frac{b - \tau}{\mu} - K(\Delta^H),$$

where we used $\tau > b + \mu \Delta^L > b + c \mu$ to simplify $\min \left\{ \frac{b+\tau}{\mu} - c \right\} = \frac{b+\tau}{\mu}$. Then, $X (\Delta^L) > X (\Delta^H)$ if and only if

$$\mu \Delta^L - K(\Delta^L) > \Delta^H + (1 - \mu) \frac{b - \tau}{\mu} - K(\Delta^H) \quad \iff \quad \tau > b + \mu \bar{\Delta}.$$

**Claim 2:** arg $\max Y (\Delta) = \Delta^L$ if $\frac{b+c}{\mu} > \bar{\Delta}$, and arg $\max Y (\Delta) = \Delta^H$ if $\frac{b+c}{\mu} \leq \bar{\Delta}$, where $\bar{\Delta}$ is defined in Lemma 4.

**Proof:** Similar to the proof of Lemma 4 hence omitted.
We are now ready to prove the lemma. Consider first the case \( \Delta < \frac{b+c}{1-\mu} \). For all \( \tau > \frac{b+c}{1-\mu} \), we have \( \bar{\pi}^{dual}(\Delta) = Y(\Delta) \), so Claim 2 implies that \( S \) optimally chooses \( \Delta^L \) and induces the reseller equilibrium. For \( \tau \leq \frac{b+c}{1-\mu} \), we have \( \bar{\pi}^{dual}(\Delta) = X(\Delta) \). If \( b + \mu \Delta < \tau \leq \frac{b+c}{1-\mu} \), Claim 1 implies that \( S \) optimally chooses \( \Delta^L \) and induces the reseller equilibrium; if \( \tau \leq b + \mu \Delta \), Claim 1 implies \( S \) optimally chooses \( \Delta^H \) and induces the intermediation equilibrium.

Next consider the case \( \Delta \geq \frac{b+c}{1-\mu} \). For all \( \tau > \frac{b+c}{1-\mu} \), we have \( \bar{\pi}^{dual}(\Delta) = Y(\Delta) \), so Claim 2 implies that \( S \) optimally chooses \( \Delta^H \) and induces the direct sales equilibrium. For \( \tau \leq \frac{b+c}{1-\mu} \leq b + \mu \Delta \), we have \( \bar{\pi}^{dual}(\Delta) = X(\Delta) \), so Claim 1 implies \( S \) optimally chooses \( \Delta^H \) and induces the intermediation equilibrium.

We can now characterize the equilibrium of the full game.

**Proposition B.4 (Dual mode equilibrium with endogenous innovation)** \( M \) sets \( \tau^{dual} = b + \mu \min \left\{ \frac{b+c}{1-\mu}, \Delta \right\} \), \( S \) participates and chooses innovation level \( \Delta^H \). In stage 3:

- If \( \bar{\Delta} \geq \frac{b+c}{1-\mu} \), then equilibrium prices are \( p^*_S = c + \Delta^H, p^*_i = \Delta^H \), and \( p^*_m = 0 \).
- If \( \bar{\Delta} \leq \frac{b+c}{1-\mu} \), then equilibrium prices are \( p^*_S = c + \Delta^H, p^*_i = c + \tau^{dual} + \Delta^H - \Delta \), and \( p^*_m = c + \tau^{dual} - \Delta \).

All regular consumers buy from \( S \) on \( M \) and direct consumers buy directly. Equilibrium profits are \( \Pi^{dual} = \tau^{dual} (1 - \mu) \) and \( \pi^{dual} = \max \left\{ \Delta^H - b - c, \Delta^H - \bar{\Delta}(1 - \mu) \right\} - K(\Delta^H) \).

**Proof.** Using Lemma B.5 and Table 3, we can write down \( M \)'s profit functions in stage 1 as follows. If \( \Delta \geq \frac{b+c}{1-\mu} \),

\[
\Pi(\tau) = \begin{cases} 
\tau (1 - \mu) & \text{if } \tau \leq \frac{b+c}{1-\mu} \\
0 & \text{if } \tau > \frac{b+c}{1-\mu} 
\end{cases}
\]

If \( \bar{\Delta} \leq \frac{b+c}{1-\mu} \),

\[
\Pi(\tau) = \begin{cases} 
\tau (1 - \mu) & \text{if } \tau \leq b + \mu \bar{\Delta} \\
(b + c - (1 - \mu) \Delta^L)(1 - \mu) & \text{if } \tau > b + \mu \bar{\Delta} 
\end{cases}
\]

Obviously, \( M \) sets \( \tau^{dual} = \min \left\{ b + \mu \bar{\Delta}, \frac{b+c}{1-\mu} \right\} \).

We now consider the effect of a ban on the dual mode, while at the same time showing that \( M \) indeed prefers operating in dual mode in the absence of the ban:

**Proposition B.5 (Ban on dual mode with endogenous innovation)**

- If \( \bar{\Delta} > \frac{b+c}{1-\mu} \) or \( \Delta^L \geq \frac{c}{1-\mu} \), a ban on the dual mode results in \( M \) choosing the marketplace mode, with \( CS_{\text{regular}}, CS, \) and \( \Pi \) decreasing; \( \pi \) increasing; and \( CS_{\text{direct}}, \Delta, \) and \( W \) not changing.
- If \( \bar{\Delta} \leq \frac{b+c}{1-\mu} \) and \( \Delta^L \leq \frac{c}{1-\mu} \), a ban on the dual mode results in \( M \) choosing the reseller mode, with \( CS_{\text{regular}}, \Pi, \Delta, \) and \( W \) decreasing; \( CS_{\text{direct}} \) increasing; \( \pi \) not changing; and \( CS \) decreasing if in addition \( \Delta^L \leq b + c \).

**Proof.** If \( \bar{\Delta} > \frac{b+c}{1-\mu} \), then \( \Pi^{\text{resell}} = 0 < b(1 - \mu) = \Pi^{\text{market}} \). If \( \Delta \leq \frac{b+c}{1-\mu} \) and \( \Delta^L \geq \frac{c}{1-\mu} \), then \( \Pi^{\text{resell}} = (b + c - (1 - \mu) \Delta^L)(1 - \mu) \leq b(1 - \mu) = \Pi^{\text{market}} \). Next,

\[
\begin{align*}
CS^{\text{dual}}_{\text{regular}} &= v + \Delta^H + b - \Delta^H = v + b > v - c = CS^{\text{market}}_{\text{regular}} \\
CS^{\text{dual}}_{\text{direct}} &= CS^{\text{market}}_{\text{direct}} = v - c \\
W^{\text{dual}} &= W^{\text{market}} = v - c + \Delta^H + (1 - \mu)b - K(\Delta^H).
\end{align*}
\]
Finally, $\pi^{\text{dual}} = \Delta^H - K(\Delta^H) - b - c < \Delta^H - K(\Delta^H) = \pi^{\text{market}}$, and $\Pi^{\text{dual}} = b + c\mu > b(1 - \mu) = \Pi^{\text{market}}$.

If $\frac{b + c}{1 - \mu} > \Delta$ and $\Delta^L \leq \frac{c}{1 - \mu}$, then $\Pi^{\text{resell}} = (b + c - (1 - \mu)\Delta^L)(1 - \mu) \geq b(1 - \mu) = \Pi^{\text{market}}$. Next,

$$
CS^{\text{resell}}_{\text{regular}} \in \left[ v - c, v - c + (1 - \mu)\Delta^L \right] \\
CS^{\text{dual}}_{\text{regular}} = v + b + \Delta^H - (c + \tau^{\text{dual}} + \Delta^H - \bar{\Delta}) = v - c + (1 - \mu)\bar{\Delta} > v - c + (1 - \mu)\Delta^L,
$$

and $CS^{\text{dual}}_{\text{direct}} = v - c < CS^{\text{resell}}_{\text{direct}}$. The comparison between $W^{\text{dual}}$ and $W^{\text{resell}}$ follows from Proposition 5 using $W^{\text{dual}}_{\Delta=\Delta^H} > W^{\text{dual}}_{\Delta=\Delta^L} \geq W^{\text{resell}}_{\Delta=\bar{\Delta}}$. Next, after substituting $\tau^{\text{dual}} = b + \mu\bar{\Delta}$ and simplifying we have $\pi^{\text{dual}} = \Delta^L\mu - K(\Delta^L) = \pi^{\text{resell}}$. Meanwhile, $\Pi^{\text{dual}} = (b + \mu\bar{\Delta})(1 - \mu) > (b + c - (1 - \mu)\Delta^L)(1 - \mu) = \Pi^{\text{resell}}$. Finally

$$
CS^{\text{dual}} = W^{\text{dual}} - \Pi^{\text{dual}} = \pi^{\text{dual}} = v - c + (1 - \mu)\Delta^L \\
CS^{\text{resell}} = W^{\text{resell}} - \Pi^{\text{resell}} = \pi^{\text{resell}} = v - c + (1 - \mu)\Delta^L - (1 - \mu)(1 - \eta)(b + c - \Delta^L).
$$

We know $\bar{\Delta} > \Delta^L$, so a sufficient condition for $CS^{\text{dual}} > CS^{\text{resell}}$ is $b + c \geq \Delta^L$.

### C Commitment to separate divisions

To analyze the model in Section 5.1, we first derive the equilibrium of the stage 3 subgame, assuming $S$ participates. Similar to the analysis of the dual mode, there are three possible types of equilibria:

- **Intermediation** equilibrium—all regular consumers buy from $S$ through the marketplace.
- **Direct sales** equilibrium—all regular consumers buy from $S$ directly.
- **Reseller** equilibrium—regular consumers sometimes buy from $R$.

**Lemma C.1 (Separation mode, intermediation)** In any intermediation equilibrium, $p^{\ast}_R = 0$, $p^{\ast}_o = \Delta$ and $p^{\ast}_i = c + \Delta$. The equilibrium exists if and only if $\tau \leq \min \left\{ \Delta - c, \frac{b + c\mu}{1 - \mu} \right\}$. Equilibrium profits are $\Pi_M = \tau(1 - \mu)$, $\Pi_R = 0$, and $\pi = \mu\Delta + (\Delta - c - \tau)(1 - \mu)$.

**Proof.** With separation, in any intermediation equilibrium, the competition for regular consumers means $S$ and $R$ necessarily set $p^{\ast}_i = \Delta$ and $p^{\ast}_o = 0$ (otherwise $R$ has an incentive to undercut), while $p^{\ast}_o = c + \Delta$. Clearly, $R$ cannot profitably deviate. To ensure the stated price profile is indeed an equilibrium, we also need to make sure that (i) $S$ is not making losses inside, which requires $\Delta - c - \tau \geq 0$; and (ii) $S$ has no incentive to set a lower $p_o$ to attract regular consumers to the direct channel, which requires

$$
\Delta - b - c \leq \mu\Delta + (1 - \mu)(\Delta - c - \tau) \iff \tau \leq \frac{b + c\mu}{1 - \mu}.
$$

Indeed, when $\tau \leq \Delta - c$, the deviation profit that $S$ can attain by setting $p_o = \Delta - b$ to attract all regular consumers to buy directly is $\Delta - b - c$. This is weakly lower than the equilibrium profit if and only if $\tau \leq \frac{b + c\mu}{1 - \mu}$. Finally, there is no other intermediation equilibrium given we ruled out all equilibria involving weakly dominated strategies.

**Lemma C.2 (Separation mode, direct sales)**
• If $\Delta < \frac{b+c}{1-\mu}$, then there is no direct sales equilibrium.

• If $\Delta \geq \frac{b+c}{1-\mu}$, then any price profile satisfying $p^*_r > \Delta$, $p^*_p = 0$ and $p^*_c = \Delta - b$ is a direct sales equilibrium. Direct sales equilibria exist if and only if $\tau \geq \frac{b+\mu \sigma}{1-\mu}$. Equilibrium profits are $\Pi_{M_0} = \Pi_R = 0$ and $\pi = \Delta - b - c$.

**Proof.** The proof of Lemma 2 applies.

As for the reseller equilibrium, the main difference with its counterpart in the dual mode is that $R$ does not have an incentive to sometimes let $S$ win the inside competition, given that it no longer profits from a transaction commission. Therefore, in any such reseller equilibrium, $S$ never makes sales inside.

**Lemma C.3** *(Separation mode, reseller)*

• If $\Delta \leq \frac{b+c}{1-\mu}$ and $\tau \geq \Delta - c$, any price profile satisfying $p^*_r \in [0, \min\{c - \Delta + \tau, c + b - (1 - \mu) \Delta\}]$, $p^*_p = p^*_c + \Delta$, and $p^*_c = c + \Delta$ is a reseller equilibrium. Equilibrium profits are $\Pi_{M_0} = 0$, $\Pi_R = p^*_c(1 - \mu)$, and $\pi = \mu \Delta$.

• If $\Delta > \frac{b+c}{1-\mu}$ or $\tau < \Delta - c$, then there is no reseller equilibrium.

**Proof.** If $\Delta > \frac{b+c}{1-\mu}$ then no reseller equilibrium exists. Therefore, we focus on the case $\Delta \leq \frac{b+c}{1-\mu}$ in what follows. Given regular consumers buy from $R$, we must have $p^*_p \geq 0$, otherwise $R$ would make a loss. We next establish the upper bound for $p^*_r$. For any given $p^*_r$ such that $M$ sells to all regular consumers, $S$ can profitably undercut by setting $p_1$ slightly below $p^*_p + \Delta$ if and only if $p^*_p > c + \tau - \Delta$. Alternatively, $S$ can undercut by setting $p_o$ slightly below $p^*_o - \Delta$ to attract regular consumers to its direct channel, which yields $p^*_r - b + \Delta - c$. This is more profitable than setting $p_o = c + \Delta$ if and only if $p^*_r > c + b - (1 - \mu) \Delta$. Thus, any

$$p^*_r \in \Phi_r \equiv [0, \min\{c - \Delta + \tau, c + b - (1 - \mu) \Delta\}]$$

with $p^*_r = p^*_p + \Delta$, and $p^*_c = c + \Delta$ can be sustained as a reseller equilibrium as long as the set $\Phi_r$ is non-empty. And the set is non-empty if and only if $\tau \geq \Delta - c$. By construction, any profile with $p^*_r \notin \Phi_r$ cannot be sustained as a reseller equilibrium.

The following table summarizes the possible equilibria that can arise if $S$ decides to participate (and given $\tau$), after applying the equilibrium selection rule used in the baseline model. Here, $\Pi_{M_0}$ and $\Pi_R$ refer to $M_0$’s and $R$’s profits. For brevity, we do not state equilibrium prices:

<table>
<thead>
<tr>
<th>$\Delta$</th>
<th>$\tau$</th>
<th>Equilibrium</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\leq \frac{b+c}{1-\mu}$</td>
<td>$\leq \Delta - c$</td>
<td>$\tau \geq \Delta - c$</td>
</tr>
<tr>
<td>$&gt; \frac{b+c}{1-\mu}$</td>
<td>$&gt; \Delta - c$</td>
<td>$\Pi_{M_0} = \Pi_R = 0$, $\pi = \mu \Delta$</td>
</tr>
</tbody>
</table>

We can then derive the overall equilibrium.
Proposition C.1 (Separation mode overall equilibrium)\( M_0 \) sets \( \tau_{sep} = \min \left\{ \Delta - c, \frac{b+c}{1-\mu} \right\} \) and \( S \) participates. In the resulting intermediation equilibrium, \( p_c^* = c + \Delta, p_{b}^* = \Delta \) and \( p_0^* = 0 \). All regular consumers buy from \( S \) on \( M_0 \) and direct consumers buy directly. Equilibrium profits are \( \Pi^\text{sep}_{M_0} = \tau_{sep} (1-\mu) \), \( \Pi^\text{sep}_R = 0 \) and \( \pi_{sep} = \max \{ \mu \Delta, \Delta - c - b \} \). Moreover, \( \Pi^\text{sep}_{M_0} + \Pi^\text{sep}_R \leq \Pi^\text{dual} \), with strict inequality if \( \Delta < \frac{b+c}{1-\mu} \).

Proof. (Proposition 13). By inspection, if \( \Delta = \frac{b+c}{1-\mu} \), we have \( \Pi^\text{sep}_{M_0} + \Pi^\text{sep}_R = (\frac{b+c}{1-\mu})(1-\mu) = \Pi^\text{dual} \); when \( \Delta < \frac{b+c}{1-\mu} \), we have \( \Pi^\text{dual} = (b + \mu \Delta)(1-\mu) > (\Delta - c)(1-\mu) = \Pi^\text{sep}_{M_0} + \Pi^\text{sep}_R \), where the last inequality is due to \( \Delta \leq \frac{b+c}{1-\mu} \iff \Delta - c < b + \mu \Delta \). ■

We can now add the separation mode into the comparison of profits and welfare across the various modes:

**Proposition C.2 (Comparisons, with separation mode).**

- **S’s profit:** \( \pi^\text{market} > \pi^\text{dual} = \pi^\text{sep} = \pi^\text{resell} \).
- **Welfare:** \( W^\text{dual} = W^\text{market} \geq W^\text{resell} \), where the inequality is strict if \( b > 0 \). \( W^\text{dual} \geq W^\text{sep} \), where the inequality is strict if \( F > 0 \).
- **Direct consumers:** \( C^\text{resell}_{\text{direct}} > C^\text{dual}_{\text{direct}} = C^\text{market}_{\text{direct}} = C^\text{sep}_{\text{direct}} \).
- **Regular consumers:** If \( \Delta > \frac{b+c}{1-\mu} \), \( C^\text{sep}_{\text{regular}} = C^\text{dual}_{\text{regular}} = C^\text{market}_{\text{regular}} > C^\text{resell}_{\text{regular}} \); if \( \Delta \leq \frac{b+c}{1-\mu} \), \( C^\text{sep}_{\text{regular}} \geq C^\text{dual}_{\text{regular}} > C^\text{resell}_{\text{regular}} \geq C^\text{market}_{\text{regular}} \), where the weak inequality is strict if \( F < \frac{b+c}{1-\mu} \).
- **Total consumer surplus:** If \( \Delta > \frac{b+c}{1-\mu} \), \( C^\text{sep} > C^\text{dual} > C^\text{market} > C^\text{resell} \); if \( \Delta \leq \frac{b+c}{1-\mu} \), \( C^\text{sep} \geq C^\text{dual} > C^\text{market} \), where the weak inequality is strict if \( \Delta < \frac{b+c}{1-\mu} \), and \( C^\text{resell} \geq C^\text{market} \).

Proof. Welfare in the dual mode and the separation mode matches that under the marketplace mode given that in all these settings, regular consumers buy \( S \) via \( M \)'s marketplace. Turning next to consumer surplus, note \( C^\text{sep}_{\text{direct}} = C^\text{dual}_{\text{direct}} = C^\text{market}_{\text{direct}} \) given \( p_c^* = c + \Delta \) in all three modes, while for regular consumers, \( C^\text{sep}_{\text{regular}} = v + b \). ■

To prove Proposition 13, we first compare \( \Pi^\text{sep} = \Pi^\text{sep}_{M_0} + \Pi^\text{sep}_R - F, \Pi^\text{resell}, \) and \( \Pi^\text{market} \) to determine which mode \( M \) chooses after the dual mode is banned. Then, we combine the choice of mode with Proposition C.2 to assess the overall impact of a ban on the dual mode:

**Proof. (Proposition 13).** By inspection, if \( \Delta > \frac{b+c}{1-\mu} \), then

\[
\Pi^\text{sep} = \left( \frac{b+c}{1-\mu} \right) (1-\mu) - F > 0 = \Pi^\text{resell}.
\]

\[
\Pi^\text{sep} = \left( \frac{b+c}{1-\mu} \right) (1-\mu) - F \geq b(1-\mu) = \Pi^\text{resell} \iff F \leq (b+c)\mu.
\]

Suppose instead \( \Delta \leq \frac{b+c}{1-\mu} \),

\[
\Pi^\text{sep} = (\Delta - c)(1-\mu) - F \geq b(1-\mu) = \Pi^\text{market} \iff \Delta \geq c + b + \frac{F}{1-\mu}.
\]

\[
\Pi^\text{sep} = (\Delta - c)(1-\mu) - F \geq (b + \mu \Delta + c - \Delta)(1-\mu) = \Pi^\text{resell} \iff \Delta \geq \frac{2c + b + F}{2 - \mu}.
\]
\[ \Pi^{\text{market}} \geq \Pi^{\text{re sell}} \iff \Delta \geq \frac{c}{1-\mu}. \]

If \( F < \mu c - b(1 - \mu) \), the ordering of these thresholds is: \( c + b + \frac{F}{1 - \mu} < \frac{2c + b + \frac{F}{1 - \mu}}{2 - \mu} < \frac{c}{1 - \mu} \leq \frac{b + c}{1 - \mu} \). If \( F \in [\mu c - b(1 - \mu), \mu (b + c)] \), the ordering of these thresholds is: \( \frac{c}{1 - \mu} \leq \frac{2c + b + \frac{F}{1 - \mu}}{2 - \mu} \leq c + b + \frac{F}{1 - \mu} \leq \frac{b + c}{1 - \mu} \).

If \( F > \mu c + \mu b \), the ordering of these thresholds is: \( \frac{c}{1 - \mu} < \frac{2c + b + \frac{F}{1 - \mu}}{2 - \mu} < \frac{b + c}{1 - \mu} < c + b + \frac{F}{1 - \mu} \). Combining the comparisons for these thresholds yields:

- Suppose \( F < \mu (b + c) - b \). A ban on dual mode results in separation mode if \( \Delta \geq \frac{1}{2 - \mu} \left(2c + b + \frac{F}{1 - \mu}\right) \); and reseller mode if \( \Delta \leq \frac{1}{2 - \mu} \left(2c + b + \frac{F}{1 - \mu}\right) \).
- Suppose \( F \in [\mu (b + c) - b, \mu (b + c)] \). A ban on dual mode results in separation mode if \( \Delta \geq c + b + \frac{F}{1 - \mu} \); marketplace mode if \( \Delta \in \left[\frac{c}{1 - \mu}, c + b + \frac{F}{1 - \mu}\right] \); reseller mode if \( \Delta \leq \frac{c}{1 - \mu} \).
- Suppose \( F > \mu (b + c) \). A ban on dual mode results in marketplace mode if \( \Delta \geq \frac{1}{1 - \mu} \); reseller mode if \( \Delta \leq \frac{c}{1 - \mu} \).

Finally, we verify that the post-intervention outcome in the separation mode results in a higher consumer surplus than the post-intervention outcome when the separation mode is unavailable. We focus on \( F < \mu (b + c) \) and \( \Delta \geq \max \left\{ \frac{1}{2 - \mu} \left(2c + b + \frac{F}{1 - \mu}\right), c + b + \frac{F}{1 - \mu} \right\} \). If \( \Delta \geq \frac{c}{1 - \mu} \), so that the marketplace mode is chosen if the separation mode is unavailable, Proposition C.2 implies \( CS^{\text{sep}} = CS^{\text{market}} \) and \( CS^{\text{sep}}_{\text{regular}} > CS^{\text{market}}_{\text{regular}} \), so \( CS^{\text{sep}} > CS^{\text{market}} \). If \( \Delta \leq \frac{c}{1 - \mu} \), so that the reseller mode is chosen if the separation mode is unavailable, we note

\[
CS^{\text{sep}} = (v + b)(1 - \mu) + (v - c)\mu \\
= v - c + (1 - \mu)(b + c) \\
\geq v - c + (1 - \mu)^2 \Delta \\
\geq v - c + (1 - \mu)^2 \Delta - (1 - \mu)(1 - \eta)(b + c - \Delta) \\
= CS^{\text{re sell}},
\]

where \( \eta \) is the probability that regular consumers buy from \( M \) in the equilibrium.

## D Competition and endogenous market structure

If both intermediaries operate as pure resellers, then in equilibrium \( \Pi_1 = \Pi_2 = 0 \) by the logic for symmetric Bertrand competition between \( M_1 \) and \( M_2 \). If both intermediaries operate as pure marketplaces both intermediaries must compete their fees down to zero in order to attract \( S \), implying \( \Pi_1 = \Pi_2 = 0 \) in the overall equilibrium. If exactly one intermediary operates as pure reseller and the other intermediary operates one of the other two modes, then the analysis proceeds as in the separation mode in Section 5.1.

When the other intermediary operates as the dual mode, the only caveat is that in \( RE \) of Table C.1, we have \( p^*_r = 0 \) exactly. Then, the overall equilibrium is described by Proposition C.1. Therefore, there are only two remaining cases to consider: when at least one intermediary operates in the dual mode, while the other intermediary operates as either (1) the dual mode or (2) the marketplace mode.

### D.1 Both \( M_1 \) and \( M_2 \) operate in dual mode

Consider the stage 3 subgame. As in the baseline model, there are three broad types of equilibria:

- intermedation equilibria (all regular consumers purchase from \( S \) through one of the marketplaces);
• direct sales equilibria (all regular consumers purchase from S directly);
• reseller equilibria (at least one of the intermediaries make a positive amount of sales to regular consumers).

We assume, without loss of generality, \( \tau_1 \leq \tau_2 \), and whenever regular consumers are indifferent between S’s product offered in both marketplaces, they purchase through M_1. All other tie-breaking rules follow from the baseline model. We first solve the equilibrium of the stage 3 subgame, assuming that S participates on both marketplaces. Let \( p^*_m \) and \( p^*_o \) denote the product prices set by M_1 and M_2, and let \( p_1 \) and \( p_2 \) be the inside prices set by S when selling through M_1 and M_2.

**Lemma D.1 (Intermediation) In any intermediation equilibrium, \( p^*_m \geq p^*_o = 0 \). The equilibrium exists if and only if \( \tau_1 \leq \min \left\{ \Delta - c, \frac{b + c \mu}{1 - \mu} \right\} \). The equilibrium profits are \( \Pi_M = \tau_1 (1 - \mu) \), \( \Pi_M = 0 \), and \( \pi = \mu \Delta + (\Delta - c - \tau_1) (1 - \mu) \).

**Proof.** Suppose \( \tau_1 < \tau_2 \). S’s price must be such that all such purchases are made through M_1, as otherwise it can profitably raise its price at M_2, \( p_2 \), to divert consumers to M_1 where the margin is higher. This implies M_2’s profit is zero in any such equilibrium, so it necessarily has an incentive to deviate to attract regular consumers with its own product, as long as \( p^*_1 > \Delta \). This implies in equilibrium \( p^*_m = \Delta, p^*_o = c + \Delta \), and M_2 sets \( p^*_o = 0 \), and M_1’s price is indeterminate and can take any value \( p^*_m \geq 0 \). Clearly, M_1 and M_2 have no incentive to deviate. To ensure the stated price profile is indeed an equilibrium, it remains to check (i) S is not making losses inside \( (\Delta - c - \tau_1 \geq 0) \); and (ii) S has no incentive to set a low \( p_0 \) to attract regular consumer to the direct channel, which requires

\[
\Delta - b - c \leq \mu \Delta + (\Delta - c - \tau_1) \iff \tau_1 \leq \frac{b + c \mu}{1 - \mu}.
\]

Finally, there is no other intermediation equilibrium given we ruled out all equilibria involving weakly dominated strategies.

Suppose \( \tau_1 = \tau_2 = \tau \). Given the symmetry, S sets the same inside prices across the two marketplaces, so \( p^*_m = p^*_o = p^*_1 \). The equilibrium profits of M_1 and M_2 are, respectively, \( \tau \cdot (1 - \mu) \) and zero given the tie-breaking rule. This implies M_2’s profit is zero in any such equilibrium, so it necessarily has an incentive to deviate to attract regular consumers with its own product. Thus, the remaining steps follow immediately from the previous paragraph.

**Lemma D.2 (Direct sales)**

- If \( \Delta < \frac{b + c \mu}{1 - \mu} \), then there is no reseller equilibrium.
- If \( \Delta \geq \frac{b + c \mu}{1 - \mu} \), then any price profile satisfying \( p^*_m = p^*_o > \Delta, p^*_m = p^*_o = 0 \), \( \pi = \Delta - b \) is a direct sales equilibrium. Direct sales equilibria exist if and only if \( \tau_1 \geq \frac{b + c \mu}{1 - \mu} \). The equilibrium profits are \( \Pi_M = \Pi = 0 \) and \( \pi = \Delta - b - c \).

**Proof.** The proof of Lemma 2 applies.

**Lemma D.3 (Reseller)**

- If \( \Delta \leq \frac{b + c \mu}{1 - \mu} \) and \( \tau_1 \geq \Delta - c \), in the reseller equilibrium, \( p^*_m = p^*_o = 0 \), \( p^*_m = p^*_o = \Delta \), and \( p^*_o = c + \Delta \) is a reseller equilibrium. All regular consumers either buy from M_1 or M_2. The equilibrium profits are \( \Pi_M = \Pi = 0 \) and \( \pi = \mu \Delta \).
- If \( \Delta > \frac{b + c \mu}{1 - \mu} \) or \( \tau_1 < \Delta - c \), there is no reseller equilibrium.
Proof. For $\Delta \leq \frac{b+c}{1-\mu}$ and $\tau_1 \geq \Delta - c$, $M_1$ and $M_2$ clearly have no incentive to deviate. $S$’s equilibrium profit is $\Delta \mu$, while its deviation profit is either $\Delta - b - c$ (from setting a low outside price) or $\Delta \mu + (1 - \mu) (\Delta - c - \tau_1)$ (from setting a low inside price), both of which are lower than the equilibrium profit. If $\tau_1 < \Delta - c$ or $\Delta > \frac{b+c}{1-\mu}$, then at least one of these two deviation becomes strictly profitable for $S$, and the equilibrium above does not exist.

Combining these lemmas, for $\tau_1 \leq \tau_2$ the equilibria of the stage 3 subgame, conditional on $S$ participating on both marketplaces, can be summarized as:

- In intermediation equilibria (IE), $\Pi_{M_1} = \tau_1 (1 - \mu), \Pi_{M_2} = 0$, and $\pi = \mu \Delta + (\Delta - c - \tau_1) (1 - \mu)$. The equilibrium exists if and only if $\tau_1 \leq \min \{\Delta - c, \frac{b+c}{1-\mu}\}$.
- In direct sales equilibria (DE), $\Pi_{M_i} = \Pi_{M_1} = 0$, and $\pi = \Delta - b - c$. The equilibrium exists if and only if $\Delta \geq \frac{b+c}{1-\mu}$ and $\tau_1 \geq \frac{b+c}{1-\mu}$.
- In reseller equilibria (RE), $\Pi_{M_1} = \Pi_{M_2} = 0$, and $\pi = \mu \Delta$. The equilibrium exists if and only if $\Delta \leq \frac{b+c}{1-\mu}$ and $\tau_1 \geq \Delta - c$.

\[
\frac{\tau_1 \leq \Delta - c}{\Delta < \frac{b+c}{1-\mu}} \quad \text{IE} \quad \frac{\tau_1 > \Delta - c}{\Delta \geq \frac{b+c}{1-\mu}} \quad \text{RE} \quad \text{and} \quad \frac{\tau_1 \leq \frac{b+c}{1-\mu}}{\Delta < \frac{b+c}{1-\mu}} \quad \text{IE} \quad \frac{\tau_1 > \frac{b+c}{1-\mu}}{\Delta \geq \frac{b+c}{1-\mu}} \quad \text{DE} \quad \text(D.1)
\]

If instead $S$ participates only on one of the marketplaces (say, $M_1$), then the analysis proceeds as if $M_2$ is operating as a pure reseller. The existing results on the separation mode (Section 5.1) then apply. And the categorization in Table D.1 also applies but with slightly different equilibrium profits:

- In intermediation equilibria (IE), $\Pi_{M_1} = \tau_1 (1 - \mu), \Pi_{M_2} = 0$, and $\pi = \mu \Delta + (1 - \mu) (\Delta - c - \tau)$. The equilibrium exists if and only if $\tau_1 \leq \min \{\Delta - c, \frac{b+c}{1-\mu}\}$.
- In direct sales equilibria (DE), $\Pi_{M_i} = \Pi_{M_1} = 0$, and $\pi = \Delta - b - c$. The equilibrium exists if and only if $\Delta \geq \frac{b+c}{1-\mu}$ and $\tau_1 \geq \frac{b+c}{1-\mu}$.
- In reseller equilibria (RE), $\Pi_{M_1} = 0$, $\Pi_{M_2} = p^*_r (1 - \mu)$, and $\pi = \mu \Delta$, where $p^*_r \in [0, \min \{c - \Delta + \tau, c + b - (1 - \mu) \Delta\}]$.

The equilibrium exists if and only if $\Delta \leq \frac{b+c}{1-\mu}$ and $\tau_1 \geq \Delta - c$.

Finally, if $S$ does not participate at all, it competes with two pure resellers. As described in the main text, this results in both intermediaries setting $p^*_{m1} = p^*_{m2} = 0$, and $S$ either sets $p^*_o = c + \Delta$ and sells only to direct consumers ($\pi = \mu \Delta$), or sets $p^*_o = \Delta - b$ and sells to all consumers ($\pi = \Delta - b - c$).

Comparing these profits, and given that $S$ is free to join both marketplaces and that $S$ breaks ties in favor of participating, we conclude that in stage 2 $S$ participates on both platforms if multihoming is costless. We know that $M_2$’s equilibrium profit (in the pricing subgame) is zero as long as $\tau_1 \leq \tau_2$. Therefore, in stage 1, for each given level of $M_1$’s commission $\tau_1 > 0$, $M_2$ has an incentive to undercut by setting $\tau_2 < \min \{\tau_1, \min \{\Delta - c, \frac{b+c}{1-\mu}\}\}$ in order to induce an intermediation equilibrium with $S$ selling on $M_2$’s marketplace. A symmetric argument implies $M_1$ has the same incentive to undercut for any given level of $\tau_2 > 0$, so in equilibrium we have $\tau_1 = \tau_2 = 0$. There is no incentive to unilaterally deviate upward from this commission level because such a deviation does not affect the equilibrium of the continuation subgame.

Note if instead multihoming is costly, then $S$ participates only on the cheaper platform (say, $M_1$). There is no equilibrium with $\tau_1 > \min \{\Delta - c, \frac{b+c}{1-\mu}\}$ because $M_1$ earns zero profit in the resulting direct
sales or reseller equilibria. For $0 < \tau_1 \leq \min \left\{ \Delta - c, \frac{b + cw}{1 - \mu} \right\}$, $M_2$ has an incentive to undercut to avoid earning zero profit. Again, $M_1$ and $M_2$ compete in commissions to attract $S$’s participation, and in equilibrium we continue to have $\tau_1 = \tau_2 = 0$.

D.2 M1 operates in dual mode and M2 operates in marketplace mode

We first consider the case of $\tau_1 > \tau_2$. Suppose $S$ participates on both marketplaces. In any equilibrium in which $S$ is making sales through one of the marketplaces, $S$’s price must be such that all such purchases are made through $M_2$. There is no equilibrium with $M_1$ facilitating any sales and so the analysis proceeds as if $S$ is not available on $M_1$ (i.e. $M_1$ operates as a pure reseller). The existing results on the separation mode (Section 5.1) applies. Therefore, in stage 2, $S$ is weakly better off from participating on both marketplaces (if multihoming is costless) or participating only on $M_2$ (if multihoming is costly).

Suppose instead $\tau_1 \leq \tau_2$. In any equilibrium in which $S$ is making sales through one of the marketplaces, $S$’s price must be such that all such purchases are made through $M_1$. Therefore, the marketplace by $M_2$ is irrelevant to the analysis, and the pricing unfolds as in the baseline model in Section 3.3. Therefore, in stage 2, $S$ is weakly better off from participating on both marketplaces (if multihoming is costless) or participating only on $M_1$ (if multihoming is costly).

We can now consider stage 1. There is no equilibrium with $\tau_2 > \min \left\{ \Delta - c, \frac{b + cw}{1 - \mu} \right\}$ because at such fee levels $S$ never sells through $M_2$, regardless of $\tau_1$. For $0 < \tau_2 \leq \min \left\{ \Delta - c, \frac{b + cw}{1 - \mu} \right\}$, $M_1$ has an incentive to undercut to avoid earning zero profit because otherwise $S$ sells to all regular consumers through $M_2$. Therefore, in equilibrium we must have $\tau_2 = 0$ and $\tau_1 = 0$. There is no incentive to unilaterally deviate upward from this commission level because such a deviation does not affect the equilibrium of the continuation subgame.

D.3 Entry decisions

We are now ready to analyze the entry and mode choice decisions of the intermediaries. We can summarize both intermediaries’ profits for all possible combinations of modes in the following table, where the first and second entries in each box represent $M_1$ and $M_2$’s profit without entry cost.

<table>
<thead>
<tr>
<th></th>
<th>$M_2$ marketplace</th>
<th>$M_2$ reseller</th>
<th>$M_2$ dual</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_1$ marketplace</td>
<td>0, 0</td>
<td>$\tau^*(1 - \mu), 0$</td>
<td>0, 0</td>
</tr>
<tr>
<td>$M_1$ reseller</td>
<td>0, $\tau^*(1 - \mu)$</td>
<td>0, 0</td>
<td>0, $\tau^*(1 - \mu)$</td>
</tr>
<tr>
<td>$M_1$ dual</td>
<td>0, 0</td>
<td>$\tau^*(1 - \mu), 0$</td>
<td>0, 0</td>
</tr>
</tbody>
</table>

where $\tau^* = \min \left\{ \Delta - c, \frac{b + cw}{1 - \mu} \right\} > 0$. Recall that $M_2$ observes $M_1$’s mode choice before making its decisions. Therefore, $M_2$ does not have incentive to enter the market as long as $M_1$ is not operating as a pure reseller given entry cost $F > 0$. Anticipating this, $M_1$ enters the market and operates in dual mode, which is indeed the most profitable mode given that $M_2$ does not enter.

E Comparison with wholesaler-retailer model

E.1 Third-party products mode

Suppose that in stage 0 $M$ chooses the third-party products mode. In this case, whenever $M$ does not sell $S$’s product, the only alternative available is the fringe suppliers’ product, which is priced at $c$. For any given wholesale price $w$ set by $S$ in stage 1, there are two possible equilibria in the pricing subgame: (i) $M$ sells the fringe suppliers’ product in equilibrium; (ii) $M$ sells $S$’s product in equilibrium. In what follows, we denote $M$’s price for $S$’s product as $p_m$ and $M$’s price for the fringe product as $p_{m'}$.  

26
The first equilibrium exists only when \( w \geq c + \Delta \). To see this, suppose to the contrary that \( w < c + \Delta \), and consider any equilibrium in which \( M \) sells fringe suppliers’ product at some price \( p_m^f \). In this case, \( M \) earns a margin of \( p_m^f - c \), but it can profitably deviate to selling \( S \)'s product at \( p_m = p_m^f + \Delta \), resulting in the same volume of sales, but a strictly higher margin of \( p_m + \Delta - w \). The following lemma follows from Proposition 2:

**Lemma E.1** (Equilibrium with \( M \) selling the fringe suppliers’ product) Suppose \( w \geq c + \Delta \). In the pricing subgame:

- If \( \Delta > \frac{b}{1-\mu} \), in the equilibrium, \( p_o^* = c + \Delta - b \) and \( p_m^f = c \). All regular consumers purchase from \( S \) directly. Equilibrium profits are \( \Pi = 0 \) and \( \pi = \Delta - b \).
- If \( \Delta \leq \frac{b}{1-\mu} \), in the mixed-strategy equilibrium, \( p_m^f \) is distributed according to c.d.f \( F_m^f \) with support \([c + b - (1 - \mu) \Delta, c + b]\), where

\[
F_m^f(p_m^f) = \frac{1}{1 - \mu} \left( 1 - \frac{\mu \Delta}{p_m^f - b + \Delta - c} \right) \text{ for } p_m^f \in [c + b - (1 - \mu) \Delta, c + b];
\]

\( p_o^* \) is distributed according to c.d.f \( F_o \) with support \([c + \mu \Delta, c + \Delta]\), where

\[
F_o(p_o^*) = \begin{cases} 
1 - \frac{c + b - (1 - \mu) \Delta}{p_o^* + b - \Delta} & \text{for } p_o^* \in [c + \mu \Delta, c + \Delta) \\
1 & \text{for } p_o^* \geq c + \Delta 
\end{cases}
\]

Equilibrium profits are \( \Pi = (b - (1 - \mu) \Delta)(1 - \mu) \) and \( \pi = \mu \Delta \).

The second equilibrium involves \( M \) selling \( S \)'s product. First, this equilibrium exists only when \( w \leq c + \Delta \). Otherwise, suppose \( w > c + \Delta \) and \( M \) sells \( S \)'s product to a positive mass of consumers at some price \( p_m \). In this case, \( M \) earns a margin of \( p_m - w \), but it can profitably deviate to selling the fringe suppliers’ product at \( p_m^f = p_m - \Delta \) to make the same amount of sales but at a strictly higher margin \( p_m - \Delta - c \). Next, we show that in general the equilibrium in which \( M \) sells \( S \)'s product cannot exist in pure strategies.

**Lemma E.2** If \( w \neq c + \Delta \), then there is no pure-strategy equilibrium with \( M \) selling \( S \)'s product. If \( w = c + \Delta \), there is a pure-strategy equilibrium with \( M \) selling \( S \)'s product to all regular consumers at \( p_m = c + \Delta + b \) and \( S \) selling its product to all direct consumers at \( p_o = c + \Delta \). Equilibrium profits are \( \Pi = b(1 - \mu) \) and \( \pi = \Delta \).

**Proof.** Suppose such an equilibrium exists. Due to the convenience benefit \( b \), in any pure-strategy equilibrium \( M \) sells \( S \)'s product to all regular consumers, which then implies \( S \) must set \( p_o = c + \Delta \) to focus on selling to direct consumers. Given \( p_o \), \( M \) does best selling \( S \)'s product at \( p_m = c + \Delta + b \), which makes all regular consumers indifferent between buying \( S \)'s product from \( M \) and buying from \( S \) directly. In this equilibrium, \( S \)'s profit is \( \pi_{eqm} = \mu \Delta + (1 - \mu)(w - c) \) given that it earns from supplying \( M \), and \( M \)'s profit is \( \Pi_{eqm} = (c + b + \Delta - w)(1 - \mu) \). If \( w > c + \Delta \), then \( M \) can profitably deviate to selling the fringe suppliers’ product at \( p_m^f = c + b \) so as to sell to all regular consumers at a strictly higher margin \( b \). If \( w < c + \Delta \), then \( S \) can deviate by slightly lowering its outside price to undercut \( M \), attracting all consumers and earning \( \pi_{dev} = \Delta + (1 - \mu)(w - c) = \pi_{eqm} \). Finally, if \( w = c + \Delta \) and \( c = 0 \), neither \( M \) nor \( S \) have an incentive to deviate from the stated equilibrium. ■

When the pure-strategy equilibrium does not exist, we obtain the following mixed-strategy equilibrium.
Lemma E.3 (Equilibrium with M selling S’s product) Suppose \( w \leq c+\Delta \). There exists a mixed-strategy equilibrium in which \( p^*_m \) is distributed according to c.d.f \( F_m \), where

\[
F_m(p^*_m) = 1 - \frac{\mu(\Delta + c + b - p^*_m)}{(1-\mu)(p^*_m - b - w)} \text{ for } p^*_m \in [c + \mu\Delta + (w - c)(1-\mu) + b, c + b + \Delta];
\]

\( p^*_o \) is distributed according to c.d.f \( F_o \), where

\[
F_o(p^*_o) = \begin{cases} 
1 - \frac{(\Delta + c - w)\mu + b}{p^*_o - w + b} & \text{for } p^*_o \in [c + \mu\Delta + (w - c)(1-\mu), c + \Delta) \\
0 & \text{for } p^*_o \geq c + \Delta.
\end{cases}
\]

Equilibrium profits are \( \Pi = ((\Delta + c - w)\mu + b)(1-\mu) \) and \( \pi = \mu\Delta + (w - c)(1-\mu) \).

**Proof.** We verify the mixed strategy equilibrium stated in the proposition. The cdf \( F_m \) is such that \( S \) is indifferent between all \( p^*_m \in [c + \mu\Delta + (w - c)(1-\mu), c + \Delta] \). When \( p^*_o = c + \Delta \), \( S \) attracts only direct consumers and obtains profit \( \mu\Delta + (w - c)(1-\mu) \). Therefore, the indifference condition is

\[
(p^*_o - c)(\mu + (1-\mu)(1 - F_m(p^*_o + b))) + (w - c)(1-\mu)F_m(p^*_o + b) = \mu\Delta + (w - c)(1-\mu).
\]

Rearranging the above expression, we can get

\[
1 - F_m(p^*_o + b) = \frac{\mu(\Delta - p^*_o + c)}{(p^*_o - w)(1-\mu)}.
\]

or after the change of variables \( p^*_m = p^*_o + b \),

\[
F_m(p^*_m) = 1 - \frac{\mu(\Delta + c + b - p^*_m)}{(1-\mu)(p^*_m - b - w)}.
\]

Then \( F_m(c + \mu\Delta + (w - c)(1-\mu) + b) = 0 \) and \( F_m(c + b + \Delta) = 1 \), so the distribution is atomless.

Meanwhile, the cdf \( F_o \) is such that \( M \) is indifferent between all \( p^*_o \in [c + \mu\Delta + (w - c)(1-\mu) + b, c + b + \Delta] \). When \( p^*_m = c + \mu\Delta + (w - c)(1-\mu) + b \), \( M \) attracts regular consumers with probability one and obtains profit \( ((\Delta + c - w)\mu + b)(1-\mu) \). Therefore, the indifference condition is

\[
(p^*_m - w)(1-\mu)(1 - F_o(p^*_m - b)) = ((\Delta + c - w)\mu + b)(1-\mu).
\]

Using the change of variables \( p^*_m = p^*_o + b \), we obtain

\[
F_o(p^*_o) = 1 - \frac{(\Delta + c - w)\mu + b}{p^*_o - w + b}.
\]

Then, \( F_o(c + \mu\Delta + (w - c)(1-\mu)) = 0 \), and

\[
\lim_{p^*_o \to c + \Delta} F_o(p^*_o) = 1 - \frac{(\Delta + c - w)\mu + b}{\Delta + c - w + b} < 1,
\]

so \( F_o \) has an atom at \( p^*_o = c + \Delta \).

Finally, we check that neither player can profitably deviate from the stated mixed strategy equilibrium. For \( S \), any \( p_o < c + \mu\Delta + (w - c)(1-\mu) \), even if it attracts all consumers, earns strictly lower profits than that obtained from selling only to direct consumers \( (\mu\Delta + (w - c)(1-\mu)) \). And any \( p_o > c + \Delta \) attracts no consumer due to the existence of fringe sellers. A similar logic applies to rule out \( M \) deviating to any \( p^*_m \notin [c + \mu\Delta + (w - c)(1-\mu) + b, c + b + \Delta] \).

We now consider \( S \)'s wholesale price decision. For \( w \leq c + \Delta \), the equilibrium in the pricing subgame has \( M \) selling \( S \)'s product, so that \( \pi(w) = \mu\Delta + (w - c)(1-\mu) \). Within this region, \( S \) clearly does best
setting the highest wholesale price. For \( w > c + \Delta \), the equilibrium in the pricing subgame has \( M \) selling the fringe suppliers’ product, so that \( \pi(w) = \max \{\mu\Delta, \Delta - b\} \) is independent of \( w \). If \( w = c + \Delta \), both types of equilibrium exists, in which case we select the equilibrium that maximizes \( S \)’s profit. Notice that \( \mu\Delta + \Delta(1 - \mu) > \max \{\mu\Delta, \Delta - b\} \). Hence, we conclude \( S \) does best setting \( w = c + \Delta \) to induce the pure-strategy equilibrium in which \( M \) sells \( S \)’s product. To summarize,

**Proposition E.1** Suppose \( M \) chooses the third-party products mode. In the overall equilibrium: \( S \) sets the wholesale price \( w = c + \Delta \), \( M \) sells \( S \)’s product to all regular consumers at \( p_m = c + \Delta + b \), and \( S \) sells its product to all direct consumers at \( p_o = c + \Delta \). Equilibrium profits are \( \Pi^{\text{third-party}} = b(1 - \mu) \) and \( \pi^{\text{third-party}} = \Delta \).

### E.2 In-house products mode

Suppose that in stage 0 \( M \) chooses the in-house products mode. In what follows, we denote \( S \)’s direct price as \( p_o \) and the price for in-house brand as \( p_m^h \). In this case, the pricing subgame unfolds as if \( M \) operates as a pure reseller in Section 3.2. Therefore, the equilibrium is described by the following lemma, which follows from Proposition 2:

**Lemma E.4** *(Equilibrium with \( M \) selling in-house brand only)*

- If \( \Delta > \frac{b + c}{1 - \mu} \), there exists a pure-strategy equilibrium in which \( p_o^* = \Delta - b \) and \( p_m^h = 0 \). All regular consumers purchase from \( S \) directly. Equilibrium profits are \( \Pi^{\text{in-house}} = 0 \) and \( \pi^{\text{in-house}} = \Delta - b - c \).
- If \( \Delta \leq \frac{b + c}{1 - \mu} \), there exists a mixed-strategy equilibrium in which \( p_m^h \) is distributed according to c.d.f \( F_m^h \) with support \([c + b - (1 - \mu)\Delta, c + b]\), where

  \[
  F_m^h(p_m^h) = \frac{1}{1 - \mu} \left( 1 - \frac{\mu\Delta}{p_m^h - b + \Delta - c} \right) \text{ for } p_m^h \in [c + b - (1 - \mu)\Delta, c + b];
  \]

  \( p_o^* \) is distributed according to c.d.f \( F_o \) with support \([c + \mu\Delta, c + \Delta]\), where

  \[
  F_o(p_o^*) = \begin{cases} 
  1 - \frac{(c + b - (1 - \mu)\Delta)}{p_o^* - b + \Delta - \mu\Delta} & \text{for } p_o^* \in [c + \mu\Delta, c + \Delta) \\
  1 & \text{for } p_o^* \geq c + \Delta
  \end{cases}.
  \]

Equilibrium profits are \( \Pi^{\text{in-house}} = (c + b - (1 - \mu)\Delta)(1 - \mu) \) and \( \pi^{\text{in-house}} = \mu\Delta \).

### E.3 Dual products mode

Suppose that in stage 0 \( M \) chooses the dual products mode. For any given wholesale price \( w \) set by \( S \) at stage 1, there are two possible equilibria.

In the first equilibrium, \( M \) sells its in-house brand only, as in Section 3.2. Therefore, the equilibrium is the same as in Lemma E.4. This equilibrium exists only when \( w \geq \Delta \). To see this, suppose to the contrary that \( w < \Delta \), and consider any equilibrium in which \( M \) sells its in-house brand at some price \( p_m^h \). In this case, \( M \) earns a margin of \( p_m^h \), but it can profitably deviate to selling \( S \)’s product at \( p_m = p_m^h + \Delta \), resulting in the same volume of sales but a strictly higher margin \( p_m^h + \Delta - w \).

The second type of equilibrium involves \( M \) selling \( S \)’s product. This equilibrium only exists if \( w \leq \Delta \). To see why, suppose \( w > \Delta \) and \( M \) sells \( S \)’s product to a positive mass of consumers at some price \( p_m \), where \( p_m \) is drawn from some possibly degenerate distribution. In this case \( M \) earns a margin of \( p_m - w \), but it can profitably deviate to selling its in-house brand at \( p_m^h = p_m - \Delta \) to make the same amount of sales but at a strictly higher margin.
Lemma E.5 If \( w \neq \Delta \) or \( c > 0 \), then there is no pure-strategy equilibrium with \( M \) selling \( S \)'s product. If \( w = \Delta \) and \( c = 0 \), there is a pure-strategy equilibrium with \( M \) selling \( S \)'s product to all regular consumers at \( p_m = c + \Delta + b \) and \( S \) selling its product to all direct consumers at \( p_o = c + \Delta \).

Proof. Suppose such an equilibrium exists. Due to the convenience benefit \( b \), in any pure-strategy equilibrium \( M \) sells \( S \)'s product to all regular consumers, which then implies \( S \) must set \( p_o = c + \Delta \) to focus on selling to direct consumers. Given \( p_o \), \( M \) does best selling \( S \)'s product at \( p_m = c + \Delta + b \) which makes all regular consumers indifferent between buying \( S \)'s product from \( M \) and buying from \( S \) directly. In this equilibrium, \( S \)'s profit is \( \pi_{eqm} = \mu\Delta + (1 - \mu)(w - c) \) and \( M \)'s profit is \( \Pi_{eqm} = (c + b + \Delta - w)(1 - \mu) \). If \( w > \Delta \), then \( M \) can profitably deviate to selling its in-house brand at \( p_m^* = c + b \) to all regular consumers, obtaining a strictly higher margin. If \( w \leq \Delta \), then \( S \) can deviate by slightly lowering its outside price to undercut \( M \), attracting all consumers and earning \( \pi_{dev} = \Delta > \mu\Delta + (1 - \mu)(w - c) = \pi_{eqm} \), where the last inequality holds whenever \( w \neq \Delta \) or \( c > 0 \). Finally, if \( w = \Delta \) and \( c = 0 \), neither \( M \) nor \( S \) have an incentive to deviate from the stated equilibrium. \(\blacksquare\)

When the pure-strategy equilibrium does not exist, the mixed-strategy equilibrium with \( M \) selling \( S \)'s product in Lemma E.3 applies.

Lemma E.6 (Equilibrium with \( M \) selling \( S \)'s product) Suppose \( w \leq \Delta \).\(^{13}\) There exists a mixed-strategy equilibrium in which \( p_m^* \) is distributed according to c.d.f \( F_m \), where

\[
F_m(p_m^*) = 1 - \frac{\mu(\Delta + c + b - p_m^*)}{(1 - \mu)(p_m^* - b - w)} \quad \text{for} \quad p_m^* \in [c + \mu\Delta + (w - c)(1 - \mu) + b, c + b + \Delta];
\]

\( p_o^* \) is distributed according to c.d.f \( F_o \), where

\[
F_o(p_o^*) = \begin{cases} 
1 - \frac{(\Delta + c - w)(\mu + b)}{p_o^* - (w + b)} & \text{for} \quad p_o^* \in [c + \mu\Delta + (w - c)(1 - \mu), c + \Delta) \\
1 & \text{for} \quad p_o^* \geq c + \Delta 
\end{cases}
\]

Equilibrium profits are \( \Pi = ((\Delta + c - w)(\mu + b)(1 - \mu) \) and \( \pi = \mu\Delta + (w - c)(1 - \mu) \).

We now consider \( S \)'s wholesale price decision. For \( w < \Delta \), the equilibrium in the pricing subgame has \( M \) selling \( S \)'s product, so that \( \pi(w) = \mu\Delta + (w - c)(1 - \mu) \). Within this region, \( M \) clearly does best setting the highest wholesale price. For \( w > \Delta \), the equilibrium in the pricing subgame has \( M \) selling the in-house brand, so that \( \pi(w) = \max \{\mu\Delta, \Delta - b - c\} \) is independent of \( w \). If \( w = \Delta \), both types of equilibrium exist, in which case we select the equilibrium that maximizes \( S \)'s profit given that it can always adjust \( w \) by an infinitesimal amount to induce the equilibrium it prefers. Note that \( \mu\Delta + (\Delta - c)(1 - \mu) > \max \{\mu\Delta, \Delta - b - c\} \). Hence, we conclude \( S \) does best by setting \( w = \Delta \) to induce the equilibrium in which \( M \) sells \( S \)'s product. To summarize,

Proposition E.2 Suppose \( M \) chooses the dual products mode. In the overall equilibrium: \( S \) sets wholesale price \( w = \Delta \), the equilibrium of the pricing game is described by Lemma E.6, and equilibrium profits are \( \Pi^{dual} = (c\mu + b)(1 - \mu) \) and \( \pi^{dual} = \Delta - c(1 - \mu) \).

E.4 Choice of mode and banning the dual products mode

Comparing across the three modes, it is obvious that \( M \) does best choosing the dual-products mode because \( \Pi^{dual} = (b + c\mu)(1 - \mu) > \max \{\Pi^{third-party}, \Pi^{in-house}\} \). Whenever the dual-products mode

\(^{13}\)Notice that if \( c = 0 \) and \( w = \Delta \), the mixed-strategy equilibrium collapses to the pure-strategy equilibrium of Lemma E.5 (\( F_m \) collapses to a single point at \( p_m = b + \Delta \) and \( F_o \) has all its mass concentrated at \( p_o = \Delta \)).
is banned, we have $\Pi_{\text{third-party}} \geq \Pi_{\text{in-house}}$ if and only if $\Delta \geq \frac{c}{1 - \mu}$, where $\Pi_{\text{third-party}} = \Pi_{\text{in-house}}$ if $\Delta = \frac{c}{1 - \mu}$.

We now provide the proof of Proposition 15 in the main text.

**Proof. (Proposition 15).** The results on profits follow from direct inspections. Consider the surplus and welfare results. Suppose $\Delta \geq \frac{c}{1 - \mu}$. Let $\eta^{\text{dual}}$ denote the probability that regular consumers buy $S$’s product from $M$ in the mixed-strategy equilibrium of the dual mode. From Lemma E.6, if $w = \Delta$ then $F_o$ has a mass point at $p_o^* = c + \Delta$, with mass $\frac{\eta^{\text{dual}}}{c + b}$. Therefore, $\eta^{\text{dual}} \geq \frac{c\mu + b}{c + b}$. We first note $W^{\text{dual}} = v + \Delta + b(1 - \mu)\eta^{\text{dual}} - c < v + \Delta + b(1 - \mu) - c = W_{\text{third-party}}$ and $CS^{\text{dual}}_{\text{direct}} = CS^{\text{third-party}}_{\text{direct}} = (v - c)\mu$.

Meanwhile,

$$CS^{\text{dual}}_{\text{regular}} = W^{\text{dual}} - \Pi^{\text{dual}} - \pi^{\text{dual}} - CS^{\text{dual}}_{\text{direct}}$$

$$= v + \Delta + b(1 - \mu)\eta^{\text{dual}} - c - (b + c\mu)(1 - \mu) - \Delta + c(1 - \mu) - (v - c)\mu$$

$$= (v - c)(1 - \mu) - b(1 - \mu)(1 - \eta^{\text{dual}}) + c(1 - \mu)^2$$

$$\geq (v - c)(1 - \mu) - c(1 - \mu)^2 \frac{b}{b + c} + c(1 - \mu)^2$$

$$\geq (v - c)(1 - \mu) = CS^{\text{third-party}}_{\text{regular}},$$

where the inequalities used $\eta^{\text{dual}} \geq \frac{c\mu + b}{c + b}$ and $\frac{b}{b + c} \leq 1$. It follows that $CS^{\text{dual}} \geq CS^{\text{third-party}}$.

Next suppose $\Delta \leq \frac{c}{1 - \mu}$. Let $\eta^{\text{in-house}}$ denote the probability that regular consumers buy $S$’s product from $M$ in the mixed-strategy equilibrium of the in-house products mode. We have $W^{\text{in-house}} = v + \Delta + (b - \Delta)(1 - \mu)\eta^{\text{in-house}} - c$. Therefore, $W^{\text{in-house}} < W^{\text{dual}}$ if and only if $(b - \Delta)\eta^{\text{in-house}} < b\eta^{\text{dual}}$.

We know that

$$\eta^{\text{dual}} \geq \frac{c\mu + b}{c + b} \geq \frac{c\mu + b - c}{c + b} = \frac{b - c(1 - \mu)}{b} \geq \frac{b - \Delta}{b}.$$ 

Therefore, $b\eta^{\text{dual}} > (b - \Delta) \geq (b - \Delta)\eta^{\text{in-house}}$, as required. To show the results on consumer surplus, we note the following two preliminary claims:

**Claim 1:** $p_o^*$ is higher in the dual mode than in the in-house products mode, in the sense of first-order stochastic dominance.

To prove this claim, substitute $w = \Delta$ in dual mode to derive the distribution of $p_o^*$ as

$$F^{\text{dual}}_o(p_o^*) \begin{cases} 1 - \frac{c\mu + b}{p_o^* + \Delta + b} & \text{for } p_o^* \in [\Delta + c\mu, c + \Delta) \\ 1 & \text{for } p_o^* \geq c + \Delta \end{cases}$$

while the distribution of $p_o^*$ in the in-house products mode is

$$F^{\text{in-house}}_o(p_o^*) = \begin{cases} 1 - \frac{c\mu + b - (1 - \mu)\Delta}{p_o^* + \Delta - b} & \text{for } p_o^* \in [c + \mu \Delta, c + \Delta) \\ 1 & \text{for } p_o^* \geq c + \Delta \end{cases}.$$

For all $p \in [c + \mu \Delta, c + \mu]$, we have $F^{\text{in-house}}_o(p) \geq 0 = F^{\text{dual}}_o(p)$; for all $p_o^* \in [\Delta + c\mu, c + \Delta)$, we have $F^{\text{in-house}}_o(p) = 1 - \frac{c\mu + b - (1 - \mu)\Delta}{p_o^* + \Delta - b} > 1 - \frac{c\mu + b - (1 - \mu)c}{p_o^* + \Delta - b} = F^{\text{dual}}_o(p)$, given $\Delta > c$; for all $p \geq c + \Delta$, we have $F^{\text{in-house}}_o(p) = F^{\text{dual}}_o(p) = 1$. We conclude $F^{\text{in-house}}_o(p) \geq F^{\text{dual}}_o(p)$ for all $p$.

**Claim 2:** Define $\bar{p}_m = p_m^* - \Delta$. Then $\bar{p}_m$ in dual mode is higher than $\bar{p}_m^*$ in the in-house products mode, in the sense of first-order stochastic dominance.

To prove this claim, we substitute $\bar{p}_m = p_m^* - \Delta$ into the distribution function in Lemma E.6 to obtain

$$\bar{F}_m(\bar{p}_m) = \frac{1}{1 - \mu} \left( 1 - \frac{c\mu}{\bar{p}_m - b} \right) \text{ for } \bar{p}_m \in [c + b - (1 - \mu)c, c + b].$$

Compare this with

$$F^{\text{h}}_m(p_m^{\text{h}*}) = \frac{1}{1 - \mu} \left( 1 - \frac{\mu \Delta}{p_m^{\text{h}}* - b + \Delta - c} \right) \text{ for } p_m^{\text{h}}* \in [c + b - (1 - \mu)\Delta, c + b].$$

31
For all \( p \in [c + b - (1 - \mu) \Delta, c + b - (1 - \mu) c] \), we have \( F_m^h(p) \geq 0 = \bar{F}_m(p) \); for all \( p \in [c + b - (1 - \mu) c, c + b] \), we have \( F_m^h(p) \geq \bar{F}_m(p) \) if and only if

\[
1 - \frac{\mu \Delta}{p - b + \Delta - c} \geq 1 - \frac{c \mu}{p - b},
\]

which is equivalent to \( p \leq c + b \). We conclude \( F_m^h(p) \geq \bar{F}_m(p) \) for all \( p \).

To show \( CS^\text{dual} > CS^\text{in-house} \), it suffices to show that \( p^*_m \) is lower in the in-house products mode, which follows directly from Claim 1 above. To show \( CS^\text{dual} > CS^\text{in-house} \), it suffices to show that (i) \( p^*_m \) is lower in the in-house products mode, and (ii) the quality-adjusted inside price \( \bar{p}_m \equiv p^*_m - \Delta \) in dual mode is higher than \( p^*_m \) in the in-house products mode. Both (i) and (ii) follow from Claims 1 and 2 above. Given both groups of consumers are better off in the dual mode, we must have \( CS^\text{dual} > CS^\text{in-house} \).

### E.5 Model with \( M \) setting wholesale prices

In this section, we consider an alternative formulation of the wholesaler-retailer model from Section 5.3 by assuming that \( M \) dictates the wholesale price. We first solve for the overall equilibrium in each of the three modes. For any given wholesale price \( w \), the pricing subgame in each of the three modes is the same as in the model presented in Section 5.3.

In the third-party products mode, \( M \) optimally sets \( w \) such that \( S \) is indifferent between supplying and not supplying \( M \). If \( S \) does not supply \( M \), the subgame unfolds as if \( M \) exclusively sources from fringe suppliers, in which case \( S \)'s equilibrium profit is max \( \{\mu \Delta, \Delta - b\} \). If \( S \) supplies \( M \), its equilibrium profit is \( \pi = \mu \Delta + (w - c)(1 - \mu) \) by Lemma E.2-E.3. Therefore, \( S \) is indifferent when \( \mu \Delta + (w - c)(1 - \mu) = \max \{\mu \Delta, \Delta - b\} \), i.e when \( w = c + \max \{\Delta - \frac{b}{1 - \mu}, 0\} \). At this \( w \), the pricing equilibrium is in mixed-strategies (Lemma E.3). Equilibrium profits are then as follows:

- If \( \Delta \leq \frac{b}{1 - \mu} \), then \( w^{\text{third-party}} = c \), \( \Pi^{\text{third-party}} = (\Delta \mu + b) (1 - \mu) \), and \( \pi^{\text{third-party}} = \mu \Delta \)
- If \( \Delta > \frac{b}{1 - \mu} \), then \( w^{\text{third-party}} = c + \Delta - \frac{b}{1 - \mu} \), \( \Pi^{\text{third-party}} = b \), and \( \pi^{\text{third-party}} = \Delta - b \).

In the in-house products mode, \( M \) does not source any third-party products so wholesale prices are irrelevant. The equilibrium is given by Lemma E.4:

- If \( \Delta \leq \frac{b + c}{1 - \mu} \), then \( \Pi^{\text{in-house}} = (c + b - (1 - \mu) \Delta) (1 - \mu) \) and \( \pi^{\text{in-house}} = \mu \Delta \)
- If \( \Delta > \frac{b + c}{1 - \mu} \), then \( \Pi^{\text{in-house}} = 0 \) and \( \pi^{\text{in-house}} = \Delta - b - c \)

Finally, for the dual-products mode, the analysis is the same as for the third-party products mode, except that whenever \( S \) does not supply \( M \) the subgame unfolds as if \( M \) exclusively sells its in-house product. In this case \( S \)'s equilibrium profit is max \( \{\mu \Delta, \Delta - c - b\} \). This means \( M \) must set a lower wholesale price compared to the third-party products mode. Therefore \( S \) is indifferent when \( \mu \Delta + (w - c)(1 - \mu) = \max \{\mu \Delta, \Delta - c - b\} \), i.e when \( w = c + \max \{\Delta - \frac{b + c}{1 - \mu}, 0\} \). At this \( w \), the pricing equilibrium is in mixed-strategies (Lemma E.3). Equilibrium profits are as follows:

- If \( \Delta \leq \frac{b + c}{1 - \mu} \), then \( w^{\text{dual}} = c \), \( \Pi^{\text{dual}} = (\Delta \mu + b) (1 - \mu) \), and \( \pi^{\text{dual}} = \mu \Delta \)
- If \( \Delta > \frac{b + c}{1 - \mu} \), then \( w^{\text{dual}} = c + \Delta - \frac{b + c}{1 - \mu} \), \( \Pi^{\text{dual}} = b + c \mu \), and \( \pi^{\text{dual}} = \Delta - b - c \).

Proposition E.3 below shows the effect of banning the dual mode in this setup. We can again compare Proposition E.3 to the baseline model (Proposition 6). First, we have a different cutoff for switching modes because \( M \) sets both the wholesale price and the retail price in the third-party products mode, so that this mode behaves very differently compared to the marketplace mode. Second, whenever the ban on the dual products mode results in \( M \) choosing the in-house products mode, consumer surplus always
increases. This result is driven by the fact that both the outside and the inside prices are higher in the dual products mode than in the in-house products mode. The outside price is higher because in the dual products mode $S$ partially internalizes the revenue of $M$’s inside sales via its wholesale price, meaning that $S$ would be less aggressive in setting its outside prices. This in turns relaxes the inter-channel competition, allowing $M$, whose price is not constrained by within-channel competition, to charge a higher inside price than the inside price it charges in the in-house products mode.

Proposition E.3 (Ban on dual products mode in the wholesaler-retailer model)

- If $\Delta \geq \frac{c}{1-\mu} - \frac{\mu b}{(1-\mu)^2}$, a ban on the dual products mode results in $M$ choosing the third-party products mode. If $\Delta \leq \frac{b}{1-\mu}$, the ban does not affect the market outcome. If $\Delta > \frac{b}{1-\mu}$, then $\Pi$, $CS_{regular}$, $CS_{direct}$, and $CS$ decrease, $\pi$ increases, and the effect on $W$ is ambiguous.

- If $\Delta \leq \frac{c}{1-\mu} - \frac{\mu b}{(1-\mu)^2}$, a ban on the dual products mode results in $M$ choosing the in-house products mode, with $\Pi$ and $W$ decreasing, $CS_{regular}$, $CS_{direct}$ and $CS$ increasing, and $\pi$ not changing.

Proof. If $\Delta > \frac{b+c}{1-\mu}$, then $\Pi_{third-party} > \Pi_{in-house}$ obviously. If $\Delta \leq \frac{b}{1-\mu}$, then $\Pi_{third-party} = (\Delta + b)(1-\mu) > (c + b - (1-\mu)\Delta)(1-\mu) = \Pi_{in-house}$. If $\frac{b}{1-\mu} < \Delta \leq \frac{c}{1-\mu} - \frac{\mu b}{(1-\mu)^2}$, then

$$\Pi_{third-party} = b > (c + b - (1-\mu)\Delta)(1-\mu) = \Pi_{in-house}$$

if and only if $\Delta > \frac{c}{1-\mu} - \frac{\mu b}{(1-\mu)^2}$. Combining all cases, we obtain $\Pi_{third-party} > \Pi_{in-house}$ if $\Delta > \frac{c}{1-\mu} - \frac{\mu b}{(1-\mu)^2}$. Suppose instead $\Delta \leq \frac{c}{1-\mu} - \frac{\mu b}{(1-\mu)^2}$. This clearly implies $\Delta \leq \frac{b}{1-\mu}$, and, given $\Delta > c$, it also implies $\Delta > \frac{b+c}{1-\mu}$. Therefore, $\Delta \leq \frac{c}{1-\mu} - \frac{\mu b}{(1-\mu)^2}$ is sufficient for $\Pi_{third-party} \leq \Pi_{in-house}$, and we note equality holds only when $\Delta = \frac{c}{1-\mu} - \frac{\mu b}{(1-\mu)^2}$. 

Suppose $\Delta \geq \frac{c}{1-\mu} - \frac{\mu b}{(1-\mu)^2}$, so that $M$ switches to the third-party products mode after the ban. We know that the firms’ equilibrium strategies are the same in the third-party products mode and in the dual mode, except that the wholesale price is strictly higher in the third-party products mode when $\Delta > \frac{b}{1-\mu}$.

In what follows, we show that both the inside and the outside prices become higher when $\Delta > \frac{b}{1-\mu}$. To see this, note that we have $\frac{c}{1-\mu} - \frac{\mu b}{(1-\mu)^2} \geq c$. And $c < \frac{c}{1-\mu} - \frac{\mu b}{(1-\mu)^2} \iff c \geq \frac{b}{1-\mu}$, which implies $\Delta > c \geq \frac{b}{1-\mu}$.

\[\begin{align*}
\frac{d}{dw} \left( \frac{1 - (\Delta + c - w)\mu + b)}{p_0^* - w + b} \right) &= \frac{(p_0^* - \Delta - c)\mu - b(1-\mu)}{(p_0^* - w + b)^2} < 0,
\end{align*}\]

where the inequality is due to $p_0^* < c + \Delta$. As for the inside price, a direct inspection reveals $F_m(p_m^*)$ is decreasing in $w$. First note the distribution domain $[c + \mu \Delta + (w - c)(1-\mu), c + \Delta]$ shifts upwards when $w$ increases, and

$w^{\text{dual}} = c$. For welfare, $W^{in-house} = v + \Delta + (b - \Delta)(1-\mu)\eta^{in-house} - c$. Therefore, $W^{in-house} < W^{\text{dual}}$ if and only if $(b - \Delta)\eta^{in-house} < b\eta^{\text{dual}}$. We know

$$\eta^{\text{dual}} \geq \frac{c\mu + b}{c + b} \geq \frac{c\mu + b - c}{c + b - c} = \frac{b - c(1-\mu)}{b} \geq \frac{b - \Delta}{b}.$$  

Therefore, $b\eta^{\text{dual}} > (b - \Delta) \geq (b - \Delta)\eta^{in-house}$, as required. To show the results on consumer surplus, we use the following two preliminary claims:

Claim 1: $p_0^* is higher in the dual mode than in the in-house products mode, in the sense of first-order stochastic dominance.

\[\begin{align*}
\text{Claim 1: } p_0^* &\geq \frac{b}{1-\mu},
\end{align*}\]
To prove this, substitute $w = c$ in dual mode to derive the distribution of $p^*_o$ as

$$F^{dual}_o = \begin{cases} 1 - \frac{\Delta \mu + b}{p^*_o - c + b} & \text{for } p^*_o \in [c + \mu \Delta, c + \Delta) \\ 1 & \text{for } p^*_o \geq c + \Delta \end{cases},$$

We wish to compare it with

$$F^{in-house}_o (p^*_o) = \begin{cases} 1 - \frac{c + b - (1 - \mu) \Delta}{p^*_o + b - \Delta} & \text{for } p^*_o \in [c + \mu \Delta, c + \Delta) \\ 1 & \text{for } p^*_o \geq c + \Delta \end{cases}.$$

We want to show $F^{in-house}_o \geq F^{dual}_o$, or $1 - \frac{c + b - (1 - \mu) \Delta}{p^*_o + b - \Delta} \geq 1 - \frac{\Delta \mu + b}{p^*_o - c + b}$, which can be shown to be algebraically equivalent to $p^*_o \geq c + \Delta \mu$, which is indeed true given the domain.

**Claim 2:** Define $\tilde{p}_m \equiv p^*_m - \Delta$. Then $\tilde{p}_m$ in dual mode follows the same distribution as $p^*_h$ in the in-house products mode.

To prove this, we substitute $\tilde{p}_m = p^*_m - \Delta$ into the distribution function to get

$$F_m (\tilde{p}_m) = \frac{1}{1 - \mu} \left( 1 - \frac{\mu \Delta}{\tilde{p}_m + \Delta - b - c} \right) \text{ for } p^*_m \in [c + b - (1 - \mu) \Delta, c + b],$$

which is exactly the same as $F^h_m (p^*_m)$.