Should Amazon be allowed to sell on its own marketplace?*

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Abstract

A growing number of intermediaries (e.g. Amazon, Apple’s Appstore, and Walmart) act as resellers on their own marketplaces. We build a model of dual marketplace and reseller intermediation to explore the implications of this practice, and the call to ban it, taking into account an intermediary’s optimal choice of mode. Our analysis shows that an outright ban tends to benefit third-party sellers at the expense of consumer surplus or welfare, even after allowing for innovation by third-party sellers. Rather than an outright ban, we show that policies that limit the imitation of highly innovative third-party products and prevent steering of buyers to the intermediary’s own products would lead to preferable outcomes.

1 Introduction

An increasing number of e-commerce players such as Amazon, Target, and Walmart, are acting both as marketplaces, i.e. enabling third party sellers to sell to consumers, and as retailers, i.e. selling products under their own name. Other notable examples include Apple’s Appstore, Google’s Playstore, Window’s Apps, Intuit’s Quickbooks Apps, Salesforce’s AppExchange, and videogame consoles like Nintendo Switch. This practice has raised regulatory concerns over the lack of a level playing field, and led to investigations in Europe and the United States, with calls from various commentators and politicians for Amazon to be forced to separate its retail business from its marketplace. And in February 2019, India introduced new laws to force the separation of the two types of businesses, leading Amazon and the Walmart-backed Flipkart to change their business practices there.

In this paper we build a tractable model of an intermediary that can adopt a dual mode, in which it sells products in its own name (i.e. as a reseller) alongside third-party seller products (i.e. as a marketplace) to explore the welfare implications of this practice. Specifically, we use the model to study how the intermediary’s optimal choice of mode changes when the dual

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mode is outlawed, and derive the implications for total welfare and consumer surplus. We also conduct a similar analysis for several alternative policy options.

A number of antitrust concerns have been raised when a dominant platform sells products in the dual mode. These all center on the possibility that the platform may want to favor its own products and so distort competition in the marketplace, leading to unfair competition.\footnote{In March 2019, U.S. Senator Elizabeth Warren published a policy proposal for curbing the power of big technology firms, which included the following statement: “Many big tech companies own a marketplace, where buyers and sellers transact, while also participating on the marketplace. This can create a conflict of interest that undermines competition. Amazon crushes small companies by copying the goods they sell on the Amazon Marketplace and then selling its own branded version.” See https://medium.com/@teamwarren/heres-how-we-can-break-up-big-tech-9ad9e0da324c} This can happen in at least two important ways. One is that the platform obtains information on the third-party sellers’ products (e.g. demand and pricing data) via its marketplace, and then uses that opportunistically to decide whether to copy and compete on the more successful offerings, potentially leading to reduced incentives for third-party sellers to invest or innovate.\footnote{See Mattioli (2020) for reports that Amazon used data from its own sellers to launch competing products.} A second channel is that the platform can steer consumers towards its own offerings (or affiliated products) rather than those offered by third-party sellers by displaying its own offerings more prominently. In Amazon’s case this can arise through its Buybox, which around 85% of consumers click on to complete their order. This allocates a seller to the consumer according to a secret algorithm that Amazon controls, and oftentimes the allocated seller is Amazon itself.

To model these practices, our analysis features an intermediary $M$ that can function as a reseller and/or a marketplace, a fringe of small third-party sellers, and an innovative seller $S$ whose product is superior to all other products. We allow for the possibility that consumers can bypass the intermediary and purchase directly from third-party sellers (i.e. from the sellers’ own websites or through some alternative channel where the sellers’ products are available). Specifically, consumers have heterogenous preference for transaction channels: some consumers obtain a convenience benefit from purchasing through the intermediary and they complete transactions through whichever channel offers them the highest utility (we call them “regular consumers”), whereas others have an overriding preference to transact with sellers directly (we call them “direct consumers”). We explore three different business models for the intermediary: pure marketplace (facilitating transactions by third-party sellers who set their own prices for these transactions), pure reseller (sourcing its own products and selling them, competing with outside sellers), and dual (operating in both modes as a reseller on the intermediary’s marketplace).

Our first finding is that the dual mode is the most profitable for the intermediary. By hosting $S$ on the marketplace, the intermediary avoids head-to-head cross-channel competition with $S$’s superior product while at the same time extracting some surplus from $S$’s product through its transaction commission. The relaxation of head-to-head cross-channel competition from hosting $S$ comes from the fact $S$ has two price instruments (its marketplace price and its direct price), allowing it to price discriminate between regular consumers through its marketplace price and direct consumers through its direct price. Even though the pure marketplace mode also offers the benefit of avoiding head-to-head competition, in dual mode the presence of $M$’s product constrains $S$’s price on the marketplace, so that for any given commission level consumers are more likely to purchase through the marketplace. This margin squeeze on $S$ allows $M$ to raise welfare.
its commission in dual mode above the commission it optimally charges in the pure marketplace mode.

We then analyze the effect of a ban on the dual mode, taking into account that $M$ endogenously decides which of the two pure modes to switch to in response to the ban. If $S$’s product is sufficiently good, $M$ switches to operate as a pure marketplace. This results in higher profits for $S$, at the expense of $M$ and consumer surplus, while total welfare remains unchanged. This is because $S$ is no longer constrained by competition with $M$’s product, so can set a higher price in both channels to fully extract the additional surplus that its superior product offers to consumers.

If instead $S$’s product is not much better than $M$’s, a ban on the dual mode results in $M$ switching to operate as a pure reseller. This results in lower total welfare because the dual mode gives consumers the option to purchase $S$’s superior product through $M$’s more convenient channel, whereas this option is unavailable in reseller mode. However, the effect on consumers is, in general, ambiguous and reflects two opposing forces. On the one hand, in the pure reseller mode the head-to-head cross-channel competition for regular consumers means $S$ sets a low price in the direct channel, resulting in a benefit to direct consumers. On the other hand, regular consumers become worse-off because they can no longer enjoy the convenience benefit of buying $S$’s product via $M$.

We then use our framework to explore two practices that have raised antitrust scrutiny. In Section 4.1, we endogenize $S$’s innovation decision and allow $M$ to imitate $S$’s innovative product and sell it at a lower cost whenever the product is hosted on $M$’s marketplace. If innovation is not cost-efficient (in a sense we will make precise in Section 4.1), a ban on the dual mode results in $M$ choosing the reseller mode, with the same qualitative implications for consumer surplus and welfare as the baseline model. Otherwise the ban on the dual mode results in the marketplace mode being chosen by $M$, which increases the resulting level of innovation because in marketplace mode $S$ fully extracts its innovation surplus and hence has the highest incentive to innovate. However, the marketplace mode prohibits the superior product from being combined with $M$’s lower cost. We show that the improvement in innovation level in marketplace mode dominates the cost saving in dual mode (so that welfare increases with the ban) if and only if the innovation is sufficiently cost-efficient. Nonetheless, $S$’s ability to capture its innovation surplus in the marketplace mode means that the ban on the dual mode always results in lower consumer surplus.

In Section 4.2, we study how $M$ can steer consumers to its advantage when it facilitates product discovery. This involves modifying our baseline model by assuming regular consumers rely on $M$’s recommendation to discover $S$’s product (i.e. the innovative new product), and they are otherwise unaware of $S$ (including $S$’s direct channel). The ability of $M$ to steer consumers means it is able to charge a higher commission in dual mode, which in turn results in a higher price on the marketplace. We consider two possible policy remedies to address the harm arising from steering. First, we show that outright banning the dual mode turns out to be ineffective, as it results in $M$ choosing the reseller mode, lowering total welfare but without improving consumer surplus or $S$’s profit. Second, we show that requiring $M$ to recommend the product that offers the highest surplus for consumers can either result in $M$ choosing the reseller mode
or continuing in the dual mode. The latter is true whenever $S$’s product is sufficiently good, and, in this case, the policy leads to higher consumer surplus and profit for $S$, while keeping the welfare unchanged. This reflects that the policy of requiring the best deal be recommended specifically addresses the negative consequences of steering, while at the same time preserving the benefits of the dual mode captured in our baseline model. Thus, we find that banning steering can be a more effective remedy than banning the dual mode.

The rest of the paper proceeds as follows. In Section 1.1 we survey the related literature. We lay out the model in Section 2 and analyze it in Section 3, where we compare the three modes that the intermediary can choose and the implications of banning the dual mode. In Section 4 we examine the implications of banning the dual mode when there is product imitation or steering by the intermediary. Section 5 explores several extensions of our framework: enforcing some separation between the marketplace and the reseller business units rather than a complete ban on the dual mode, allowing for competing intermediaries that can endogenously choose the mode of their operations, and comparing the marketplace-reseller dual mode analyzed here to the more traditional case in which the intermediary is a retailer (like a supermarket) that can offer its own in-house brands alongside products sourced from third-party suppliers. Finally, in Section 6 we briefly conclude.

1.1 Related literature

A recent strand of literature has emerged that compares the platform business model with various alternative models: marketplace or reseller (Hagiu and Wright, 2015a), platform or vertically integrated firm (Hagiu and Wright, 2015b and 2018), agency or wholesale pricing (Johnson, 2017). In these papers, the key distinction between the business models is the delegation of control rights over key factors that are relevant for total demand, e.g. prices and marketing choices. This literature does not consider the possibility of the dual mode, in which a single firm operates a marketplace and offers its own product to compete with third-party sellers on its marketplace.

Somewhat closer is the literature that considers whether a platform should offer its own products or services (dual mode in our terminology). For example, Hagiu and Spulber (2013) consider a platform facing the chicken-and-egg coordination problem in user participation, showing that this problem can be mitigated by introducing first-party content alongside third-party content. Farrell and Katz (2000) and Jiang, Jerath, and Srinivasan (2011) analyze platform owners that face a tradeoff between extracting rents and motivating innovation by third-party complementors. Zhu and Liu (2018) empirically investigate this question, showing that Amazon is more likely to compete with its marketplace sellers in product categories that are more successful in terms of sales. A distinction relative to our paper is that this literature assumes that all products or services have to be sold through the platform and does not consider the surplus and welfare implications of the dual mode.

Our analysis of banning the dual mode when there is steering (Section 4.2) is quite related to the analysis by De Cornière and Taylor (2019), which considers a vertically-integrated intermediary that biases its recommendations in favor of its subsidiary seller at the expense of

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3Hagiu et al. (2020) consider the opposite situation of a traditional firm hosting rivals to become a platform.
third-party sellers. Divestiture (i.e. banning the dual mode) always results in both the intermediary and the seller coexisting and operating independently in their setup. Among several results, they show that divestiture can increase consumer surplus if sellers’ strategic instruments are such that an increase in firms’ markup decreases the net utility offered to consumers (e.g. pure price competition), but such biases can be harmful in the opposite case (e.g. pure quality competition). We focus on pure price competition but endogenize the intermediary’s post-ban choice of business model, showing that the intermediary may sometimes choose to become a pure marketplace or a pure reseller, so that banning the dual mode has an ambiguous effect on consumer surplus in general.

An important part of the mechanism driving our results rests on the ability of (regular) consumers to choose which channel to buy from, i.e. through the intermediary or directly. This relates to some recent work that explicitly models the direct purchase channel option, e.g. Edelman and Wright (2015), Wang and Wright (2020) and Ronayne and Taylor (2019) among others, but none of these papers consider an intermediary’s ability to operate in dual mode.

In our paper, a platform that operates in dual mode can be viewed as a vertically integrated firm that uses the upstream input (the facilitation of transactions through the marketplace) to offer downstream products (selling its product through the marketplace) that compete with other downstream sellers. The literature on vertical foreclosure has studied how the upstream market power leads to negative effects on downstream competition (e.g. Rey and Tirole, 2007). Our setting is different in several respects, perhaps most importantly in that the marketplace is not an essential facility in our setting, so third-party sellers can still sell to consumers directly.

2 Model setup

Suppose each consumer wants to buy one unit of one product where there is a continuum (measure one) of consumers. Transactions can be performed directly or through an intermediary $M$.

Each product category is supplied by $n \geq 2$ identical “fringe sellers”, and the products are each valued at $v$ by consumers. In addition, there is a superior seller $S$ that has innovated the product such that its product is valued at $v + \Delta > v$. The marginal costs of $S$ and fringe sellers are constant at $c \geq 0$. Depending on the mode of operation, the intermediary $M$ may be able to operate as a reseller and sell its own private label products to consumers. In the baseline version of the model we assume $M$ cannot copy $S$’s product, and like the fringe firms, its product is also valued at $v$. However, due to economies of scale $M$ has a cost advantage over sellers and has potentially a lower marginal cost, which we normalize to zero so that parameter $c$ captures the relative cost efficiency of $M$. By setting $c = 0$ we can handle the case that $M$ faces the same costs as other sellers.

There are two types of consumers. They differ in the benefit they get from using $M$ to complete transactions. A fraction $0 < \mu < 1$ of consumers dislike using $M$ to make transactions.

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4See also the first section of Calvano and Polo (forthcoming) for a comprehensive survey on the economic literature of biased intermediation by digital platforms.

5For a discussion of Amazon’s sourcing of its private label products, see: https://onezero.medium.com/amazon-finally-reveals-who-makes-its-branded-products-13e68913c770
Specifically, we assume these consumers face a sufficiently large inconvenience/transaction cost of using \( M \) for a transaction so using \( M \) is never a relevant consideration for them.\(^6\) We call them direct consumers: we have in mind that they always buy directly from the sellers (the sellers’ own websites) or from alternative channels in which the sellers’ products are available (e.g. other platforms or retail stores). The remaining fraction \( 1 - \mu \) of consumers enjoy a convenience benefit \( b \geq 0 \) of using \( M \) to make transactions, which captures the various transaction cost savings these consumers obtain from using \( M \) to complete their transaction. We call these regular consumers. In Section A of the Online Appendix we extend our benchmark analysis to allow for a continuum of consumer types that vary in their draws of \( b \). The existence of some direct consumers is key to ensuring that in dual mode \( M \) can sometimes extract more than its transaction benefit \( b \) in fees without causing all consumers to buy from \( S \). The basic idea is that because of direct consumers, \( S \) will sometimes prefer to exploit these consumers rather than trying to induce regular consumers to buy outside by lowering its price in the direct channel to compete with \( M \)’s own good. Without some direct consumers in our model, \( M \) would never choose to operate in the dual mode, and so the question of whether to ban the dual mode would not arise.

Consumers always have an outside option of not buying anything, which gives zero value. We assume \( v > c \) so consumers always buy something. We also assume \( \Delta > c \) so that \( S \)’s product innovation is a more important dimension than \( M \)’s cost efficiency, as otherwise in dual mode there can be no equilibrium where \( S \) makes any sales via \( M \), and the dual mode simply reduces to the reseller mode. Finally, we assume that one of the two parameters \( b \) and \( c \) is positive to exclude the uninteresting case where \( M \)’s profit is the same in all modes because \( M \) can never charge a positive price and attract any demand in any of the modes since it doesn’t offer any efficiency benefit.

Whenever \( M \)’s mode includes a marketplace, it charges a commission \( \tau \) to third-party sellers for each transaction facilitated. Third-party sellers (including \( S \)) can choose whether to participate on \( M \)’s marketplace, and whenever they do, can price discriminate between consumers that come to it through the marketplace and consumers that come to it through the direct channel. We posit that all third-party sellers always participate on \( M \)’s marketplace if they are indifferent. Given there are always two or more identical fringe sellers competing in the direct channel and on the marketplace, following the standard Bertrand logic, we take as given that fringe sellers always price at marginal cost, i.e. \( c \) if selling directly and \( c + \tau \) if selling on a marketplace, regardless of how \( S \) and \( M \) price. Thus, when we characterize equilibria, we take these fringe seller prices as given.\(^7\)

Throughout, we solve for subgame perfect Nash equilibria. To ensure equilibria are well defined, we assume that consumers break ties in favor of \( S \)’s product whenever they are indifferent between multiple products including \( S \)’s product, and then break ties in favor of \( M \)’s product whenever they are indifferent between that and the fringe sellers’ products. Note these

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\(^6\)A sufficient condition is that the transaction or inconvenience cost these consumers face from using \( M \) for a transaction exceeds \( c \) (in the baseline model) or \( \Delta \) (in the extended model of Section 4.1). Alternatively, it could be that these consumers are unaware of the existence of \( M \).

\(^7\)Thus, throughout the paper, we rule out equilibria supported by fringe suppliers pricing below cost, i.e. playing weakly dominated strategies.
tie-breaking rules do not apply when regular consumers are indifferent between S’s products sold in different channels, in which case we state the tie-breaking rule as part of the equilibrium construction.

Finally, whenever there are multiple equilibria in any subgame that are payoff equivalent for S, but payoff ranked by M, we select the one preferred by M, and similarly, whenever there are multiple equilibria in any subgame that are payoff equivalent for M, but payoff ranked by S, we select the one preferred by S.

3 Banning dual mode in the baseline model

In this section we specify the equilibrium outcomes under each mode, and then compare these outcomes to determine the effects of banning the dual mode in our baseline model.

3.1 Marketplace

Suppose the intermediary M acts as a pure marketplace and sets a commission $\tau \geq 0$. **Timing:** (1) $\tau$ is set; (2) S chooses whether to participate; (3) sellers set prices simultaneously, both inside (on M) and outside (when selling directly to consumers).

Suppose S has joined the marketplace. S’s inside price and outside price, denoted $p_i$ and $p_o$, are bounded above by fringe sellers’ price after adjusting for S’s superior quality product; i.e. $p_i \leq c + \tau + \Delta$ and $p_o \leq c + \Delta$. Clearly, given our tie breaking rule, S does best by setting its inside and outside prices so these constraints bind, implying $p_i^* = c + \Delta + \tau$, $p_o^* = c + \Delta$. Regardless of $\tau$, provided $\tau \leq v - c$, S earns profit $\Delta$ from participating. However, if M sets $\tau > b$ then all regular consumers would purchase from S directly. Therefore, the highest commission that M can set is constrained by the direct channel constraint, so $\tau \leq b$.

If S does not join the marketplace, it has to compete with fringe sellers listed on M as well as fringe sellers at the outside channel. As before, S’s outside price is bounded by $c + \Delta$, so M can never set $\tau$ above $b$, without causing all regular consumers to buy outside. Thus, regular consumers buy from S outside if and only if $p_o \leq c + \Delta + \tau - b$. Thus, in equilibrium S either sets $p_o^* = c + \Delta + \tau - b$ to sell to all consumers, or sets $p_o^* = c + \Delta$ and sells only to direct consumers. For both possibilities, S’s non-participation profit is weakly lower than $\Delta$, and so it always joins M.

Given the equilibrium in the pricing subgames characterized above, we have:

**Proposition 1 (Marketplace)** In the equilibrium, M sets $\tau^{market} = b$, S sets $p_i^* = c + \Delta + b$ and $p_o^* = c + \Delta$. All regular consumers purchase from S through the marketplace. The equilibrium profits of M and S are $\Pi^{market} = b (1 - \mu)$ and $\pi^{market} = \Delta$ respectively.

Note in particular that M obtains the same profit regardless of the level of $\Delta$. The reason is that S can fully extract the value of its innovation (inside and outside the platform). Meanwhile, M only extracts surplus up to the transaction benefit it offers.
3.2 Reseller

Suppose $M$ acts as a pure reseller and sets its price $p_m$. **Timing:** (1) $M$ and third-party sellers set prices simultaneously.

There are two possible types of equilibria in the reseller mode depending on whether $M$ makes any sales. First, if $\Delta$ is large, there is a unique pure-strategy equilibrium in which $S$ makes all the sales, with $p_o^* = \Delta - b$ and $p_m^* = 0$. Here $S$ sells to all consumers and earns $\pi^{\text{resell}} = \Delta - b - c$, while $M$ earns $\Pi^{\text{resell}} = 0$. Here, $S$ has no incentive to deviate to $p_o = c + \Delta$ to exploit the direct consumers, as long as $\Delta \geq \frac{b+c}{1-\mu}$. On the other hand, when $\Delta < \frac{b+c}{1-\mu}$, $S$ would have an incentive to deviate from this pure-strategy equilibrium by setting $p_o = c + \Delta$ to exploit direct consumers. In this case, given that the direct consumers are essentially “captive” to $S$, the usual logic of the Bertrand-Edgeworth cycle implies that there can be no pure-strategy equilibrium, and the equilibrium has to be in mixed-strategies in which $S$ exploits direct consumers. In this case, given that the direct consumers are essentially “captive” to $S$, the usual logic of the Bertrand-Edgeworth cycle implies that there can be no pure-strategy equilibrium, and the equilibrium has to be in mixed-strategies in which $M$ sometimes makes a positive amount of sales. In the special case of $\Delta = \frac{b+c}{1-\mu}$, these two types of equilibria co-exist and our selection rule implies the equilibrium with $M$ earning a positive profit is selected. Formally,

**Proposition 2 (Reseller)**

- If $\Delta > \frac{b+c}{1-\mu}$, in the equilibrium $p_o^* = \Delta - b$ and $p_m^* = 0$. All regular consumers purchase from $S$ directly. Equilibrium profits are $\Pi^{\text{resell}} = 0$ and $\pi^{\text{resell}} = \Delta - b - c$.

- If $\Delta \leq \frac{b+c}{1-\mu}$, in the mixed-strategy equilibrium, $p_m^*$ is distributed according to c.d.f $F_m$ with support $[c + b - (1 - \mu) \Delta, c + b]$, where

\[
F_m(p_m^*) = \frac{1}{1 - \mu} \left( 1 - \frac{\mu \Delta}{p_m^* - b + \Delta - c} \right) \quad \text{for} \quad p_m^* \in [c + b - (1 - \mu) \Delta, c + b]; \quad (1)
\]

$p_o^*$ is distributed according to c.d.f $F_o$ with support $[c + \mu \Delta, c + \Delta]$, where

\[
F_o(p_o^*) = \begin{cases} 
1 - \left( \frac{c + b - (1 - \mu) \Delta}{p_o^* - b + \Delta} \right) & \text{for} \quad p_o^* \in [c + \mu \Delta, c + \Delta) \\
1 & \text{for} \quad p_o^* \geq c + \Delta 
\end{cases}. \quad (2)
\]

Equilibrium profits are $\Pi^{\text{resell}} = (c + b - (1 - \mu) \Delta) (1 - \mu)$ and $\pi^{\text{resell}} = \mu \Delta$. This is the unique equilibrium when $\Delta < \frac{b+c}{1-\mu}$.

Here, $\Delta - \frac{b+c}{1-\mu}$ captures $S$’s incentive to dominate the market (i.e. selling to all consumers). For $\Delta > \frac{b+c}{1-\mu}$, the overall efficiency of $S$’s product is much higher than $M$’s and the mass of regular consumers is sufficiently large, so that the pricing equilibrium involves $S$ taking the whole market rather than exploiting direct consumers. All regular consumers buy from $S$, resulting in $M$ earning zero profit. For $\Delta \leq \frac{b+c}{1-\mu}$, either (i) the efficiency of $S$’s product is too small or (ii) the mass of regular consumers is too small, so that $S$ is unwilling to take the whole market with a low price, and so both $M$ and $S$ make a positive amount of sales to regular consumers in the mixed strategy equilibrium. Consider the extreme case in which $\mu \to 0$, so the condition for the mixed strategy equilibrium to prevail converges to $\Delta \leq b + c$. When $\mu \to 0$, the...
c.d.f. $F_m$ has all its mass concentrated at the lower bound $c + b - \Delta$. In this case, $M$’s product has a higher overall efficiency so it takes the whole market, and $S$’s profit converges to zero. At the other extreme, if $\mu \to 1$, then $S$ chooses to exploit direct consumers by pricing with a distribution that converges to the price $c + \Delta$, and $M$ competes to attract regular consumers by pricing with a distribution that converges to the price $c + b$, with $M$’s profit converging to zero.

### 3.3 Dual marketplace and reseller mode

Now suppose $M$ acts as both a marketplace and reseller, meaning regular consumers can choose between buying from $M$ (as a seller) and buying from third-party sellers who participate on $M$’s marketplace. **Timing:** (1) $\tau$ is set; (2) $S$ chooses whether to participate; (3) All sellers, $S$, and $M$ set prices simultaneously.

#### 3.3.1 Post-participation pricing subgame

We solve by backwards induction, considering first the stage-3 pricing subgame in which $S$ participates on $M$’s marketplace. There are three possible types of equilibria in this subgame depending on the purchase decisions of regular consumers in equilibrium:

- **Intermediation** equilibrium: all regular consumers buy from $S$ through the marketplace.
- **Direct sales** equilibrium: all regular consumers buy from $S$ directly.
- **Reseller** equilibrium: a positive fraction of regular consumers buy from the reseller $M$ and the equilibrium can be in mixed strategies.

First, consider the intermediation equilibrium in which regular consumers buy from $S$ through the marketplace. Then $S$’s equilibrium inside price satisfies $p^*_i \leq \tau + \Delta$, because otherwise $M$ could earn a higher margin than $\tau$ by undercutting $p^*_i$ and selling itself. Given that the marginal costs of $S$ and $M$ are $c + \tau$ and 0 respectively, in principle any inside-price profile $p^*_i \in [\max\{c + \tau, \Delta\}, \tau + \Delta]$ and $p^*_m = p^*_i - \Delta$, in which $S$ sells to all regular consumers, is a possible candidate equilibrium. Even though $M$’s marginal cost is zero, it does not necessarily want to undercut $S$ by setting $p^*_m < p^*_i - \Delta$ because $p^*_i \leq \tau + \Delta$ implies that undercutting leads to a smaller margin than the commission $\tau$ collected. For each candidate equilibrium it remains to check whether $S$ can profitably deviate to induce regular consumers to the outside channel. Naturally, such a deviation is not profitable as long as $\tau$ is not too high.

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8When $\mu$ is exactly zero, there is also a pure-strategy equilibrium: the standard asymmetric Bertrand outcome, in which $p^*_m = c + b - \Delta$ and $p^*_i = c$, along with the mixed-strategy equilibrium.

9Strictly speaking, there can be other equilibria involving $p^*_m < 0$. However, these equilibria involve $M$ playing weakly dominated strategies. We rule out such equilibria because they involve $M$ setting a price that it would prefer to change if some consumers actually purchased from it (i.e. off the equilibrium path).

10This does not mean $p^*_m < \tau$ is weakly dominated. This is because in cases where $\tau$ is high and $p^*_m$ is low so that consumers buy from $S$ directly, $M$ earns zero commission and it can profitably set a price below $\tau$ to attract regular consumers.
Lemma 1 (Dual mode, intermediation) Fix $\tau$ and define $\hat{p} = c + \Delta + \frac{b - \tau (1 - \mu)}{\mu}$. Any price profile satisfying $p_i^* \in [\max\{c + \tau, \Delta\}, \min\{\hat{p}, \tau + \Delta\}]$, $p_m^* = p_i^* - \Delta$, and $p_o^* = c + \Delta$ is an intermediation equilibrium. The intermediation equilibria exist if and only if $\tau \le b + \mu \min\{\frac{b + c}{1 - \mu}, \Delta\}$. The equilibrium profits are $\Pi = \tau (1 - \mu)$ and $\pi = \mu \Delta + (1 - \mu) (p_i^* - c - \tau)$.

Next consider the direct sales equilibrium. In such an equilibrium, $M$ earns zero profit given all sales occur directly. Given this, it will undercut and attract consumers back to the marketplace as long as $p_o^* > \Delta - b$. Consequently, such equilibria necessarily have $p_o^* = \Delta - b$, $p_m^* = 0$ and $p_i^* > \Delta$, and $S$ earns $\Delta - b - c$ in equilibrium. It remains to check whether $S$ has an incentive to deviate to $p_o = c + \Delta$ to fully exploit the direct consumers (earning profit $\mu \Delta$) instead of setting the low equilibrium price that attracts all consumers to the direct channel. Comparing $S$’s equilibrium profit $\Delta - b - c$ against the relevant deviation profit, such a deviation is profitable only if $\Delta$ or $\tau$ is small:

Lemma 2 (Dual mode, direct sales)

- If $\Delta \ge \frac{b + c}{1 - \mu}$ and $\tau \ge \frac{b + c}{1 - \mu}$, then any price profile satisfying $p_i^* > \Delta$, $p_m^* = 0$, and $p_o^* = \Delta - b$ is a direct sales equilibrium. The equilibrium profits are $\Pi = 0$ and $\pi = \Delta - b - c$.

- If $\Delta \le \frac{b + c}{1 - \mu}$ or $\tau < \frac{b + c}{1 - \mu}$, then there is no direct sales equilibrium.

The derivation of the reseller equilibrium follows the case in which $M$ operated as a pure reseller, but with two important differences: (i) if $M$ wins the inside competition and makes any sales, its price $p_m$ is capped by a tighter constraint $\min\{c + \tau - \Delta, c + b\}$; (ii) $M$ can sometimes let $S$ win the inside competition by setting $p_m = p_i - \Delta$ to earn commissions instead of making sales by itself, and this strategy is feasible as long as $\tau$ is not too high so that in equilibrium $S$ is still willing to make sales inside. However, if $\tau$ is sufficiently high, $S$’s inside product is irrelevant, and the outcome is the same as the reseller mode.

Lemma 3 (Dual mode, reseller)

- If $\Delta \ge \frac{b + c}{1 - \mu}$ or $\tau < b + \mu \Delta$, then there is no reseller equilibrium.

- If $\Delta \le \frac{b + c}{1 - \mu}$ and $\tau \in [b + \mu \Delta, b + \Delta)$, then in the mixed-strategy equilibrium, $p_i^* = c + \tau$; $p_m^*$ is distributed according to c.d.f $F_m$ with support $[c + b - (1 - \mu) \Delta, c + \tau - \Delta]$, where

$$F_m(p_m^*) = \begin{cases} \frac{1}{1 - \mu} \left( 1 - \frac{\mu \Delta}{p_m^* - b + \Delta - c} \right) & \text{for } p_m^* \in [c + b - (1 - \mu) \Delta, c + \tau - \Delta] \\ 1 & \text{for } p_m^* \ge c + \tau - \Delta \end{cases}$$

$p_o^*$ is distributed according to c.d.f $F_o$ with support $[\mu \Delta + c, c + \tau - b] \cup \{c + \Delta\}$, where

$$F_o(p_o^*) = \begin{cases} 1 - \frac{c + b - (1 - \mu) \Delta}{c + \mu \Delta + c + \tau - b} & \text{for } p_o^* \in [\mu \Delta + c, c + \tau - b] \\ 1 - \frac{c + b - (1 - \mu) \Delta}{\tau} & \text{for } p_o^* \in [c + \tau - b, c + \Delta] \\ 1 & \text{for } p_o^* \ge c + \Delta \end{cases}.$$
In equilibrium $S$ makes some inside sales. Regular consumers buy from $S$ directly when they are indifferent between $S$’s product in multiple channels. The equilibrium profits are $\Pi = (b + c - (1 - \mu) \Delta) (1 - \mu)$ and $\pi = \mu \Delta$.

- If $\Delta \leq \frac{b + c}{1 - \mu}$ and $\tau \geq b + \Delta$, then any price profile satisfying $p^*_i > c + b + \Delta$, with $p^*_o$ and $p^*_m$ being distributed according to c.d.f (1) and (2), is a mixed-strategy reseller equilibrium. In equilibrium $S$ makes no inside sales, and the equilibrium profits are $\Pi = (b + c - (1 - \mu) \Delta) (1 - \mu)$ and $\pi = \mu \Delta$.

To summarize the stage-3 equilibria in the subgame with $S$’s participation, in the table below we list down the relevant range of parameters and state the corresponding equilibria that exist:

- In the intermediation equilibria (IE), $\Pi = \tau (1 - \mu)$ and $\pi = \mu \Delta + (1 - \mu)(p^*_i - c - \tau)$, $p^*_i \in \left[\max\{c + \tau, \Delta\}, \min\{\hat{p}, \tau + \Delta\}\right]$, and
  $$\max_{p^*_i} \pi = \mu \Delta + (1 - \mu) \min\left\{\Delta + \frac{b - \tau}{\mu}, \Delta - c\right\}.$$

- In the direct sales equilibria (DE), $\Pi = 0$ and $\pi = \Delta - b - c$.

- In the reseller equilibria (RE), $\Pi = (b + c - (1 - \mu) \Delta) (1 - \mu)$ and $\pi = \mu \Delta$.

<table>
<thead>
<tr>
<th>$\tau$</th>
<th>$\Delta$</th>
<th>IE</th>
<th>IE,RE*</th>
<th>RE</th>
<th>DE,RE*</th>
</tr>
</thead>
<tbody>
<tr>
<td>$&lt; b + \mu \Delta$</td>
<td>$\leq \frac{b + c}{1 - \mu}$</td>
<td>IE</td>
<td>IE,RE*</td>
<td>RE</td>
<td>DE,RE*</td>
</tr>
<tr>
<td>$= b + \mu \Delta$</td>
<td>$\frac{b + c}{1 - \mu}$</td>
<td>IE</td>
<td>IE,DE,RE*</td>
<td>DE</td>
<td>RE*</td>
</tr>
<tr>
<td>$&gt; b + \mu \Delta$</td>
<td>$&gt; \frac{b + c}{1 - \mu}$</td>
<td>IE</td>
<td>IE,DE*</td>
<td>DE</td>
<td></td>
</tr>
</tbody>
</table>

(3) * $\pi$ is the same across all coexisting equilibria

Notice that whenever at least two out of the three types of equilibria coexist, $S$’s profit does not vary across co-existing equilibria. Therefore, based on our selection rule, we select the intermediation equilibrium whenever it is available, which is the best for $M$, followed by the reseller equilibrium and the direct sales equilibrium.

### 3.3.2 Overall equilibrium

If $S$ does not participate, the pricing subgame unfolds as if $M$ operates as a pure reseller, except that its price is bounded by $p^*_{o,m} \leq c + \min\{\tau, b\}$ due to the existence of fringe sellers both on the marketplace and outside. The equilibrium characterization in this case is largely similar to Proposition 2: if $\Delta$ is large, $S$ sells to all regular consumers; if $\Delta$ is small, $M$ sells to some regular consumers. In the proof of the next proposition, we show that in the equilibrium of the subgame following non-participation, $S$’s profit is $\pi_{np} = \max\{\mu \Delta, \Delta - b - c\}$. Compared
to the post-participation profit described by Table 3, it follows that S always weakly prefers to participate.

We can now derive the overall equilibrium (including M’s choice of \( \tau \) in stage 1):

**Proposition 3 (Dual mode equilibrium)** M sets \( \tau^{\text{dual}} = b + \mu \min \left\{ \frac{b+c}{1-\mu}, \Delta \right\} \) and S participates. In the resulting intermediation equilibrium:

- If \( \Delta > \frac{b+c}{1-\mu} \), the equilibrium prices are \( p^*_o = c + \Delta, p^*_i = \Delta, \) and \( p^*_m = 0 \).
- If \( \Delta \leq \frac{b+c}{1-\mu} \), the equilibrium prices are \( p^*_o = c + \Delta, p^*_i = c + \tau^{\text{dual}}, \) and \( p^*_m = c + \tau^{\text{dual}} - \Delta \).

All regular consumers buy from S on M and direct consumers buy directly. The equilibrium profits are \( \Pi^{\text{dual}} = \tau^{\text{dual}} (1-\mu) \) and \( \pi^{\text{dual}} = \max \{ \Delta - b - c, \mu \Delta \} \).

In this equilibrium, the competition with M’s own product on the marketplace effectively imposes a cap on S’s inside price. By combining this cap with a higher commission, M can squeeze S’s margin while still maintaining S’s incentive to sell through its marketplace.

### 3.4 Comparisons of the different modes

Before determining which mode M prefers, there is one other mode that we need to rule out first. Specifically, M could choose to own independent marketplace and reseller divisions, but with separate teams that are not allowed to communicate or coordinate with each other. The result would be equivalent to having separate competing marketplace and reseller intermediaries. As we show in Section 5.1, in this case, the separate reseller would earn zero profits and so has no incentive to keep operating. Indeed, taking into account any, arbitrarily small, fixed costs that the reseller would face to operate, we can rule out this outcome as part of any equilibrium. We return to this possibility in Section 5.1 to capture the scenario in which M, facing a ban on the dual mode, can however commit to operate the two separate divisions (for example, because this can sometimes allow it to extract more in its marketplace division, and it can use some of its profit from its marketplace division to cover the fixed costs of operating its reseller division).

We first compare the equilibrium profits of M and S across the three modes.

**Proposition 4 (Profits comparison)**

- M’s profit: \( \Pi^{\text{dual}} > \max \{ \Pi^{\text{market}}, \Pi^{\text{resell}} \} \). Moreover, \( \Pi^{\text{market}} \geq \Pi^{\text{resell}} \) if and only if \( \Delta \geq \frac{c}{1-\mu} \), with the inequality strict if \( b > 0 \) and \( \Delta > \frac{c}{1-\mu} \).
- S’s profit: \( \pi^{\text{market}} > \pi^{\text{dual}} = \pi^{\text{resell}} \).

Comparing first across the two pure modes, we see that the marketplace mode is better for M if (i) M’s cost efficiency c is weak relative to S’s innovation surplus \( \Delta \), and (ii) the mass of regular consumers is large so that S sets its price aggressively when competing against the reseller given it cannot earn very much profit from exploiting direct consumers. When these
conditions hold, $M$ is better off operating as a marketplace to avoid head-to-head competition with $S$.

Compared to the marketplace mode, the dual mode allows $M$ to charge a higher commission $\tau$ and still have $S$ sell through its marketplace to regular consumers (indeed, $\tau_{\text{dual}} > b = \tau_{\text{market}}$). To see the intuition for this result, note that in both the dual mode and the marketplace mode, $M$’s commission is constrained by the possibility that $S$’s outside price induces regular consumers to purchase from $S$ directly. However, for any given $\tau$, the inside price is lower in the dual mode than in the marketplace mode due to the competition with $M$’s own product, which means it is more difficult for $S$ to shift regular consumers from the marketplace to the direct channel. This relaxed constraint on $M$’s commission allows it to charge a higher commission in dual mode without consumers preferring to buy outside. Notably, this conclusion can still be valid even if $c = 0$ (i.e. when $M$ has no cost advantage in dual mode). And $\tau_{\text{dual}} > \tau_{\text{market}}$ immediately implies that $\Pi_{\text{dual}} > \Pi_{\text{market}}$.

The reason $\Pi_{\text{dual}} > \Pi_{\text{resell}}$ follows a similar logic. In principle, $M$ can always replicate the outcome of the reseller mode by setting a very high $\tau$ in the dual mode to deter $S$ from participating. However, in the dual mode the margin squeeze on $S$’s inside sales means $M$ uses its commission to extract some of $S$’s added value over $M$’s product, i.e. $\Delta - c$. Therefore, $M$ would actually have an incentive to keep $S$ on its marketplace. Reflecting this point, the gap in profit is $\Pi_{\text{dual}} - \Pi_{\text{resell}} = (1 - \mu) (\Delta - c)$, which is increasing in the added value of $S$’s product.

Comparing the three modes from $S$’s perspective, the marketplace remains the best for $S$ since it can fully extract the value of its innovation on both channels and does not face any direct competition from $M$. Meanwhile, the dual mode is better than the reseller mode for $S$, because when $M$ hosts $S$, it becomes less aggressive in its pricing (since it now collects commissions), which relaxes cross-channel competition.

Next consider consumer surplus and welfare, where the consumer effects for direct consumers and for regular consumers are distinguished given they can be affected in different ways:

**Proposition 5 (Consumer surplus and welfare).**

- **Welfare:** $W_{\text{dual}} = W_{\text{market}} \geq W_{\text{resell}}$, where the inequality is strict if $b > 0$.

- **Direct consumers:** $CS_{\text{resell}}_{\text{direct}} > CS_{\text{dual}}_{\text{direct}} = CS_{\text{market}}_{\text{direct}}$.

- **Regular consumers:** If $\Delta > \frac{b + c}{1 - \mu}$, $CS_{\text{dual}}_{\text{regular}} = CS_{\text{resell}}_{\text{regular}} > CS_{\text{market}}_{\text{regular}}$; if $\Delta \leq \frac{b + c}{1 - \mu}$, $CS_{\text{dual}}_{\text{regular}} > CS_{\text{resell}}_{\text{regular}} > CS_{\text{market}}_{\text{regular}}$.

- **Total consumer surplus:** If $\Delta > b + c$, $CS_{\text{resell}} > CS_{\text{dual}} > CS_{\text{market}}$; if $\Delta \leq b + c$, $CS_{\text{dual}} \geq CS_{\text{resell}} > CS_{\text{market}}$, where the weak inequality is strict if $\Delta < b + c$.

The welfare result is intuitive. In dual mode, given that $S$ participates and in equilibrium all regular consumers buy from $S$ through the marketplace, the total welfare generated is the same as in the marketplace mode, which is higher than the welfare generated in reseller mode. The equality $W_{\text{dual}} = W_{\text{market}}$ obtained here is an artefact of the assumption of discrete consumer types and it does not generally hold when we allow for continuous consumer types based on their
draw \( b \) of the convenience benefit (or cost) of using \( M \). The analysis of this extension is provided in Section A of the Online Appendix, where we show that the only welfare difference between the dual mode and the marketplace mode is due to a possible distortion arising from cross-channel price differences. To the extent that \( S \)'s price is lower in one channel than another, this will induce too many consumers to buy in the channel they do not prefer, potentially foregoing their convenience benefit (or cost) from using \( M \). Both modes potentially involve distortions. The marketplace involves the inside price being set higher than the outside price, while in the dual mode this price difference is smaller or even reverses. Provided that the price difference is not reversed in the dual mode, it results in a smaller distortion. Otherwise, the comparison in welfare is ambiguous.

Returning to our setting with discrete consumer types, we find that the marketplace mode is the worst overall for consumers. In this case, \( S \) fully extracts the value of its innovation (inside and outside the platform) and \( M \) extracts surplus up to the transaction benefit it offers. The other modes leave some of the efficiencies provided by the firms with consumers, and so provide greater consumer surplus. Even though the commission is generally higher in dual mode, the on-platform competition sufficiently suppresses \( S \)'s inside margin so that the resulting inside price is still lower than in the marketplace mode. Thus, \( M \) and consumers benefit from the margin squeeze conducted in the dual mode, at the expense of \( S \).

To compare consumer surplus in the reseller mode with the dual mode, note in the dual mode \( S \) can price discriminate between regular consumers and direct consumers by charging a low inside price and a high direct price. This implies that direct consumers become worse off while regular consumers become better off compared to the reseller mode, with total consumer surplus being higher in dual mode if \( \Delta \leq b + c \), but lower in dual mode if \( \Delta > b + c \).

### 3.4.1 Banning dual mode

A policy that bans the dual mode can result in two possible market structures, depending on whether \( M \) chooses to operate in the marketplace mode or the reseller mode. To analyze this, suppose there is a prior period (period zero) in which \( M \) chooses which mode to operate in. Using our results characterizing the equilibrium level of \( M \)'s profit for each different mode, we first compare \( \Pi_{\text{resell}} \) and \( \Pi_{\text{market}} \) to determine which mode \( M \) would choose as a result of the dual mode is banned. Then, we combine this choice of mode from Proposition 4 with the resulting surplus comparisons in Proposition 5 to assess the overall impact of a ban on the dual mode.\[^{11}\]

**Proposition 6** (Ban on dual mode)

- If \( \Delta \geq \frac{c}{1 - \mu} \), a ban on the dual mode results in \( M \) choosing the marketplace mode, with \( \Pi \), \( CS_{\text{regular}} \), and \( CS \) decreasing; \( \pi \) increasing; and \( CS_{\text{direct}} \) and \( W \) not changing.
- If \( \Delta \leq \frac{c}{1 - \mu} \), a ban on the dual mode results in \( M \) choosing the reseller mode, with \( \Pi \), \( CS_{\text{regular}} \), and \( W \) decreasing; \( CS_{\text{direct}} \) increasing, \( \pi \) not changing; and \( CS \) decreasing if \( \Delta < b + c \), not changing if \( \Delta = b + c \), and increasing if \( \Delta > b + c \).

\[^{11}\text{Note in Proposition 6, the boundary case } \Delta = \frac{c}{1 - \mu} \text{ covers both possibilities since } M \text{ is indifferent between the two modes, so both of the outcomes listed in the proposition are possible.}\]
Proposition 6 shows that in this benchmark setting, a ban on dual mode results in (weakly) lower welfare, and in most cases, lower consumer surplus. Welfare will be strictly lower whenever \( M \) switches to the reseller mode after the ban, reflecting the loss in convenience benefits for regular consumers who prefer to buy \( S \) through \( M \). If the proportion of direct consumers \( \mu \) is small then \( \Delta \leq \frac{c}{1-\mu} \) implies \( \Delta < b + c \), in which case Proposition 6 implies that a ban on dual mode always lowers total consumer surplus. The only way in which banning the dual mode increases total consumer surplus is when there is a sufficient proportion of direct consumers, and \( S \) has an intermediate level of efficiency advantage \( \Delta \in \left[ b + c, \frac{c}{1-\mu} \right] \). Intuitively, in the reseller mode, the head-to-head competition between \( M \) and \( S \) means some of the surplus from \( \Delta \) is left with consumers, but at the same time regular consumers are unable to enjoy the convenience benefits from \( M \) facilitating transactions. Whenever \( \Delta > b + c \), the gain in surplus from \( \Delta \) dominates, and so the reseller mode would lead to a higher total consumer surplus. However, if \( \Delta \) is too high or \( \mu \) is too low, consumers do not capture this benefit since \( M \) prefers to operate as a marketplace. Reflecting this logic, the parameter range for which the ban improves consumer surplus becomes larger when \( b \) is low or \( \mu \) is high, i.e. when the convenience benefit offered by the platform is relatively unimportant.

4 Practices under scrutiny

In this section we explore the two major practices arising from Amazon’s use of dual mode that policymakers have been scrutinizing—Amazon imitating third-party products and Amazon steering consumers to its own products.

4.1 Innovation and product imitation

Suppose that the innovation \( \Delta \) is chosen by \( S \) instead of being exogenously fixed. In this case, \( M \)’s mode of operation affects \( S \)’s innovation decision. Let \( K(\Delta) \) be the innovation cost for arbitrary innovation level \( \Delta \geq 0 \), where \( K(\cdot) \) is increasing, continuously differentiable, and strictly convex. We define \( \Delta^H \) and \( \Delta^L \) such that \( K'(\Delta^H) = 1 \) and \( K'(\Delta^L) = \mu \), and assume \( \Delta^L > c \) so that \( \Delta^H > \Delta^L > c \). These particular innovation levels play a key role in characterizing the equilibrium outcome. We also assume \( \mu \Delta^L \geq K(\Delta^L) \) to ensure \( S \) is profitable in the equilibrium below. The timing for each pure mode remains the same as the baseline model except that \( S \) chooses \( \Delta \) at the same time as it decides whether to participate in case \( M \) operates as a marketplace, and before \( M \) sets its price in case \( M \) operates as a reseller.

In the marketplace mode, \( S \)’s profit function in the innovation stage is \( \bar{\pi}_{market}(\Delta) = \Delta - K(\Delta) \), which is independent of \( \tau \). Given that \( S \) extracts the entire innovation surplus, it optimally chooses \( \Delta_{market} = \Delta^H \). In the reseller mode, by Proposition 2, we have

\[
\bar{\pi}_{resell}(\Delta) = \begin{cases} 
\mu \Delta - K(\Delta) & \text{for } \Delta \leq \frac{b+c}{1-\mu} \\
\Delta - b - c - K(\Delta) & \text{for } \Delta > \frac{b+c}{1-\mu} 
\end{cases}.
\]  

(4)

\( \bar{\pi}_{resell} \) is continuous but it may not be single-peaked depending on other parameters, and the
profit-maximizing $\Delta$ can be characterized as:\(^{12}\)

**Lemma 4 (Innovation level in the reseller mode).**
Denote

$$
\bar{\Delta} = \frac{\Delta^H - \mu \Delta^L - (K(\Delta^H) - K(\Delta^L))}{1 - \mu} \in (\Delta^L, \Delta^H).
$$

- Suppose $\bar{\Delta} > \frac{b+c}{1-\mu}$: in equilibrium $\Delta^{resell} = \Delta^H$. The equilibrium profits are $\Pi^{resell} = 0$ and $\pi^{resell} = \Delta^H - b - c - K(\Delta^H)$.

- Suppose $\bar{\Delta} \leq \frac{b+c}{1-\mu}$: in equilibrium $\Delta^{resell} = \Delta^L$. The equilibrium profits are $\Pi^{resell} = (1-\mu) (c + b - (1-\mu) \Delta^L)$ and $\pi^{resell} = \mu \Delta^L - K(\Delta^L)$.

Here, the condition $\bar{\Delta} > \frac{b+c}{1-\mu}$ is obtained from comparing the maximized interior value of the two expressions in (4), i.e. $\bar{\Delta} > \frac{b+c}{1-\mu}$ if and only if $\mu \Delta^L - K(\Delta^L) < \Delta^H - b - c - K(\Delta^H)$. Hence, analogous to the baseline model, $\bar{\Delta} - \frac{b+c}{1-\mu}$ measures $S$’s incentive to take the whole market rather than exploit the direct consumers. However, here we need to take into account the endogeneity of $S$’s innovation level. Notice $\bar{\Delta} > \frac{b+c}{1-\mu}$ is more likely if (i) $b + c$ is small; (ii) $\mu$ is small; or (iii) $K(\Delta^H) - K(\Delta^L)$ is small, all of which means it is more profitable for $S$ to set $\Delta^H$ and sell to all consumers, relative to setting $\Delta^L$ and focusing on selling to direct consumers. Conversely, if $\bar{\Delta} > \frac{b+c}{1-\mu}$ then $S$ is better off setting $\Delta^H$ and dominating the market.

Consider the dual mode. We assume whenever $S$ is available on the platform, $M$ can imitate $S$’s superior product and thereby also offer consumers the additional surplus $\Delta$. This reflects that by hosting the third-party, $M$ can accurately monitor sales and observe which seller is being valued more highly by consumers, and copy the relevant product features from that seller.\(^{13}\) $M$ cannot imitate $S$’s product if $S$ does not participate. The modified timing under dual mode is: (1) $M$ sets and announces $\tau$ whenever applicable; (2) $S$ chooses $\Delta$ and whether to participate; (3) $M$ chooses whether to imitate $S$’s product if $S$ participates, and then the pricing subgame unfolds as in the baseline model. Note this timing implicitly assumes that $M$ is unable to commit to not imitate.

We first solve for the pricing in stage 3 assuming product imitation occurs. In this case, $M$’s product is exactly the same as $S$, and recall that $M$’s marginal cost is zero, which is lower than $S$’s cost $c$. It follows that there can be no intermediation equilibrium, because in any such equilibrium $M$ necessarily has an incentive to undercut below $S$’s effective marginal cost for intermediated sales, i.e. $c + \tau$. The only possible equilibrium in the subgame is a reseller equilibrium in which $M$ makes all the inside sales, with $\Pi = (c + \min\{b + \mu \Delta, \tau\}) (1-\mu)$ and $\pi = \mu \Delta$. Comparing this equilibrium with the equilibria in Lemmas 1 - 3, it follows that $M$ always wants to imitate $S$’s product at the beginning of stage 3.

In the presence of product imitation, $S$ does not always find it profitable to join the marketplace. Specifically, for each given $\Delta$, if $\Delta > \frac{b+c}{1-\mu}$ then $S$’s non-participation profit is $\Delta - b - c$.

\(^{12}\)Without loss of generality, we assume that $S$ chooses the highest $\Delta$ whenever it is indifferent between multiple choices of $\Delta$.

\(^{13}\)In Section B of the Online Appendix, we allow for imperfect imitation (i.e. letting the value of $M$’s product after imitation be between $v$ and $v + \Delta$) and show that this does not change the main insights obtained in this section.
which is higher than its participation profit and so it does not participate (we denote this outcome as NP). If instead \( \Delta \leq \frac{b+c}{1-\mu} \), S’s non-participation profit is \( \Delta \mu \), which is the same as its participation profit with imitation. Based on our selection rule, we select the equilibrium in which S breaks the tie in favor of participating. This reflects that M is better off as a result of S participating, and therefore could always offer a small transfer to S to ensure it participates. Combining both cases, S’s expected profit in stage 2 is

\[
\hat{\pi}^{\text{dual}}(\Delta) = \begin{cases} 
\mu \Delta - K(\Delta) \ (\text{RE}) & \text{for } \Delta \leq \frac{b+c}{1-\mu} \\
\Delta - b - c - K(\Delta) \ (\text{NP}) & \text{for } \Delta > \frac{b+c}{1-\mu}.
\end{cases}
\]

This function is the same as equation (4) so that S’s optimal \( \Delta \) can be characterized similarly to Lemma 4. Then, the overall equilibrium in dual mode is:

**Proposition 7** (Dual mode equilibrium with product imitation) M sets \( \tau^{\text{dual}} = b + \mu \Delta^L \).

- If \( \hat{\Delta} > \frac{b+c}{1-\mu} \), S sets \( \Delta = \Delta^H \) and does not participate in stage 2. In stage 3, \( p^*_o = \Delta^H - b \) and \( p^*_m = 0 \), all regular consumers buy from S directly, while \( \Pi^{\text{dual}} = 0 \) and \( \pi^{\text{dual}} = \Delta^H - b - c - K(\Delta^H) \).

- If \( \hat{\Delta} \leq \frac{b+c}{1-\mu} \), S sets \( \Delta = \Delta^L \) and participates in stage 2. In stage 3, \( p^*_o = c + \Delta^L \) and \( p^*_m = p^*_i = c + \tau^{\text{dual}} \), all regular consumers buy from M, while \( \Pi^{\text{dual}} = (1-\mu) \left( b + c + \mu \Delta^L \right) \) and \( \pi^{\text{dual}} = \mu \Delta^L - K(\Delta^L) \).

Notably, if \( \hat{\Delta} > \frac{b+c}{1-\mu} \), the possibility of product imitation deters S from participating in the marketplace in case M operates in dual mode. Instead, S sets a high innovation level such that all regular consumers end up buying from it directly, resulting in M earning zero profit in the dual mode. This reflects M’s inability to commit to not imitate S’s product.\(^{14}\) In this case, M prefers the marketplace mode. On the other hand, if \( \hat{\Delta} \leq \frac{b+c}{1-\mu} \), S is willing to participate under dual mode, and M prefers the dual mode over the other two modes.

Banning the dual mode in the presence of product imitation has the following implications:\(^{15}\)

**Proposition 8** (Ban on dual mode with product imitation) Define the relative welfare improvement from innovation as

\[
\Psi \equiv \Delta^H - K(\Delta^H) - (\Delta^L - K(\Delta^L)).
\]

- **If** \( \Delta > \frac{b+c}{1-\mu} \), a ban on the dual mode has no effect.
- **If** \( \Delta \leq \frac{b+c}{1-\mu} \) and \( \Delta^L \geq \frac{c}{1-\mu} \), a ban on the dual mode results in M choosing the marketplace mode, with \( \Pi \), \( CS_{\text{regular}} \), and \( CS \) decreasing; \( \pi \) and \( \Delta \) increasing; \( CS_{\text{direct}} \) not changing; and \( W \) decreasing if \( \Psi < c(1-\mu) \), not changing if \( \Psi = c(1-\mu) \), and increasing if \( \Psi > c(1-\mu) \).

\(^{14}\) A similar observation is made in Muthers and Wismer (2012), in which the intermediary may benefit from committing not to compete with third-party sellers because doing so leads to more participation and ex-ante investment by sellers.

\(^{15}\) In Section B of the Online Appendix, we also consider the effect of banning dual mode when M cannot imitate S’s product. We find qualitatively the same results as in Proposition 6, except that the choice of the pure mode after the ban will generally be different because the cutoff between the two pure modes is different.
\[
\text{If } \bar{\Delta} \leq \frac{b+c}{1-\mu} \text{ and } \Delta^L \leq \frac{c}{1-\mu}, \text{ a ban on the dual mode results in } M \text{ choosing the reseller mode, with } \Pi, \text{ CS}_{\text{regular}}, \text{ and } W \text{ increasing; CS}_{\text{direct}} \text{ decreasing; } \pi \text{ and } \Delta \text{ not changing; and } CS \text{ decreasing if } \Delta^L < b+c, \text{ not changing if } \Delta^L = b+c, \text{ and increasing if } \Delta^L > b+c.
\]

Surprisingly, even though we allow for the possibility of \( M \) to freely imitate \( S \)'s superior product in dual mode, and we take into account the effect of this through \( S \)'s choice on how much to innovate, a ban on dual mode is not necessarily good for consumers or welfare. Specifically, note that if \( \bar{\Delta} > \frac{b+c}{1-\mu} \), \( M \) always prefers the marketplace mode (since in dual mode, \( S \) would not participate) and hence the ban has no effect. If \( \bar{\Delta} \leq \frac{b+c}{1-\mu} \) and \( \Delta^L \leq \frac{c}{1-\mu} \), the ban results in \( M \) choosing the reseller mode, with qualitative implications that are the same as the second part of Proposition 6, in which welfare is strictly lower.

The interesting case occurs when \( \bar{\Delta} \leq \frac{b+c}{1-\mu} \) and \( \Delta^L \geq \frac{c}{1-\mu} \), whereby the ban results in \( M \) choosing the marketplace mode, with qualitative implications similar to the first part of Proposition 6 except that welfare can increase after the ban. This reflects that \( S \) has less incentive to innovate in dual mode, given \( M \)'s imitation results in \( S \) not always selling to all regular consumers. The exact welfare loss from reduced innovation is \(-\Psi\). At the same time, imitation allows \( S \)'s innovation to be disseminated, so that in equilibrium regular consumers purchase the superior product of \( M \) that has the added advantage of being more cost-efficient, the welfare improvement of which is \( c(1-\mu) \). Therefore, when \( \Psi < c(1-\mu) \), welfare decreases following a ban on the dual mode, and vice-versa.

A related question concerns the effect of banning imitation while still allowing \( M \) to operate in dual mode.

**Proposition 9 (Ban on product imitation):**

- If \( \bar{\Delta} > \frac{b+c}{1-\mu} \), a ban on imitation results in \( M \) switching from the marketplace to the dual mode (i.e. the ban makes dual mode viable), with \( \Pi, \text{ CS}_{\text{regular}}, \) and \( \text{CS increasing; } \pi \text{ decreasing; and } \text{CS}_{\text{direct}}, \Delta, \) and \( W \) not changing.

- If \( \bar{\Delta} \leq \frac{b+c}{1-\mu} \), a ban on imitation results in \( M \) continuing the dual mode, with \( \Delta, \text{ CS}_{\text{regular}}, \) and \( \text{CS increasing; } \pi \) and \( \text{CS}_{\text{direct}} \) not changing; \( W \) decreasing if \( \Psi < c(1-\mu) \), not changing if \( \Psi = c(1-\mu) \), and increasing if \( \Psi > c(1-\mu) \); and \( \Pi \) decreasing if \( \Psi < c\left(\frac{\mu-1}{\mu}\right) \), not changing if \( \Psi = c\left(\frac{\mu-1}{\mu}\right) \), and increasing if \( \Psi > c\left(\frac{\mu-1}{\mu}\right) \).

The proposition has an interesting implication. Recall that we can loosely interpret \( \bar{\Delta} \) and \( \Psi \) as measures of the value of innovation (both measures increase if the difference \( \Delta^H - \Delta^L \) is high relative to the cost difference \( K(\Delta^H) - K(\Delta^L) \)). From the proposition, when \( \bar{\Delta} > \frac{b+c}{1-\mu} \) or \( \Psi > c\left(\frac{\mu-1}{\mu}\right) \) (i.e. when innovation is valuable), \( M \) benefits from a ban on imitation because doing so ensures \( S \) participates and sets a high innovation level, which \( M \) can partially extract through its commission. In contrast, if \( \bar{\Delta} \leq \frac{b+c}{1-\mu} \) and \( \Psi \leq c\left(\frac{\mu-1}{\mu}\right) \), \( M \) does not benefit much from ensuring \( S \)'s high innovation. Instead, it is better off directly imitating \( S \)'s (low-innovation) product and making all the sales by itself (given that \( S \) participates). Thus, \( M \) would want to support a ban on imitation that only rules out imitating products involving sufficiently large
innovations. In practice, $M$ could try to implement such a policy by building up a reputation for only copying products based on relatively minor innovations.

Comparing Propositions 8 and 9, we note that banning imitation always results in $M$ operating in dual mode while banning dual mode outright results in $M$ switching to either the marketplace mode or the reseller mode. This comparison implies $W$ and $CS_{\text{regular}}$ are weakly higher and $CS_{\text{direct}}$ is weakly lower when imitation is banned relative to the outcome when the dual mode is banned, suggesting banning imitation under dual mode may be better than banning dual mode altogether. We provide some further thoughts on this comparison in the concluding section.

4.2 Product recommendations and steering

So far, we have assumed that consumers are aware of all available products in the market and then select their preferred purchase channel. However, in practice, many consumers rely on marketplaces like Amazon to find out about the existence of new products. The marketplace, in turn, can manipulate what consumers are able to discover on the marketplace through its recommendation algorithm (i.e. to steer consumers).

In this section, we modify our assumptions about regular consumers from the baseline specification so as to study the issue of platform steering. We assume that regular consumers (mass $1 - \mu$) rely on $M$’s recommendation to find out about products. Specifically, after all prices are set, $M$ selects a product (or none) to recommend to regular consumers. In addition to the fringe products in the direct channel, which can be interpreted as their de facto outside option, regular consumers only know about the product that is recommended by $M$. Therefore, if fringe products or $M$’s own product (in the reseller mode or dual mode) is recommended, regular consumers only choose between the recommended product on $M$ and the fringe products in the direct channel. If instead $S$’s product is recommended, then regular consumers also become aware of its existence in the direct channel and its associated price. They are free to choose which channel to buy $S$’s product from (and still enjoy transaction benefit $b$ if they buy it through $M$), or they can buy the fringe product directly. Meanwhile, direct consumers (mass $\mu$) behave the same as in the baseline model, i.e. they know about all products available in the direct channel and only buy directly.

Consider $M$’s profit-maximizing recommendation decision in the marketplace mode after prices are set. Since the transaction commission $\tau$ is uniform, $M$ simply recommends whichever inside product results in transactions on the marketplace (i.e. is preferred by regular consumers over products in the direct channel). Formally, this means it (i) recommends $S$ if $\Delta + b - p_i \geq \max \{\Delta - p_o, -c\}$ and $b - c - \tau < -c$; and (ii) recommends the fringe product if $\Delta + b - p_i < \max \{\Delta - p_o, -c\}$ and $b - c - \tau \geq -c$. In case (i), $M$ is strictly better off recommending $S$ because otherwise consumers would buy the fringe product directly, while in case (ii), $M$ is strictly better off recommending the fringe product because otherwise consumers would buy

16Alternatively, we can assume that regular consumers are also unaware of fringe suppliers’ products if these products are unavailable on $M$. As will be clear below, this change does not affect the analysis of the marketplace mode or the dual mode, but it would mean that in the reseller mode $M$ can extract the entire regular consumer surplus. This simply shifts the comparisons below in favor of (against) pure reseller mode in terms of $M$’s profit (consumer surplus), without affecting the overall insights.
order to avoid paying the commission. Here, $M$'s commission level is set so high, in the pricing subgame $S$ was capped by the so-called “showrooming constraint” (Wang and Wright, 2020): when $\tau$ is too high, in the pricing subgame $S$ sets prices to attract consumers to its direct channel in order to avoid paying the commission. In such a case (which also arises below in the dual mode), we assume that $M$ breaks the tie in favor of recommending the product that provides the highest surplus to consumers.

In the Appendix we show this recommendation rule implies the equilibrium characterization of the marketplace mode is the same as in the baseline model, i.e. Proposition 1 still holds. In particular, in equilibrium we have $\tau^{market} = b$, $p_i^* = c + \Delta + b$, $p_o^* = c + \Delta$, $\Pi^{market} = b(1 - \mu)$, and $\pi^{market} = \Delta$.

For the reseller mode, $M$ necessarily recommends its own product to regular consumers. Since these consumers compare only $M$’s product with the fringe sellers’ direct products, $M$ can charge up to $p_m^* = c + b$. Likewise, $S$ sets its outside price as high as possible to extract surplus from direct consumers, subject to the competitive constraint imposed by fringe sellers, so $p_o^* = c + \Delta$. The equilibrium profits of $M$ and $S$ are $\Pi^{resell} = (b + c)(1 - \mu)$ and $\pi^{resell} = \mu \Delta$ respectively.

In the dual mode, for each given $\{p_i, p_o, p_m, \tau\}$, $M$ recommends $S$ if (i) $S$’s inside product is preferred by regular consumers over the products available in the direct channel (formally, $\Delta + b - p_i \geq \max \{\Delta + p_m, -c\}$; and (ii) the commission is higher than the margin $M$ could earn from selling by itself (formally, $p_m \leq \tau$ or $b - p_m \leq \max \{\Delta + p_o, -c\}$). Otherwise, $M$ is better off recommending its own product or one of the fringe suppliers’ product. Then, the equilibrium in the dual mode is characterized as follows:

**Proposition 10 (Dual mode with steering)** In the pricing subgame:

- If $\tau < b + c$, then $p_m^* = b + c$, $p_o^* = c + \Delta$, $p_i^*$ can be any value, and $M$ recommends its own product to all regular consumers. Equilibrium profits are $\Pi = (c + b)(1 - \mu)$ and $\pi = \mu \Delta$.

- If $\tau \in [b + c, b + \Delta]$, then $p_i^* = b + c + \Delta$, $p_o^* = c + \Delta$, $p_m^*$ can be any value, and $M$ recommends $S$ to all regular consumers. Equilibrium profits are $\Pi = \tau(1 - \mu)$ and $\pi = \mu \Delta + (1 - \mu)(\Delta + b - \tau)$.

- If $\tau > b + \Delta$, then $p_m^* = b + c$, $p_o^* = c + \Delta$, $p_i^* > b + c + \Delta$, and $M$ recommends its product to all regular consumers. Equilibrium profits are $\Pi = (b + c)(1 - \mu)$ and $\pi = \mu \Delta$.

In the overall equilibrium, $M$ sets $\tau^{dual} = b + \Delta$. The equilibrium profits are $\Pi^{dual} = (b + \Delta)(1 - \mu)$ and $\pi^{dual} = \mu \Delta$.

When regular consumers rely on $M$’s recommendation, in the dual mode $M$ sets a high commission that extracts all the innovation surplus and convenience benefit. Recall that in the baseline model where regular consumers know about all products, $M$’s commission level was capped by the so-called “showrooming constraint” (Wang and Wright, 2020): when $\tau$ is too high, in the pricing subgame $S$ sets prices to attract consumers to its direct channel in order to avoid paying the commission. Here, $M$ can steer consumers, so $S$ knows if it were to
set such prices then \( M \) would recommend its own product instead of \( S \)’s. The threat of not being recommended means \( S \) does not attempt to attract consumers to the direct channel even when the commission \( \tau \) is high. Even though in the overall equilibrium \( M \) still recommends \( S \), the possibility of biased product recommendations (off-equilibrium) removes the showooming constraint on \( M \)’s commission. Moreover, given \( M \) bases its recommendation on the revenue generated rather than what is best for consumers, price competition in the marketplace is weak, leading to a high inside price \( p^*_i = b + c + \Delta \).

There are two distinct approaches to address the negative implications of steering in the dual mode. Consider the direct structural approach of banning the dual mode. We first note that \( M \) always prefers the reseller mode post-ban, i.e. \( \Pi^{resell} \geq \Pi^{market} \), where the inequality is strict if \( c > 0 \). This is because when consumers rely on \( M \) to find out about products, in reseller mode \( M \) can avoid competition with \( S \), which recall was the main reason \( M \) could prefer the marketplace mode over the reseller mode in the baseline model. Then, comparing the market outcomes of reseller mode and dual mode, we obtain:

**Proposition 11** (Ban on dual mode with steering) A ban on the dual mode results in \( M \) choosing the reseller mode, with \( \Pi \) and \( W \) decreasing; \( \pi \), \( CS \), \( CS_{regular} \), and \( CS_{direct} \) remaining unchanged.

Instead of banning the dual mode, another possible intervention is to require \( M \) to provide objective and unbiased recommendations, i.e. always recommend the product that provides the highest surplus to consumers, so as to rule out steering:

**Proposition 12** (Objective recommendations)

- If \( \min \left\{ \frac{(b+c)\mu}{1-\mu}, \mu \Delta \right\} < c \), requiring objective recommendations results in \( M \) choosing the reseller mode, with \( \Pi \) and \( W \) decreasing; \( \pi \), \( CS \), \( CS_{regular} \), and \( CS_{direct} \) remaining unchanged.

- If \( \min \left\{ \frac{(b+c)\mu}{1-\mu}, \mu \Delta \right\} \geq c \), requiring objective recommendations results in \( M \) continuing to choose the dual mode, with \( \Pi \) decreasing; \( \pi \), \( CS \), and \( CS_{regular} \) increasing; \( W \) and \( CS_{direct} \) remaining unchanged.

When steering is banned, \( M \) prefers operating as a pure reseller if \( \min \left\{ \frac{(b+c)\mu}{1-\mu}, \mu \Delta \right\} < c \). The condition is more likely to be satisfied if \( c \) is large (provided \( \mu < 1/2 \)), or if \( \Delta \), \( b \), or \( \mu \) are small. This reflects that in dual mode \( M \) effectively promotes \( S \) to regular consumers who were previously unaware of \( S \)’s existence. Since the threat of steering is absent, \( S \) can potentially set a low outside price to induce these consumers to purchase through the outside channel. This showooming constraint caps \( M \)’s ability to extract innovation surplus through its commission. Hence, it prefers to directly profit from its own efficiency advantage (selling as a pure reseller) when \( c \) is large. If instead \( \min \left\{ \frac{(b+c)\mu}{1-\mu}, \mu \Delta \right\} \geq c \), then \( M \) continues to operate in dual mode even after steering is banned. In this case the intervention eliminates the negative consequences of steering (high commission and high inside prices), while at the same time consumers, \( S \), and
welfare still benefit from having $S$ selling on the marketplace (i.e. the benefits of the dual mode discussed in the baseline model).

Comparing Propositions 8 and 12, we note the two interventions lead to different outcomes when $\min\left\{\frac{(b+c)\mu}{1-p}, \mu\Delta\right\} \geq c$. In particular, $CS_{\text{regular}}$, $CS$, $\Pi$, $\pi$, and $W$ are all weakly higher when steering is banned, compared to the case of an outright ban on the dual mode. Hence, requiring objective recommendations may be a more targeted remedy in addressing biased recommendations.

5 Extensions

We consider several different extensions of the baseline model to explore how the effects of banning the dual mode change. All derivations and proofs of propositions are relegated to the Online Appendix.

5.1 Commitment to separate divisions

A less drastic regulatory alternative to fully banning the dual mode would be to require that $M$ runs the marketplace and the reseller as independent divisions if it wants to continue adopting the dual mode. These divisions would involve separate teams that are not allowed to communicate or coordinate with each other.\(^{17}\) In this setting, the dual mode means having separate, competing marketplace and reseller intermediaries, except that $M$ owns both and considers their joint profits. Thus, although in equilibrium it will turn out that the reseller division does not make any profits, $M$ may still want to commit to operate it (and cover its fixed costs) if that allows $M$ to extract larger profits from its marketplace division.

To make things clear, we call the new dual mode under which $M$ runs a marketplace and a reseller as separate divisions as the ”separation mode”. In the separation mode, we label the marketplace as $M_0$ and the reseller as $R$. All other assumptions remain the same as in the baseline model: $R$ has a cost of zero, purchases from $R$ or sellers selling through $M_0$ provide convenience benefits $b$ to regular consumers, and direct consumers never purchase from $R$ or through $M_0$.

Timing: (1) $M_0$ sets $\tau$; (2) sellers (including $S$) simultaneously choose whether to participate; (3) $S$, $R$, and fringe sellers set prices simultaneously. Here, there is no reason for $R$ to sell on $M_0$ given it offers the same benefit directly, has the same underlying cost of zero, and competes for the same regular consumers, with the only difference being that selling on $M_0$ involves an additional cost of $\tau$. Thus, without loss of generality, we can assume $R$ does not participate on $M_0$.

In Section C of the Online Appendix, we derive the overall equilibrium of the separation mode as: $M_0$ sets $\tau_{sep} = \min\left\{\Delta - c, \frac{b+c\mu}{1-p}\right\}$ and $S$ participates. In the equilibrium, $S$ sets outside and inside prices at $p^*_o = c + \Delta$ and $p^*_r = \Delta$, while $R$ sets its price at $p^*_r = 0$. All regular consumers buy from $S$ on $M_0$, and all direct consumers buy directly from $S$. Equilibrium profits are $\Pi^*_{M_0} = \tau_{sep} (1 - \mu)$, $\Pi^*_{R} = 0$, and $\pi^*_{sep} = \max\{\mu\Delta, \Delta - c - b\}$.

\(^{17}\)This treatment is analogous to “legal unbundling” commonly observed in network industries and other markets with regulated access (see Höffer and Kranz, 2011; Cremer, Crémer and De Donder, 2006).
Comparing $M$’s profit in the dual mode and the separation mode, we find $\Pi^{\text{dual}} \geq \Pi^{\text{sep}}_M + \Pi^{\text{sep}}_R$, with strict inequality if $\Delta < \frac{b + c}{1 - \mu}$. On this range of $\Delta$, we have $\tau^{\text{sep}} = \Delta - c < b + \mu \Delta = \tau^{\text{dual}}$, i.e. the marketplace collects a lower commission in separation mode. To see why, recall that in dual mode, when $\tau > \Delta - c$, we have $p^*_i = c + \tau$ and $p^*_m = c + \tau - \Delta > 0$, and this is an equilibrium because $M$ has no incentive to undercut further given that it is earning its commission. However, in separation mode, $p^*_i = c + \tau$ and $p^*_r = c + \tau - \Delta$ does not constitute an equilibrium because $R$ does not internalize the revenue from the commission and hence it does want to undercut. The competition with $R$ implies a stronger “margin squeeze” on $S$’s inside price, relative to the squeeze in dual mode. Consequently, the marketplace cannot set its commission above $\Delta - c$ because it needs to take into account that $S$ may make a negative margin from inside sales. The lower commission in separation mode reflects the inability of $M_0$ and $R$ to internalize each other’s profit (as compared to the dual mode).

We are now ready to examine the effect of a ban on dual mode. Assume that the separation mode involves some additional fixed cost $F > 0$ for $M$ to set up the two separate divisions (e.g. separate websites, separate teams). As a result, $\Pi^{\text{sep}} = \Pi^{\text{sep}}_M + \Pi^{\text{sep}}_R - F = \tau^{\text{sep}} (1 - \mu) - F$.

**Proposition 13** *(Ban on dual mode when $M$ can commit to separation mode)*

- If $F < \mu (b + c)$ and $\Delta \geq \max \left\{ \frac{1}{2 - \mu} \left( 2c + b + \frac{F}{1 - \mu} \right), c + b + \frac{F}{1 - \mu} \right\}$, then a ban on the dual mode results in $M$ choosing the separation mode, with $CS_{\text{regular}}$ and $CS$ increasing, $CS_{\text{direct}}$ and $\pi$ not changing, and $\Pi$ and $W$ decreasing.

- If $\Delta \leq \min \left\{ \frac{1}{2 - \mu} \left( 2c + b + \frac{F}{1 - \mu} \right), c + b + \frac{F}{1 - \mu} \right\}$, then a ban on the dual mode results in $M$ choosing the reseller mode, with $CS_{\text{regular}}$ decreasing, $CS_{\text{direct}}$ increasing, $\pi$ not changing; $\Pi$ and $W$ decreasing; and $CS$ decreasing if $\Delta < b + c$ and increasing if $\Delta \geq b + c$.

- For all other parameter values, a ban on the dual mode results in $M$ choosing the marketplace mode, with $CS_{\text{regular}}$, $CS$ and $\Pi$ decreasing; $\pi$ increasing; and $CS_{\text{direct}}$ and $W$ not changing.

Relative to the baseline model, the new possibility considered here is that $M$ can choose the separation mode after the ban on its dual mode, which it will do when $F$ is low and $\Delta$ is high. Compared to the dual mode, the separation mode always results in higher consumer surplus. Intuitively, in the separation mode, the stronger margin squeeze leads to an even lower inside price. However, the separation mode is less welfare-efficient due to the fixed cost of having two separate modes.

Comparing the post-intervention outcome here with Proposition 6, it can be shown that the separation mode (whenever it is chosen by $M$) always leads to a higher post-intervention total consumer surplus, compared to the case where $M$ can only choose between marketplace and reseller modes. In this sense, a softer version of breaking up Amazon — by allowing it to operate two independent divisions — is preferable to a complete breakup from consumers’ perspective.
5.2 Competition and endogenous market structure

In the baseline model, there is always a single intermediary that optimally chooses to operate in dual mode, absent any policy restrictions. In this section, we consider the entry decisions of multiple intermediaries and show how the market structure assumed in our baseline model can arise endogenously.

Suppose there are two homogenous intermediaries $M_1$ and $M_2$. In stage 0, $M_1$ first chooses whether to enter the market, which requires incurring some small fixed cost $F > 0$, and if it enters, the mode it wants to operate in. After observing $M_1$’s choice, $M_2$ also chooses whether to enter, which requires incurring the same small fixed cost $F$.\footnote{Alternatively, we can assume that $M_1$ and $M_2$ make simultaneous entry decisions, which does not affect the result we obtain.} After the entry stages, both intermediaries choose their decision variables according to our existing timing assumptions, competing between themselves and with $S$. Specifically, if both intermediaries enter the market, the timing is modified as: (1) for every $i \in \{1, 2\}$ such that $M_i$ does not operate as a pure reseller, $M_i$ sets commission $\tau_i$; (2) for every $i \in \{1, 2\}$ such that $M_i$ does not operate as a pure reseller, $S$ chooses whether to participate on marketplace $i$; (3) All sellers, $S$, and every $M_i$ that is not operating as a pure marketplace for $i = 1, 2$ simultaneously set prices. Note $S$ is free to join one or both marketplaces (when both are available), and regular consumers can freely choose between all channels. We could also allow $S$ to face a multihoming cost: this does not affect the main conclusion below.

We start with the following observations. First, if both intermediaries operate as pure resellers, then in equilibrium both intermediaries earn zero profit by the logic of symmetric Bertrand competition. Second, if both intermediaries operate in the marketplace mode or in dual mode, then $S$ optimally sells to regular consumers exclusively through whichever intermediary sets a lower commission because $S$ can always adjust its relative prices to divert all regular consumers to the higher-margin channel. Anticipating this, both intermediaries compete their fees down to zero. Third, if exactly one intermediary operates as a pure reseller and the other intermediary operates in one of the other two modes, then the analysis proceeds as in the separation mode in Section 5.1, with the equilibrium profit of the pure reseller being zero.

Completing the discussion above, in Section C of the Online Appendix we derive the following table that summarizes both intermediaries’ post-entry profits for all possible combinations of modes, where the first and second entries in each box represent $M_1$ and $M_2$’s profits, gross of the entry cost $F$.

<table>
<thead>
<tr>
<th></th>
<th>$M_2$ marketplace</th>
<th>$M_2$ reseller</th>
<th>$M_2$ dual</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_1$ marketplace</td>
<td>0, 0</td>
<td>$\tau_1^*(1 - \mu), 0$</td>
<td>0, 0</td>
</tr>
<tr>
<td>$M_1$ reseller</td>
<td>0, $\tau_1^*(1 - \mu)$</td>
<td>0, 0</td>
<td>$0, \tau_1^*(1 - \mu)$</td>
</tr>
<tr>
<td>$M_1$ dual</td>
<td>0, 0</td>
<td>$\tau_1^*(1 - \mu), 0$</td>
<td>0, 0</td>
</tr>
</tbody>
</table>

Clearly, once $M_1$ has entered the market in dual mode (the most profitable mode), the fixed cost $F > 0$ means $M_2$ has no incentive to enter.

If the dual mode is banned, then $M_1$ optimally enters as a pure marketplace. Doing so has
two competitive advantages over entering as a pure reseller. First, there is a first-mover advantage due to fees being set before prices. Second, in the equilibrium of the post-entry subgame the intermediary that creates higher total value attracts all regular consumers, and operating in marketplace mode indeed creates a higher value (than the reseller mode) by combining S’s superior product with M1’s convenience benefit. Thus, when there is potential competition between intermediaries, banning the dual mode results in M1 always choosing the marketplace mode.

To summarize the results from this section:

**Proposition 14 (Multiple intermediaries)**

- In the overall equilibrium M1 enters the market and operates in dual mode, while M2 does not enter. The equilibrium prices are therefore the same as in Proposition 3.
- If the dual mode is banned, M1 enters in marketplace mode, leading to lower CS$_{regular}$, CS and Π; higher π; and unchanged CS$_{direct}$ and W.

### 5.3 Comparison with wholesaler-retailer model

At a high level, Amazon’s practice of selling on its own marketplace appears similar to that of retailers (e.g. Costco, Home Depot, Target, Trader Joe’s, WalMart) that offer their own in-house brands alongside products sourced from third-party suppliers. In this section, we discuss the key differences between these two practices and why the lessons from one do not translate to the other.

A key difference in the wholesaler-retailer structure compared to Amazon’s use of dual mode is the degree of competition between intermediaries. Amazon is widely perceived to dominate online retail sales in the U.S., whereas there is strong competition between supermarket chains in most local U.S. markets. As highlighted in the previous section, the effect of banning the dual mode can be quite different with and without competing intermediaries, suggesting one reason why the lessons from retailers’ use of house brands do not translate to Amazon’s use of dual mode.

Arguably a more fundamental difference between the two structures is that in the wholesaler-retailer structure the intermediary sets retail prices for all products regardless of whether they are in-house brands or sourced from third parties, whereas when Amazon uses the dual mode, third-party sellers on the marketplace maintain control of their prices. To analyze the implications of this difference, we first lay down a model of the wholesaler-retailer structure that captures this feature. Suppose M is a retailer, and it can source products from S and fringe sellers. In addition, M can also source from its own in-house brand, the marginal cost of which is set at zero as in the baseline model. Then, M sells all sourced products through its own sales channel, competing against the direct channels of S and the fringe sellers. Thus, M is now a multi-product firm setting prices for all its products.

M can choose to operate in three possible modes. Corresponding to marketplace mode, reseller mode and dual mode in our benchmark setting, we distinguish between the “third-party products” mode (M sources products from third-parties exclusively), the “in-house products”
mode (M source products from its in-house brand exclusively) and the “dual products” mode (M sources both types of products). The specifications of consumer types and utilities are the same as in the baseline model. To make things comparable to the baseline model, we adopt the following timing: (0) M chooses the mode of operation; (1) S and fringe sellers set their wholesale prices simultaneously; (2) M, S, and fringe sellers compete in retail prices.\(^{19}\)

In Section E of the Online Appendix, we derive the equilibrium of this wholesaler-retailer model under each of the three modes, and show that M prefers the dual products mode in the absence of any intervention. Then, banning the dual-products mode in this wholesaler-retailer model has the following implications:

**Proposition 15 (Ban on dual products mode in the wholesaler-retailer model)**

- If \(\Delta \geq \frac{c_1}{1-\mu}\), a ban on the dual products mode results in M choosing the third-party products mode, with \(\Pi, CS_{\text{regular}}, \text{and } CS\) decreasing; \(\pi\) and \(W\) increasing; and \(CS_{\text{direct}}\) not changing.

- If \(\Delta \leq \frac{c_1}{1-\mu}\), a ban on the dual products mode results in M choosing the in-house products mode, with \(\Pi, \pi, \text{and } W\) decreasing; \(CS_{\text{regular}}, CS_{\text{direct}}, \text{and } CS\) increasing.

A few remarks are in order. First, the cutoff in \(\Delta\) that determines M’s choice of mode after the ban is the same as in the baseline model (Proposition 6). This reflects that, in both third-party products mode and in-house products modes, the market outcomes (in terms of retail prices and split of profits) are the same in the baseline model and in the wholesaler-retailer context.

Second, in the wholesaler-retailer model, a new result is that whenever the ban on the dual products mode results in M choosing the third-party products mode, total welfare increases. Indeed, the dual products mode involves a mixed-strategy equilibrium, in which not all regular consumers purchase S’s product sold by M. This reflects that M decides all retail prices in the inside channel so there is no competition in the inside channel at the retail level, and that S has an incentive to exploit direct consumers in the outside channel. This parallels the feature of the reseller mode in the baseline mode that M and S must randomize over their respective prices in any equilibrium. The fact M’s inside price is sometimes high deters regular consumers from buying inside, generating welfare losses.

Third, whenever the ban on the dual products mode results in M choosing the in-house products mode, consumer surplus always increases. This result is driven by the fact that both outside and inside prices are higher in the dual products mode than in the in-house products mode. The outside price is higher because in the dual products mode S partially internalizes the revenue of M’s inside sales via its wholesale price, meaning that S would be less aggressive in setting its outside prices. This in turns relaxes the inter-channel competition, allowing M, whose price is not constrained by within-channel competition, to charge a higher inside price than the inside price in the in-house products mode.

\(^{19}\)In Section E.5 of the Online Appendix we consider an alternative setup in which in stage (1) M sets the wholesale price paid by M to third-party suppliers in case it wants to source from them. Given the wholesale price offered by M, suppliers just decide whether to supply M or not. We show that the overall insights are broadly similar in this case.
6 Discussion and conclusion

The practice of platforms selling products or services in their own name alongside similar offerings from third-party sellers is increasingly widespread. Indeed, such dual mode intermediation has clear benefits, which are captured by our benchmark model. It is therefore not surprising that there are now companies like Mirakl (https://www.mirakl.com/), which help retailers create marketplaces for third-party sellers to sell alongside the retailers’ own products. Mirakl’s customers include Best Buy, Carrefour, Darty, Office Depot, Urban Outfitters and others.

However, dual mode intermediation has also raised concerns from competition authorities regarding the possibility of distorting competition to the disadvantage of third-party sellers, particularly when the intermediary is a dominant e-commerce firm like Amazon. While such concerns are valid, our analysis suggests that a structural remedy such as an outright ban on the dual mode tends to benefit third-party sellers at the expense of consumer surplus or total welfare. The main reason for this is that in the dual mode the presence of the intermediary’s products constrains the pricing of the third-party sellers on its marketplace, which benefits consumers. Furthermore, when the third-party products do not offer high net surplus, we have shown that a ban on operating in the dual mode results in the intermediary becoming a pure reseller which leads to lower consumer surplus and total welfare.

One may expect that these results would be overturned once the third-party sellers’ incentives to innovate and the possibility for the intermediary to copy such innovations in the dual mode are taken into account. Perhaps surprisingly, even in this richer setting, a ban on the dual mode is not necessarily good for consumers or welfare, mainly because the intermediary already has a clear incentive to take into account the impact of its actions on third-party seller innovation incentives. Specifically, if the third-party innovation is sufficiently valuable, the intermediary would choose the marketplace mode as a way to commit not to expropriate the sellers’ innovation so as to ensure the participation of highly innovative sellers. In this case, a ban on the dual mode has no effect on consumer surplus or welfare. On the other hand, if the third-party innovation is not so valuable, the intermediary would adopt the reseller mode, so that a ban on the dual mode has similar (negative) implications as in the baseline setting.

Rather than banning the dual mode outright, we have shown that policy interventions that target specific behaviors by the intermediary are preferable. Namely, banning the imitation of (highly) innovative third-party products by the intermediary restores sellers’ incentive to innovate while still preserving the various benefits associated with the dual mode. Similarly, a ban on the intermediary steering consumers towards its own products preserves the benefits of the dual mode while preventing the intermediary from extracting excessively high commissions from third-party sellers.

Of course, a downside of these types of behavioral policy remedies (relative to a broad stroke ban on dual mode intermediation) is that they require continued monitoring of the intermediary’s conduct to be effective. For example, banning imitation only for highly innovative third-party products would be hard to implement in practice. This is despite the fact that, as shown in our paper, the intermediary has an incentive to commit itself not to imitate highly innovative third-party products in order to preserve their incentives to innovate and participate, and so would potentially benefit from an appropriately implemented ban. The difficulty comes
from the fact that in practice, the intermediary’s own employees (working in its in-house products division) may want to opportunistically make use of data from its marketplace division. Interestingly, Amazon has an internal policy forbidding the use of non-public data about specific sellers to launch its own in-house products, and yet, as noted in Mattioli (2020), there are reports of its employees violating the policy. This suggests regulators may require the relevant intermediaries (e.g. Amazon) to maintain a “Chinese wall” between their respective private label and marketplace divisions. Similarly, to prevent opportunistic steering, the intermediary may be required to provide public APIs that allow approved outsiders (e.g. policy makers or researchers) to audit their recommendation algorithms.

7 Appendix

7.1 Proofs for the baseline model

7.1.1 Proof of Proposition 2

In this proof we let \( r \in [0, 1] \) denote the mass of regular consumers who break ties in favor of \( M \)'s product when they are indifferent between \( M \)'s and \( S \)'s products. We first note the following two lemmas.

**Lemma 5** In reseller mode, there is no pure-strategy equilibrium when \( \Delta < \frac{b+c}{1+\mu} \).

**Proof.** Suppose \( S \) sets a deterministic price \( p^*_o \). If \( p^*_o = c + \Delta \), then, \( M \) must optimally set \( p^*_m = c + b \) (if \( r = 1 \)) or \( p^*_m = c + b - \epsilon \) for \( \epsilon > 0 \) small (if \( r < 1 \)) to attract all regular consumers. In both cases, given \( \Delta > 0 \) and \( c + b > 0 \), \( S \) can profitably deviate by undercutting with \( p_o = p^*_o - \epsilon \) to attract all regular consumers. If \( p^*_o < c + \Delta \) and \( S \) is not attracting any regular consumers, then \( S \) can deviate from \( p^*_o \) by setting \( p_o = c + \Delta \), earning a higher margin selling only to direct consumers. Suppose instead \( p^*_o < c + \Delta \) and \( S \) is attracting some regular consumers. If \( p^*_o > \Delta - b \), \( M \) can profitably undercut by setting \( p_m = p^*_o + b - \Delta - \epsilon \) (for \( \epsilon > 0 \) small) to attract all regular consumers; If \( p^*_o \leq \Delta - b \), \( S \)'s equilibrium profit is \( p^*_o - c \leq \Delta - b - c \), and it can profitably deviate by setting \( p_o = c + \Delta \), exploiting direct consumers to earn \( \mu \Delta > \Delta - b - c \). ■

**Lemma 6** In reseller mode, there is no equilibrium in which \( M \) sells to all regular consumers.

**Proof.** In any such equilibrium, \( S \) necessarily sets \( p^*_o = c + \Delta \) given that \( M \), by assumption, is selling to all regular consumers. Then, \( M \) must optimally set \( p^*_m = c + b \) (if \( r = 1 \)) or \( p^*_m = c + b - \epsilon \) for \( \epsilon > 0 \) small (if \( r < 1 \)). In both cases, given \( \Delta > 0 \), \( S \) can profitably deviate by undercutting with \( p_o = p^*_m - \epsilon \) to attract all regular consumers. ■

Consider the equilibrium where \( S \) makes all the sales and \( p^*_o = \Delta - b \) and \( p^*_m = 0 \). From the text, the price profile is sustainable as an equilibrium if and only if \( \Delta \geq \frac{b+c}{1+\mu} \). When \( \Delta < \frac{b+c}{1+\mu} \), Lemma 5 implies there is no other pure strategy with \( S \) making all the sales.

We next consider the other type of equilibrium where \( M \) sometimes makes a positive amount of sales, noting that such equilibrium is necessarily in mixed strategies due to Lemma 6. We first suppose \( \Delta \leq \frac{b+c}{1+\mu} \) and verify the mixed strategy equilibrium stated in the proposition. The cdf of \( p^*_m \), \( F_m \), is such that \( S \) is indifferent for all \( p^*_m \in [c + \mu \Delta, c + \Delta] \). Notice when \( p^*_m = c + \Delta \), \( S \) attracts only direct consumers and obtains profit \( \mu \Delta \). Therefore, the indifference condition is

\[
(p^*_o - c) (\mu + (1 - \mu) (1 - F_m (p^*_m + b - \Delta))) = \mu \Delta.
\]

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Letting \( p_m^* = p_o^* + b - \Delta \), and rearranging, we get

\[
F_m(p_m^*) = \frac{1}{1-\mu} \left( 1 - \frac{\mu \Delta}{p_m^* - b + \Delta - c} \right),
\]

which is exactly the cdf stated in the proposition. Note \( F_m(c + b - (1 - \mu) \Delta) = 0 \) and

\[
\lim_{p_m^* \rightarrow c+b} F_m(p_m^*) = 1,
\]

so \( F_m \) has no mass point. The cdf of \( p_o^* F_o \), is such that \( M \) is indifferent for all \( p_m^* \in [c + b - (1 - \mu) \Delta, c + b] \). Notice when \( p_m^* = c + b - (1 - \mu) \Delta \), \( M \) attracts regular consumers with probability one and obtains profit \((c + b - (1 - \mu) \Delta)(1 - \mu)\). Therefore, the indifference condition is

\[
p_m^* (1 - \mu) (1 - F_o(p_m^* - b + \Delta)) = (c + b - (1 - \mu) \Delta)(1 - \mu).
\]

Letting \( p_o^* = p_m^* - b + \Delta \), and rearranging, we get

\[
F_o(p_o^*) = 1 - \left( \frac{c + b - (1 - \mu) \Delta}{p_o^* + b - \Delta} \right).
\]

Notice \( F_o(c + \mu \Delta) = 0 \) and

\[
\lim_{p_o^* \rightarrow c+\Delta} F_o(p_o^*) = \frac{(1 - \mu) \Delta}{c + b} \leq 1,
\]

given \( \Delta \leq \frac{b + c}{1 - \mu} \). Hence, \( F_o \) has a mass point \( \Pr(p_o^* = c + \Delta) = 1 - \frac{(1 - \mu) \Delta}{c + b} \) if \( \Delta < \frac{b + c}{1 - \mu} \). This is exactly the cdf stated in the proposition. Given that \( F_m \) has no mass point, any \( r \in [0, 1] \) can be supported in equilibrium.

Finally, we check that from the stated mixed strategy equilibrium each player cannot profit from a deviation outside of its support. For \( S \), any \( p_o < c + \mu \Delta \) earns profit strictly less than \( \mu \Delta \) even if it attracts all consumers and so is dominated by \( p_o = c + \Delta \); while any \( p_o > c + \Delta \) attracts no consumers due to the existence of fringe sellers. For \( M \), any \( p_m < c + b - (1 - \mu) \Delta \) is strictly dominated by \( p_m = c + b - (1 - \mu) \Delta \geq 0 \) given that \( M \) already attracts regular consumers with probability one at this price; while any \( p_m > c + b \) attracts no consumers due to the existence of fringe sellers.

### 7.1.2 Proof of Lemma 1

Given all regular consumers buy from \( S \) through the marketplace, we must have \( p_i^* \geq c + \tau \) as otherwise \( S \) makes strict losses. Meanwhile, given that \( p_m^* \geq 0 \) must hold, the best response for \( S \) must have \( p_i^* \geq \Delta \) since at \( p_i^* = \Delta \) it can already attract all regular consumers so would do strictly worse setting a price below this. Therefore, \( p_i^* \geq \max \{ \Delta, c + \tau \} \). We next establish the upperbound for \( p_i^* \). If \( S \) sets its outside price just below \( p_o = p_i^* - b \) to attract all regular consumers to the direct channel, its profit would be just below \( p_i^* - b - c \), while if \( S \) sets \( p_o = c + \Delta \) to sell only to direct consumers its profit is \( \mu \Delta + (p_i^* - c - \tau)(1 - \mu) \). Comparing these two profit expressions, \( S \) does not induce showroming if and only if the candidate equilibrium inside price of \( S \) is

\[
p_i^* \leq c + \Delta + \frac{b - (1 - \mu) \tau}{\mu} \equiv \hat{\tau}.
\]

Meanwhile, if \( p_i^* \leq \tau + \Delta \) then \( M \) has no incentive to undercut since all regular consumers are buying through the marketplace in equilibrium and undercutting would require setting \( p_m \leq p_i^* - \Delta \leq \tau \), i.e. a lower margin than what it is earning from fees. Any \( p_i^* > \tau + \Delta \) cannot be part of the equilibrium because \( M \) would want to undercut the price and get a higher margin selling itself than the commission.
\( \tau \) collected. Therefore, in any intermediation equilibrium, \( p^*_i \leq \min \{ \hat{\varphi}, \tau + \Delta \} \), and it can be verified that \( \min \{ \hat{\varphi}, \tau + \Delta \} \leq b + c\mu + \Delta < b + c + \Delta \) so regular consumers indeed do not go outside for \( p^*_i \leq \min \{ \hat{\varphi}, \tau + \Delta \} \) and \( p^*_o = c + \Delta \). Therefore, any
\[
p^*_i \in \Phi_i \equiv \left[ \max \left\{ c + \tau, \Delta \right\}, \min \{ \hat{\varphi}, \tau + \Delta \} \right]
\] (7.6)
with \( p^*_m = p^*_i - \Delta \) and \( p^*_o = c + \Delta \) can be sustained as an intermediation equilibrium as long as the set \( \Phi_i \) is non-empty. By construction, any profile with \( p^*_i \notin \Phi_i \) cannot be an intermediation equilibria. Note that \( \hat{\varphi} \geq \Delta \Leftrightarrow \tau \leq \frac{b + c}{1 - \mu} \) while \( \hat{\varphi} \geq c + \tau \Leftrightarrow \tau \leq b + \mu \Delta \). When \( \Delta < \frac{b + c}{1 - \mu} \), \( \tau \leq b + \mu \Delta \) implies \( \tau \leq \frac{b + c \mu}{1 - \mu} \), so that the set \( \Phi_i \) is non-empty if and only if \( \tau \leq b + \mu \Delta \). When \( \Delta \geq \frac{b + c}{1 - \mu} \), \( \tau \leq \frac{b + c \mu}{1 - \mu} \) implies \( \tau \leq b + \mu \Delta \), so that the set \( \Phi_i \) is non-empty if and only if \( \tau \leq \frac{b + c \mu}{1 - \mu} \).

### 7.1.3 Proof of Lemma 2

Following the discussion in main text, such equilibria necessarily have \( p^*_m = \Delta - b \), \( p^*_m = 0 \) and \( p^*_i \geq \Delta \), and \( S \) earns \( \pi^* = \Delta - b - c \) in equilibrium. When \( \Delta < \frac{b + c}{1 - \mu} \), \( S \) can profitably deviate by setting \( p_o = c + \Delta \) and a high \( p_i \) (i.e. deviate to let \( M \) win inside), earning deviation profit \( \mu \Delta > \Delta - b - c \). When \( \Delta \geq \frac{b + c}{1 - \mu} \) and \( \tau < \frac{b + c \mu}{1 - \mu} \), \( S \) can profitably deviate by setting \( p_o = c + \Delta \) and \( p_i = \Delta \) (i.e. deviate by attracting consumers to the marketplace), earning deviation profit
\[
\begin{align*}
\mu \Delta + (\Delta - c - \tau) (1 - \mu) &> \mu \Delta + \left( \Delta - c - \frac{b + c \mu}{1 - \mu} \right) (1 - \mu) \\
&= \Delta - b - c = \pi^*,
\end{align*}
\]
where the inequality uses \( \tau < \frac{b + c \mu}{1 - \mu} \). Finally, when \( \Delta \geq \frac{b + c}{1 - \mu} \) and \( \tau \geq \frac{b + c \mu}{1 - \mu} \), \( S \)'s deviation profit from setting \( p_o = c + \Delta \) (while setting either \( p_i = \Delta \) or a high \( p_i \) that lets \( M \) win) is at most
\[
\begin{align*}
\mu \Delta + \max \{ \Delta - c - \tau, 0 \} (1 - \mu) &\leq \max \left\{ \mu \Delta + \left( \Delta - c - \frac{b + c \mu}{1 - \mu} \right) (1 - \mu), \mu \Delta \right\} \\
&= \max \{ \Delta - b - c, \mu \Delta \} \\
&\leq \Delta - b - c = \pi^*.
\end{align*}
\]

### 7.1.4 Proof of Lemma 3

We first note the following preliminary lemma:

**Lemma 7** In dual mode, there is no pure-strategy equilibrium in which \( M \) makes all the sales to regular consumers.

**Proof.** Suppose to the contrary, there is such an equilibrium. Then we must have \( p^*_m \leq p^*_i - \Delta \), \( p^*_i \leq c + \tau \) (otherwise \( S \) will slightly undercut inside), and \( p^*_m \leq c + b \) (otherwise regular consumers will buy from fringe sellers outside). Moreover, \( p^*_m \) must be such that \( S \) prefers to set \( p^*_o = c + \Delta \), selling exclusively to the direct consumers, i.e.
\[
p^*_m - b + \Delta - c \leq \mu \Delta.
\] (7.7)

If \( p^*_i - \Delta > c + b \), then \( M \) can deviate from (7.7) by raising its price to \( p_m = b + c - \epsilon \) (for some small \( \epsilon > 0 \)) while still attracting regular consumers, earning deviation profit
\[
\Pi' = (b + c - \epsilon) (1 - \mu) > (b + c - (1 - \mu) \Delta) (1 - \mu) \geq p^*_m (1 - \mu) = \Pi^*.
\]
where the first inequality uses $\Delta > 0$ while the second inequality uses (7.7). If $p^*_i - \Delta \leq c + b$, then $M$ can deviate from (7.7) by raising its price to $p_m \geq p^*_i - \Delta$, letting $S$ sell inside, and earning deviation profit

$$\Pi' = \tau (1 - \mu) \geq (p^*_i - c) (1 - \mu) \geq (p^*_m + \Delta - c) (1 - \mu) > \Pi^*,$$

where the first two inequalities use $p^*_i \leq c + \tau$ and $p^*_m \leq p^*_i - \Delta$, while the last inequality uses $\Delta - c > 0$.

To prove Lemma 3, we now construct the mixed-strategy equilibrium with $M$ making a positive amount of sales. Similar to Proposition 2, if $\Delta > \frac{c + \mu}{1 - \mu}$ then there is no equilibrium with $M$ selling to any regular consumers. Therefore, we focus on $\Delta \leq \frac{c + \mu}{1 - \mu}$ in what follows. There are two possible candidate equilibria.

(1) Candidate reseller equilibrium such that $S$ never makes sales inside. Note $p^*_i > c + b + \Delta$ means that regular consumers prefer fringe sellers’ product sold directly over $S$’s product sold through the marketplace, so $M$’s price is only constrained by $p_m \leq c + b$. Therefore, the verification of the mixed strategy in this case is exactly the same as the second part of Proposition 2. It remains to check that:

(i) $M$ cannot deviate from this mixed strategy equilibrium by letting $S$ win inside, which is true because $p^*_i > c + b + \Delta$ means that if $M$ deliberately lets $S$ win inside then all regular consumers purchase from fringe sellers outside; (ii) $S$ has no incentive to deviate and make sales inside. To check (ii), recall in equilibrium $S$ earns $\mu \Delta$. Given $p^*_m \leq c + b$, $S$ can profitably deviate inside if and only if it can set $p_i \in (c + \tau, c + b + \Delta]$ (so that there is a positive probability it makes a positive profit) together with $p_o = c + \Delta$. The set $(c + \tau, c + b + \Delta]$ is empty if and only if $\tau \geq b + \Delta$. Any deviation $p_i \not\in (c + \tau, c + b + \Delta]$ either makes no profit or does not affect purchase decisions. We thus conclude that the (mixed-strategy) equilibrium exists if and only if $\tau \geq b + \Delta$.

(2) Candidate reseller equilibrium such that $S$ sometimes makes sales inside. Recall we are focusing on $\tau \in [b + \mu \Delta, b + \Delta)$, and the domains for mixed strategies stated in the lemma are:

$$p^*_m \in \left[ c + b - (1 - \mu) \Delta, c + \tau - \Delta \right],$$

$$p^*_o \in \left[ \mu \Delta + c, c + \tau - b \right] \cup \left\{ c + \Delta \right\},$$

where $p^*_i = c + \tau$. (Note that all values of $p^*_m$ such that $p^*_m \geq c + \tau - \Delta$ are outcome equivalent because if $M$ sets these prices then $S$ will make all inside sales, in which case $M$ earns only from collecting commission $\tau$. Therefore, it is without loss of generality to rule out $p^*_m > c + \tau - \Delta$.)

We first verify the mixed strategy equilibrium stated in the proposition. The cdf of $p^*_m$ is such that $S$ is indifferent for all $p^*_m \in [\mu \Delta + c, c + \tau - b] \cup \{ c + \Delta \}$. Notice when $p^*_o = c + \Delta$, $S$ never attracts regular consumers given $p^*_m \leq c + \tau - \Delta < c + b$, so when $p^*_o = c + \Delta$, the outside channel attracts only direct consumers and $S$ obtains profit $\mu \Delta$. Therefore, the indifference condition is

$$(p^*_o - c) (\mu + (1 - \mu) (1 - F_m (p^*_o + b - \Delta))) = \mu \Delta.$$ 

Letting $p^*_o = p^*_m - b + \Delta$, and rearranging, we get

$$F_m (p^*_m) = \frac{1}{1 - \mu} \left( 1 - \frac{\mu \Delta}{p^*_m - b + \Delta - c} \right).$$

Note $F_m (c + b - (1 - \mu) \Delta) = 0$ and

$$\lim_{p^*_m \nearrow c + \tau - \Delta} F_m = \frac{1}{1 - \mu} \left( 1 - \frac{\mu \Delta}{\tau - b} \right) < 1$$

given $\tau < b + \Delta$. Hence, $F_m$ has a mass point $\Pr (p^*_m = c + \tau - \Delta) = \frac{\mu}{1 - \mu} \left( \frac{\Delta}{\tau - \sigma} - 1 \right)$. 31
The cdf of \( p^*_o, F_o \), is such that \( M \) is indifferent for all \( p^*_m \in [c + b - (1 - \mu) \Delta, c + b] \). Note that \( F_o \) has two mass points: \( \rho_b \equiv \Pr(p^*_o = c + \Delta) \) and \( \rho_l \equiv \Pr(p^*_o = c + \tau - b) \), and they need to be such that \( M \) is indifferent between three price levels:

- \( p_m = c + b - (1 - \mu) \Delta \), and attracts all regular consumers with probability one. This gives \( M \) expected profit \((c + b - (1 - \mu) \Delta)(1 - \mu)\).
- \( p_m = c + \tau - \Delta \), letting \( S \) win inside. Here, recall that in the equilibrium specification consumers break ties in favor of \( S \)'s direct channel, so if \( p_m = c + \tau - \Delta \), regular consumers buy from \( S \) inside only when \( p_o = c + \Delta \). This gives \( M \) expected profit \( \tau p_b (1 - \mu) \).
- \( p_m = c + \tau - \Delta - \epsilon \) for \( \epsilon < 0 \), so that \( M \) makes sales whenever \( p^*_o = c + \Delta \) or \( p^*_o = c + \tau - b \), earning expected profit \((c + \tau - \Delta)(\rho_o + \rho_l)(1 - \mu)\).

Equating these three profit expressions and eliminating the common factor:

\[
\tau \rho_h = \frac{(c + b - (1 - \mu) \Delta)}{c + \tau - \Delta}, \quad \rho_l = \frac{c + b - (1 - \mu) \Delta}{c + \tau - \Delta}, \quad \rho_h = \rho_l.
\]

Given \( \Delta \leq \frac{b + c}{1 + \mu} \) and \( \tau \geq b + \mu \Delta \), obviously \( \rho_h \in [0, 1] \) and \( \rho_l \leq 1 \). Moreover, \( \tau \geq b + \mu \Delta \implies \tau \geq \Delta + \epsilon > 0 \), so \( \rho_l \geq 0 \). It remains to verify the cdf for \( p^*_o \in [\mu \Delta + c, c + \tau - b] \). The indifference condition is

\[
p_m \left(1 - F_o(p_m - b + \Delta)\right) = c + b - (1 - \mu) \Delta,
\]

\[
\implies F_o(p_o) = 1 - \frac{c + b - (1 - \mu) \Delta}{p_o - \Delta + b}.
\]

Obviously, \( F_o(\mu \Delta + c) = 0 \), and

\[
\lim_{p_o \searrow c + \tau - b} F_o = 1 - \frac{c + b - (1 - \mu) \Delta}{c + \tau - \Delta},
\]

which is readily verified to equal to \( 1 - \rho_h - \rho_l \).

Next, we check that from the stated mixed strategy equilibrium each player cannot profit from a deviation outside of its support. For \( M \), any \( p_m < c + b - (1 - \mu) \Delta \) is weakly dominated by \( p_m = c + b - (1 - \mu) \Delta \geq 0 \) given that \( p^*_c \geq c + \mu \Delta \) and \( F_o \) has no mass point at \( p^*_o = c + \mu \Delta \); while any \( p_m > c + \tau - \Delta \) means \( S \) wins the inside competition, which is equivalent to setting \( p_m = c + \tau - \Delta \). For \( S \), any \( p_o < c + \mu \Delta \) earns profit strictly less than \( \mu \Delta \); any \( p_o > c + \Delta \) attracts no consumer due to the existence of fringe sellers selling directly; while any \( c + \tau - b, c + \Delta \) does not attract any regular consumers given \( p^*_o \leq c + \tau - \Delta \) and is dominated by \( p_o = c + \Delta \). Finally, \( S \) has no incentive to deviate by changing \( p_i \) (or changing both prices together) because any \( p_i < p^*_i = c + \tau \) leads to a loss, while any \( p_i > p^*_i \) does not change its profit.

Furthermore, notice the mass points \( \Pr(p^*_o = c + \tau - b) \) and \( \Pr(p^*_o = c + \tau - \Delta) \) mean that there is a strictly positive probability for regular consumers to be indifferent between \( S \)'s product in both channels. If a positive mass of regular consumers break ties in favor of \( S \)'s inside product, then \( S \) can profitably deviate by shifting mass from \( p^*_o = c + \tau - b \) to price just below that, so that all regular consumers buy from \( S \) directly whenever \( p^*_m = c + \tau - \Delta \) is realized, yielding \( S \) a margin of \( \tau - b > 0 \).

In this case, \( F_o \) cannot be part of the equilibrium.

Notice if \( \tau \rightarrow b + \Delta \), the two mass points \( \Pr(p^*_o = c + \Delta) \) and \( \rho_l \equiv \Pr(p^*_o = c + \tau - b) \) combine, in which \( \rho_h + \rho_l = \frac{(c + b - (1 - \mu) \Delta)}{c + \tau - \Delta} \), while the mass point \( \Pr(p^*_o = c + \tau - \Delta) = \frac{c + \ell}{\tau - \Delta} (\mu \Delta + 1) \rightarrow 0 \). Therefore, the equilibrium outcome is continuous with the pure reseller equilibrium obtained when \( \tau \geq b + \Delta \).

Likewise, when \( \tau \rightarrow b + \mu \Delta \), then \( \Pr(p^*_o = c + \tau - \Delta) \rightarrow 0 \), i.e. \( M \) always lets \( S \) win inside, so the
equilibrium outcome is continuous with the intermediation equilibrium obtained when \( \tau \leq b + \mu \Delta \). Finally, when \( \tau < b + \mu \Delta \), in the mixed strategy equilibrium the support for \( F_m, [c+b-(1-\mu)\Delta, c+\tau-\Delta] \), become an empty set. In that case, Lemma 7 implies there is no other reseller equilibrium.

### 7.1.5 Proof of Proposition 3

We first derive the equilibrium of the non-participation subgame. The only difference with Proposition 2 is that \( M \)'s price is bounded by \( p_m \leq c + \min \{ \tau, b \} \) due to the competition with fringe sellers selling inside and outside.

**Lemma 8 (Dual mode, non-participation)**

- If \( \Delta > \frac{b+c}{1-\mu} \), then in the equilibrium \( p_m^* = \Delta - b \) and \( p_m^* = 0 \). All regular consumers purchase from \( S \) directly. The equilibrium profits are \( \Pi^{np} = 0 \) and \( \pi^{np} = \Delta - b - c \).

- If \( \Delta \leq \frac{b+c}{1-\mu} \) and \( \tau > b - (1-\mu)\Delta \), then in the mixed-strategy equilibrium, \( p_m^* \) is distributed according to c.d.f \( F_m \) with support \( [c+b-(1-\mu)\Delta, c+\min \{ \tau, b \}] \), where

\[
F_m (p_m^*) = \begin{cases} 
\frac{1}{1-\mu} \left( 1 - \frac{\mu \Delta}{\mu \Delta - b - \Delta - c} \right) & \text{for } p_m^* \in [c+b-(1-\mu)\Delta, c+\min \{ \tau, b \}] \\
1 & \text{for } p_m^* \geq c + \min \{ \tau, b \}
\end{cases}
\]

\( p_m^* \) is distributed according to c.d.f \( F_o \) with support \( [c+\mu\Delta, c+\Delta+\min \{0, \tau-b\}] \cup \{c+\Delta\} \), where

\[
F_o (p_o^*) = \begin{cases} 
1 - \frac{c+b-(1-\mu)\Delta}{p_o^*+b-\Delta} & \text{for } p_o^* \in [c+\mu\Delta, c+\Delta+\min \{0, \tau-b\}] \\
1 - \frac{c+b-(1-\mu)\Delta}{1+c+\tau} & \text{for } p_o^* \in (c+\Delta+\min \{0, \tau-b\}, c+\Delta] \\
1 & \text{for } p_o^* \geq c + \Delta
\end{cases}
\]

The equilibrium profits are \( \Pi^{np} = (c+b-(1-\mu)\Delta)(1-\mu) \) and \( \pi^{np} = \mu \Delta \).

**Proof.** The first part follows from the first part of Proposition 2. For the second part, we verify the mixed strategy equilibrium. If \( \tau \geq b \), the derivation is the same as the second part of Proposition 2. We consider \( \tau \in (b-(1-\mu)\Delta, b) \) in what follows, in which case

\[
p_m^* \in [c+b-(1-\mu)\Delta, c+\tau] \\
p_o^* \in [c+\mu\Delta, c+\Delta+\tau-b] \cup \{c+\Delta\}.
\]

The cdf of \( p_m^*, F_m \), is such that \( S \) is indifferent for all \( p_m^* \in [c+\mu\Delta, c+\Delta+\tau-b] \cup \{c+\Delta\} \). Notice when \( p_o^* = c+\Delta \), \( S \) attracts only direct consumers (given \( p_m^* < c+b \)) and obtains profit \( \mu \Delta \). Therefore, the indifference condition is

\[
(p_o^* - c) \left( \mu + (1-\mu) (1 - F_m (p_o^* + b - \Delta)) \right) = \mu \Delta.
\]

Letting \( p_m^* = p_o^* + b - \Delta \), and rearranging, we get

\[
F_m (p_m^*) = \frac{1}{1-\mu} \left( 1 - \frac{\mu \Delta}{p_m^*-b+\Delta-c} \right),
\]

which is exactly the cdf stated in the proposition. Note \( F_m (c+b-(1-\mu)\Delta) = 0 \) and

\[
\lim_{p_m^* \to c+\tau} F_m (p_m^*) = \frac{1}{1-\mu} \left( 1 - \frac{\mu \Delta}{\tau-b+\Delta} \right) < 1,
\]

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so \( F_m \) has a mass point \( \Pr (p_m^* = c + \tau) = \frac{\mu}{1 - \mu} \left( \frac{\Delta}{c + \Delta} - 1 \right) \).

The cdf of \( p_m^* \), \( F_o \), is such that \( M \) is indifferent for all \( p_m^* \in [c + b - (1 - \mu) \Delta, c + \tau] \). Notice when \( p_m^* = c + b - (1 - \mu) \Delta, M \) attracts regular consumers with probability one and obtains profit \((c + b - (1 - \mu) \Delta)(1 - \mu)\). Therefore, the indifference condition is

\[
p_m^*(1 - \mu)((1 - F_o (p_m^* - b + \Delta))) = (c + b - (1 - \mu) \Delta)(1 - \mu).
\]

Letting \( p_o^* = p_m^* - b + \Delta, \) and rearranging, we get

\[
F_o (p_o^*) = 1 - \left( \frac{c + b - (1 - \mu) \Delta}{p_o^* + b - \Delta} \right).
\]

Notice \( F_o (c + \mu \Delta) = 0 \) and

\[
\lim_{p_o^* \to c + \Delta + \tau - b} F_o (p_o^*) = 1 - \left( \frac{c + b - (1 - \mu) \Delta}{c + \tau} \right) < 1,
\]
given \( \tau > b - (1 - \mu) \Delta \). Hence, \( F_o \) has a mass point \( \Pr (p_o^* = c + \Delta) = \frac{c + b - (1 - \mu) \Delta}{c + \tau} \). This is exactly the cdf stated in the proposition.

Finally, we check that from the stated mixed strategy equilibrium each player cannot profit from a deviation outside of its support. For \( S \), any \( p_o < c + \mu \Delta \) earns profit strictly less than \( \mu \Delta \) even if it attracts all consumers so it is strictly dominated by \( p_o = c + \Delta; \) any \( p_o > c + \Delta \) attracts no consumers due to the existence of fringe sellers; while any \( p_o \in (c + \Delta + \tau - b, c + \Delta) \) does not attract any regular consumers and so is strictly dominated by \( p_o = c + \Delta \). For \( M \), any \( p_m < c + b - (1 - \mu) \Delta \) is weakly dominated by \( p_m = c + b - (1 - \mu) \Delta \geq 0 \) given that \( p_m^* \geq c + \mu \Delta \) and \( F_o \) has no mass point at \( p_o^* = c + \mu \Delta \); while any \( p_m > c + \tau \) attracts no consumers due to the existence of fringe sellers in the marketplace.

Combining the cases of \( \tau \in (b - (1 - \mu) \Delta, b) \) and \( \tau > b \) gives the characterization in the second case of the lemma.

For the third case, when \( p_o^* = c + \tau \leq c + b - (1 - \mu) \Delta, S \) has no incentive to deviate from \( p_o^* = c + \Delta \) because its deviation profit is no higher than \( p_o^* - b + \Delta - c \leq \mu \Delta \), i.e. lower than its equilibrium profit. \( M \) has no incentive to deviate given the constraint \( p_m \leq c + \tau \).

Therefore, regardless of \( \tau \) and equilibrium selection, in all equilibrium in the stage 3 subgame following S’s participation, S’s profit is weakly higher than \( \pi^{np} = \max \{\mu \Delta, \Delta - b - c\}, \) so it always participates.

From Table 3, we know \( M \) does best by setting the highest \( \tau \) as long as it still induces the intermediation equilibrium in the stage 3 subgame. Hence, \( \tau^{\text{dual}} = b + \mu \Delta \) when \( \Delta \leq \frac{b + c}{1 - \mu}, \) and \( \tau^{\text{dual}} = \frac{b + c a}{1 - \mu} \) when \( \Delta > \frac{b + c}{1 - \mu} \).

### 7.1.6 Proof of Proposition 4

We have \( \tau^{\text{dual}} = \min \left\{ \frac{b + c a}{1 - \mu}, b + \mu \Delta \right\} > b, \) so \( \Pi^{\text{dual}} > \Pi^{\text{market}} \). When \( \Delta > \frac{b + c}{1 - \mu}, \) \( \Pi^{\text{dual}} = b + c \mu > 0 = \Pi^{\text{resell}} \). When \( \Delta \leq \frac{b + c}{1 - \mu}, \) \( \Pi^{\text{dual}} = (b + \mu \Delta)(1 - \mu) > (b + \mu \Delta + c - \Delta)(1 - \mu) = \Pi^{\text{resell}} \). Moreover, if \( \Delta > \frac{b + c}{1 - \mu}, \) \( \Pi^{\text{resell}} = 0 < \Pi^{\text{market}} \); while if \( \Delta \leq \frac{b + c}{1 - \mu} \), then \( \Pi^{\text{resell}} \leq \Pi^{\text{market}} \) if and only if \( \Delta \geq \frac{c}{1 - \mu} \). Given \( b \geq 0, \) we can combine both possibilities to conclude that \( \Pi^{\text{resell}} \leq \Pi^{\text{market}} \) if and only if \( \Delta \geq \frac{c}{1 - \mu} \).

Finally, comparisons for S’s profit follow from the fact \( \pi^{\text{market}} = \Delta > \pi^{\text{dual}} = \max \{\mu \Delta, \Delta - b - c\} = \pi^{\text{resell}} \).

### 7.1.7 Proof of Proposition 5

We first compare welfare between the two pure modes. Let \( 0 \leq \eta < 1 \) denote the probability that regular consumers buy from \( M \) in the equilibrium in reseller mode. The associated welfare is \( W^{\text{resell}} = \)
The inequalities follow from \( \Delta > 0 \), where \( \Delta \) by CS in the marketplace mode is simply consumers buy \( S \) Welfare in the dual mode matches that under the marketplace mode given that in both settings, regular consumers buy \( S \) via \( M \)'s marketplace.

Turning next to consumer surplus, we first compare the two pure modes. Overall consumer surplus in the marketplace mode is simply \( CS_{\text{market}} = v - c \) since the surplus from \( \Delta \) and \( b \) is fully extracted by \( S \) and \( M \) respectively. In the reseller mode (i) direct consumers are better off given \( p^*_n < c + \Delta \) with positive probability; (ii) regular consumers are better off given \( p^*_o < c + \Delta \) with positive probability. Therefore, in the reseller mode both groups of consumers earn expected surplus \( CS_{\text{resell}} > v - c = CS_{\text{market}} \). Furthermore, note \( CS^\text{dual}_{\text{direct}} = CS^\text{market}_{\text{direct}} \) given \( p^*_n = c + \Delta \) in both modes, while \( p^*_o < c + \Delta \) with positive probability in the reseller mode so \( CS^\text{dual}_{\text{direct}} \) is strictly higher in reseller mode.

For regular consumers, if \( \Delta > b + c \frac{1}{1-\mu} \) then

\[
\begin{align*}
CS^\text{resell}_{\text{regular}} &= v + \Delta - (\Delta - b) = v + b \\
CS^\text{market}_{\text{regular}} &= v - c \\
CS^\text{dual}_{\text{regular}} &= v + \Delta + b - \Delta = v + b.
\end{align*}
\]

If \( \Delta \leq b + c \frac{1}{1-\mu} \), in the reseller mode we can bound \( p^*_o \in [c + \mu \Delta, c + \Delta] \), so

\[
\begin{align*}
CS^\text{resell}_{\text{regular}} &\in [v - c, v - c + (1 - \mu) \Delta] \\
CS^\text{market}_{\text{regular}} &= v - c \\
CS^\text{dual}_{\text{regular}} &= v + \Delta + b - (c + b + \mu \Delta) = v - c + (1 - \mu) \Delta.
\end{align*}
\]

The inequalities follow from \( \Delta > 0 \) and considering \( \Delta \leq b + c \frac{1}{1-\mu} \). For total consumer surplus, the only non-trivial case is when \( \Delta \leq b + c \frac{1}{1-\mu} \), in which \( CS^\text{resell}_{\text{direct}} > CS^\text{dual}_{\text{direct}} \) and \( CS^\text{dual}_{\text{regular}} > CS^\text{resell}_{\text{regular}} \). We can calculate

\[
\begin{align*}
CS^\text{resell} &= W^\text{resell} - \Pi^\text{resell} - \pi^\text{resell} \\
&= v - c + (1 - \mu)^2 \Delta + (1 - \mu)(1 - \eta)(\Delta - b - c),
\end{align*}
\]

and comparing it to \( CS^\text{dual} = v - c + (1 - \mu)^2 \Delta \) gives the result.

### 7.2 Proofs for Section 4.1

#### 7.2.1 Proof of Lemma 4

For all \( \frac{b + c}{1 - \mu} \leq \Delta^L \), \( \bar{\pi}^\text{resell} (\Delta) \) has exactly one interior peak point at \( \Delta = \Delta^H > \Delta^L \geq \frac{b + c}{1 - \mu} \), so \( S \) optimally chooses \( \Delta^H \). For all \( \frac{b + c}{1 - \mu} \geq \Delta^H \), \( \bar{\pi}^\text{resell} (\Delta) \) has exactly one interior peak point at \( \Delta = \Delta^L < \Delta^H \leq \frac{b + c}{1 - \mu} \), so \( S \) optimally chooses \( \Delta^L \). For \( \Delta^L < \frac{b + c}{1 - \mu} < \Delta^H \), \( \bar{\pi}^\text{resell} (\Delta) \) has two interior peak points:

\[
\begin{align*}
\max_{\Delta \leq \frac{b + c}{1 - \mu}} \bar{\pi}^\text{resell} (\Delta) &= \mu \Delta^L - K(\Delta^L) \quad \text{and} \\
\max_{\Delta > \frac{b + c}{1 - \mu}} \bar{\pi}^\text{resell} (\Delta) &= \Delta^H - b - c - K(\Delta^H),
\end{align*}
\]

where \( \bar{\pi}^\text{resell} (\Delta^L) > \bar{\pi}^\text{resell} (\Delta^H) \) if and only if

\[
\frac{b + c}{1 - \mu} > \frac{\Delta^H - \mu \Delta^L - (K(\Delta^H) - K(\Delta^L))}{1 - \mu} \equiv \bar{\Delta}.
\]
It is straightforward to verify that \( \hat{\Delta} \in [\Delta^L, \Delta^H] \), using that \( \Delta^L = \arg \max \{ \mu \Delta - K(\Delta) \} \) and \( \Delta^H = \arg \max \{ \Delta - K(\Delta) \} \). Note if \( \frac{b+c}{\mu} = \Delta \), then \( \pi^{\text{resell}}(\Delta^L) = \pi^{\text{resell}}(\Delta^H) \) so that \( S \) is indifferent, in which case our equilibrium selection rule implies that the equilibrium with innovation \( \Delta^L \) is selected.

### 7.2.2 Proof of Proposition 7

We first formally state the equilibrium of the pricing subgame, conditioned on \( S \) participating.

**Lemma 9** In stage 3 of the dual mode with imitation. For each given \( (\tau, \Delta) \):

- If \( \tau > b + \mu \Delta \), in the mixed-strategy equilibrium, \( p_m^* \) is distributed according to c.d.f \( F_m \) with support \([c + \mu \Delta + b, \bar{p}_m]\), where \( \bar{p}_m = c + \min \{ \tau, \Delta + b \} \) and

\[
F_m(p_m^*) = \begin{cases} 
\frac{1}{1-\mu} \left( 1 - \frac{\mu \Delta}{\bar{p}_m - b - c} \right) & \text{for } p_m^* \in [c + \mu \Delta + b, \bar{p}_m), \\
1 & \text{for } p_m^* \geq \bar{p}_m 
\end{cases}
\]

\( p_m^* \) is distributed according to c.d.f \( F_o \) with support \([c + \mu \Delta, \bar{p}_m - b] \cup \{ c + \Delta \} \), where

\[
F_o(p_o^*) = \begin{cases} 
1 - \left( \frac{c + b + \mu \Delta}{\bar{p}_o^* + b} \right) & \text{for } p_o^* \in [c + \mu \Delta, \bar{p}_m - b) \\
1 - \left( \frac{c + b + \mu \Delta}{\bar{p}_o^*} \right) & \text{for } p_o^* \in (\bar{p}_m - b, c + \Delta) \\
1 & \text{for } p_o^* \geq c + \Delta 
\end{cases}
\]

while \( p_i^* \geq c + \tau \). The equilibrium profits are \( \Pi = (c + b + \mu \Delta)(1 - \mu) \) and \( \pi = \mu \Delta \).

- If \( \tau \leq b + \mu \Delta \), in the equilibrium, \( p_o^* = c + \Delta \) and \( p_m^* = p_i^* = c + \tau \). All regular consumers purchase from \( M \) if \( c > 0 \) and purchase from \( S \) inside if \( c = 0 \).\(^{20}\) The equilibrium profits are \( \Pi = (c + \tau)(1 - \mu) \) and \( \pi = \mu \Delta \).

**Proof.** Consider \( \tau \leq b + \mu \Delta \). Given \( p_m^* = c + \tau \leq c + b + \mu \Delta \), \( S \) has no incentive to deviate to a low outside price to attract all consumers to the direct channel because the profit from doing so is \( \tau - b \), which is lower than its equilibrium profit \( \mu \Delta \). Likewise, \( S \) has no incentive to deviate by changing \( p_i \) (or changing both prices together) because any \( p_i < p_i^* = c + \tau \) leads to a loss, while any \( p_i > p_i^* \) does not change its profit. Meanwhile, \( M \) has no incentive to deviate given the constraint \( p_m \leq p_i^* = c + \tau \), and \( p_m^* = c + \tau \geq 0 \) so \( M \) is profitable. Finally, any equilibrium with \( p_i^* < c + \tau \) and \( p_o^* = c + \Delta \) is ruled out because such pairs of prices are weakly dominated by \( p_i^* = c + \tau \) and \( p_o^* = c + \Delta \).

When \( \tau > b + \mu \Delta \), the pure-strategy equilibrium stated above does not exist because \( S \) can profitably deviate by attracting all consumers to the outside channel. In this case, there is no pure-strategy equilibrium in the pricing subgame. Suppose to the contrary, there is an equilibrium with \( S \) setting a deterministic price \( p_o^* \). If \( p_o^* = c + \Delta \), then, \( M \) must optimally set \( p_m^* = c + b \) (if \( r = 1 \)) or \( p_m^* = c + b - \epsilon \) for \( \epsilon > 0 \) small (if \( r < 1 \)) to attract all regular consumers. In both cases, given \( \Delta > 0 \) and \( c + b > 0 \), \( S \) can profitably deviate by undercutting with \( p_o = p_m^* - \epsilon \) to attract all regular consumers. If \( p_o^* < c + \Delta \) and \( S \) is not attracting any regular consumers, then \( S \) can deviate from \( p_o^* \) by setting \( p_o = c + \Delta \), earning a higher margin selling only to direct consumers. Suppose instead \( p_o^* < c + \Delta \) and \( S \) is attracting some regular consumers. Then for all \( p_o^* \geq c \), \( M \) can profitably undercut by setting \( p_m^* = c + b - \epsilon > 0 \) to attract all regular consumers. Combining all cases, we conclude there is no pure-strategy equilibrium when \( \tau > b + \mu \Delta \).

We now verify the mixed-strategy equilibrium stated in the statement of the lemma. The c.d.f of \( p_m^* \), \( F_m \), is such that \( S \) is indifferent for all \( p_m^* \in [c + \mu \Delta, \bar{p}_m - b] \cup \{ c + \Delta \} \), where \( \bar{p}_m = c + \min \{ \tau, \Delta + b \} \).

\(^{20}\)This follows from the tie-breaking rules stated in Section 2. Nonetheless, the equilibrium profit is independent of the tie-breaking rule.
Later, we will verify that \( p^n_m \) has no mass point at \( p^*_m = c + \Delta + b \), and so when \( S \) sets \( p^*_o = c + \Delta \) it attracts only direct consumers and obtains profit \( \mu \Delta \). Therefore, the indifference condition is

\[
(p^*_o - c) (\mu + (1 - \mu) (1 - F_m (p^*_o + b))) = \mu \Delta.
\]

Letting \( p^*_m = p^*_o + b \), and rearranging, we get

\[
F_m (p^*_m) = \frac{1}{1 - \mu} \left( 1 - \frac{\mu \Delta}{p^*_m - b - c} \right),
\]

which is exactly the cdf stated in the proposition. Note \( F_m (c + b + \mu \Delta) = 0 \) and

\[
\lim_{p^*_m \to p_m} F_m (p^*_m) = \frac{1}{1 - \mu} \left( 1 - \frac{\mu \Delta}{\min \{\tau, \Delta + b\} - b} \right).
\]

If \( \tau \geq \Delta + b \) then (7.8) equals one, so \( F_m \) has no mass point at \( p^*_m = c + \Delta + b \); otherwise if \( \tau \in (b + \mu \Delta, b + \Delta) \), then (7.8) equals \( \frac{1}{1 - \mu} \left( 1 - \frac{\mu \Delta}{\tau - \mu} \right) \in (0, 1) \), in which case \( F_m \) has a mass point \( \Pr (p^*_m = c + \tau) \) at \( p^*_m = c + \tau < c + \Delta + b \).

The cdf of \( p^*_o \), \( F_o \), is such that \( M \) is indifferent for all \( p^*_m \in [c + \mu \Delta + b, \bar{p}_m] \). Notice when \( p^*_m = c + \mu \Delta + b \), \( M \) attracts regular consumers with probability one and obtains profit \( (c + b + \mu \Delta) (1 - \mu) \). Therefore, the indifference condition is

\[
p^*_m (1 - \mu) (1 - F_o (p^*_m - b)) = (c + b + \mu \Delta) (1 - \mu).
\]

Letting \( p^*_o = p^*_m - b \), and rearranging, we get

\[
F_o (p^*_o) = 1 - \left( \frac{c + b + \mu \Delta}{p^*_o + b} \right).
\]

Notice \( F_o (c + \mu \Delta) = 0 \) and

\[
\lim_{p^*_o \to p_m - b} F_o (p^*_o) = 1 - \frac{c + b + \mu \Delta}{c + \min \{\tau, \Delta + b\}} \in (0, 1).
\]

Hence, \( F_o \) has a mass point \( \Pr (p^*_o = \bar{p}_m - b) = \frac{c + b + \mu \Delta}{c + \min \{\tau, \Delta + b\}} \). This is exactly the cdf stated in the proposition.

Finally, we check that no player can profitably deviate from outside of its support in the stated mixed strategy equilibrium. For \( S \), any \( p_o < c + \mu \Delta \) earns profit strictly less than \( \mu \Delta \) even if it attracts all consumers, hence it is strictly dominated by \( p_o = c + \Delta \); any \( p_o > c + \Delta \) attracts no consumer due to the existence of fringe sellers; while any \( p_o \in (c + \tau - b, c + \Delta) \) does not attract any regular consumers. \( S \) has no incentive to deviate by changing \( p_i \) (or changing both prices together) because any \( p_i < p^*_i = c + \tau \) leads to a loss, while any \( p_i > p^*_i \) does not change its profit. For \( M \), any \( p_m < c + b + \mu \Delta \) is weakly dominated by \( p_m = c + b + \mu \Delta \) given that \( M \) already attracts regular consumers with probability one at this price; while any \( p_m \) does not attract any consumer due to the existence of \( S \) in the marketplace.

Finally, note this mixed-strategy equilibrium exists as long as \( \tau \geq \mu \Delta \). If \( \tau < b + \mu \Delta \), the support for \( p^*_m \), that is, \([c + \mu \Delta + b, c + \tau] \) becomes an empty set, i.e., there is no \( p_m \) that can make \( S \) indifferent between setting \( c + \Delta \) and prices that are lower.

We can now derive \( S \)'s innovation decision in stage 2. For \( \Delta \leq \frac{b + c}{1 - \mu} \), \( S \)'s non-participation profit is \( \Delta \mu \), which is the same as its participation profit with imitation, so \( S \) breaks the tie in favor of participating. For \( \Delta > \frac{b + c}{1 - \mu} \), \( S \)'s non-participation profit is \( \Delta - b - c \), which is higher than its participation profit, and so it does not participate (we denote this outcome as NP). Combining both cases, \( S \)'s expected profit in
stage 2 is

\[ a_{dual}(\Delta) = \begin{cases} 
\mu \Delta - K(\Delta) \ (RE) & \text{for } \Delta \leq \frac{b+c}{1-p} \\
\Delta - b - c - K(\Delta) \ (NP) & \text{for } \Delta > \frac{b+c}{1-p}.
\end{cases} \]

This function is the same as the one in the case of the reseller mode. Therefore, the following observation follows immediately from Lemma 4:

- suppose \( \Delta > \frac{b+c}{1-p} \): \( S \) sets \( \Delta = \Delta^H \) and does not participate. All regular consumers buy from \( S \) directly. The equilibrium profits are \( \Pi = 0 \) and \( \pi = \Delta^H - b - c - K(\Delta^H) \).
- suppose \( \Delta \leq \frac{b+c}{1-p} \): \( S \) sets \( \Delta = \Delta^L \) and participates. All regular consumers either buy from \( M \) or buy directly from \( S \), so \( S \) does not sell on \( M \) in equilibrium. The equilibrium profits are \( \Pi = (1 - \mu) \left(c + \min \{\tau, b + \mu \Delta^L\}\right) \) and \( \pi = \mu \Delta^L - K(\Delta^L) \).

Then, the overall equilibrium in dual mode can be derived as stated in Proposition 7. Specifically, given that \( \tau \) does not influence participation, \( M \) sets \( \tau_{dual} = b + \mu \Delta^L \) to maximize its profit in the case where \( S \) participates (i.e. Lemma 9). The complete equilibrium characterization then follows from Lemma 8 (if \( S \) does not participate) and Lemma 9 (if \( S \) participates). This proves Proposition 7.

### 7.2.3 Proof of Proposition 8

We focus on \( \Delta \leq \frac{b+c}{1-p} \) in what follows. In this case, \( \Pi^{\text{dual}} = (1 - \mu) \left(b + c + \mu \Delta^L\right) \) which is higher than \( \Pi^{\text{resell}} = (1 - \mu)(b + c - (1 - \mu)\Delta^L) \) and \( \Pi^{\text{market}} = (1 - \mu)b \). Next, \( \Delta^{\text{dual}} = \Delta^{\text{resell}} = \Delta^L < \Delta^H = \Delta^{\text{market}} \). For welfare:

\[ W^{\text{market}} = v + \Delta^H + (1 - \mu)b - c - K(\Delta^H) \]
\[ W^{\text{dual}} = v + \Delta^L + (1 - \mu)b - c - K(\Delta^L), \]

given that \( M \) sells to all regular consumers in dual mode. Therefore, \( W^{\text{market}} > W^{\text{dual}} \) if and only if \( \Psi \equiv \Delta^H - K(\Delta^H) - (\Delta^L - K(\Delta^L)) > c(1 - \mu) \). Meanwhile \( W^{\text{dual}} > W^{\text{resell}} \) follows from the baseline model (since these two modes have the same \( \Delta \)). In the dual mode, \( p^*_n = c + \tau^{\text{dual}} = c + \mu \Delta^L + b \), so when \( \Delta^L \geq \frac{c}{1-p} \), we have \( CS^{\text{dual}}_{\text{regular}} = v - c + (1 - \mu) \Delta^L > CS^{\text{market}}_{\text{regular}} = v - c \). When instead \( \Delta^L \leq \frac{c}{1-p} \), the distribution support \( p^*_n \in [c + \mu \Delta^L, c + \Delta^L] \) in reseller mode implies \( CS^{\text{resell}}_{\text{regular}} < v - c + (1 - \mu) \Delta^L \), so it follows that \( CS^{\text{dual}}_{\text{regular}} > CS^{\text{resell}}_{\text{regular}} \). Meanwhile \( p^*_n = c + \Delta \) in both the marketplace mode and the dual mode, so \( CS^{\text{dual}}_{\text{direct}} = CS^{\text{dual}}_{\text{direct}} = v - c < CS^{\text{resell}}_{\text{direct}} \). It follows that \( CS^{\text{dual}} > CS^{\text{market}} \). Next,

\[ CS^{\text{resell}} = W^{\text{resell}} - \Pi^{\text{resell}} - \pi^{\text{resell}} = v - c + (1 - \mu)^2 \Delta^L + (1 - \mu)(1 - \eta)(\Delta^L - b - c), \]

while

\[ CS^{\text{dual}} = (1 - \mu)CS^{\text{dual}}_{\text{regular}} + \mu CS^{\text{dual}}_{\text{direct}} = v - c + (1 - \mu)^2 \Delta^L. \]

So \( CS^{\text{dual}} \leq CS^{\text{resell}} \) if and only if \( \Delta^L - b - c \geq 0 \).

### 7.2.4 Proof of Proposition 9

In this proof, we use superscript \( dual (I) \) to denote the equilibrium of the dual mode with imitation, and \( dual (NI) \) to denote the equilibrium of the dual mode without imitation. In Section B.4 of the Online Appendix, we derive the equilibrium in the dual mode without imitation:

- \( M \) sets \( \tau^{\text{dual}(NI)} = b + \mu \min \left\{ \frac{b+c}{1-p}, \Delta \right\} \), \( S \) participates and chooses innovation level \( \Delta^H \).
- If \( \Delta \geq \frac{b+c}{1-p} \), the equilibrium prices are \( p^*_n = c + \Delta^H \), \( p^*_i = \Delta^H \), and \( p^*_m = 0 \).
• If $\bar{\Delta} \leq \frac{b+c}{1-\mu}$, the equilibrium prices are $p^*_o = c + \Delta^H$, $p^*_i = c + \tau_{\text{dual}(NI)} + \Delta^H - \bar{\Delta}$, and $p^*_m = c + \tau_{\text{dual}} - \bar{\Delta}$.

• All regular consumers buy from $S$ on $M$ and direct consumers buy directly.

• The equilibrium profits are $\Pi_{\text{dual}(NI)} = \tau_{\text{dual}(NI)} (1-\mu)$ and $\pi_{\text{dual}(NI)} = \max \{ \Delta^H - b - c, \Delta^H - \bar{\Delta}(1-\mu) \} - K(\Delta^H)$.

When $\bar{\Delta} > \frac{b+c}{1-\mu}$, the profit from the dual mode without imitation is $\Pi_{\text{dual}(NI)} = b + c \mu \geq b (1-\mu) = \Pi_{\text{market}}$. So the ban on imitation means $M$ switches from the marketplace mode to the dual mode without imitation. We have

$$ CS_{\text{dual}(NI)}^{\text{regular}} = v + \Delta^H + b - \Delta^H = v + b > v - c = CS_{\text{market}} $$

$$ CS_{\text{dual}(NI)}^{\text{direct}} = CS_{\text{market}} = v - c $$

$$ W_{\text{dual}(NI)} = W_{\text{market}} = v - c + \Delta^H + (1-\mu) b - K(\Delta^H). $$

Finally, $\pi_{\text{dual}(NI)} = \Delta^H - K(\Delta^H) - b - c < \Delta^H - K(\Delta^H) = \pi_{\text{market}}$.

For $\Delta \leq \frac{b+c}{1-\mu}$, profit from the dual mode without imitation is $\Pi_{\text{dual}(NI)} = (1-\mu) \{ b + \mu \bar{\Delta} \} > \max \{ \Pi_{\text{market}}, \Pi_{\text{resell}} \} = \max \{ (1-\mu) b, (1-\mu) \{ b + c - (1-\mu) \Delta^L \} \}$, where the inequality utilizes $\mu \bar{\Delta} > \mu \Delta^L > c - (1-\mu) \Delta^L$. So the ban on imitation means $M$ continues with dual mode, but without imitation. Note $W_{\text{dual}(NI)} = W_{\text{market}}$, and so $W_{\text{dual}(NI)} > W_{\text{dual}(I)}$ by Proposition 8. Meanwhile, $CS_{\text{dual}(NI)}^{\text{direct}} = CS_{\text{dual}(I)}^{\text{direct}} = v - c$ and $CS_{\text{dual}(NI)}^{\text{regular}} = v - c + (1-\mu) \bar{\Delta} > v - c + (1-\mu) \Delta^L = CS_{\text{dual}(I)}$. Combining both comparisons yields $CS_{\text{dual}(NI)} > CS_{\text{dual}(I)}$. Next,

$$ \pi_{\text{dual}(NI)} = \max \{ \Delta^H - b - c, \Delta^H - (1-\mu) \bar{\Delta} \} - K(\Delta^H) $$

$$ = \Delta^H - (1-\mu) \bar{\Delta} - K(\Delta^H) $$

$$ = \mu \Delta^L - K(\Delta^L) = \pi_{\text{dual}(I)}. $$

Finally, $\Pi_{\text{dual}(I)} = (1-\mu) \{ c + b + \mu \Delta^L \}$, which is higher than $\Pi_{\text{dual}(NI)} = (b + \mu \bar{\Delta}) (1-\mu)$ if and only if

$$ b + \frac{\mu}{1-\mu} \{ \Delta^H - \Delta^L \mu - (K(\Delta^H) - K(\Delta^L)) \} \leq c + b + \mu \Delta^L $$

$$ \iff \Psi \equiv \Delta^H - \Delta^L - (K(\Delta^H) - K(\Delta^L)) \leq c \left( \frac{1-\mu}{\mu} \right). $$

### 7.3 Proofs for Section 4.2

#### 7.3.1 Proof of Proposition 10

For $\tau < b + c$, any $p_m \in (\tau, b + c]$ ensures $M$ is recommended (by itself in the recommendation stage), and it does best setting $p_m = b + c$, i.e. the highest price that avoid consumers switching to the fringe suppliers’ product in the direct channel. Any deviation of setting $p_m \leq \tau$ means $M$ earns at most the margin $\tau$. Meanwhile, given $p_m^* = b + c$, $S$ knows its inside product is never recommended and so sets $p_o^* = c + \Delta$ to focus on exploiting direct consumers.

For $\tau > \Delta + c$, recall if $S$ wants to make any sales inside, its inside price must be at most $p_i = b + c + \Delta$ due to the existence of fringe suppliers’ direct product and so, $p_i - c - \tau < 0$. This means making sales inside is never profitable for $S$, so for any $p_i^*$ it sets, it would also set $p_i^* > p_o^* + b$ to ensure it does not make sales inside. Given this, the recommendation rule means $M$ never recommends $S$. Hence, we have $p_m^* = b + c$ and $p_o^* = c + \Delta$ as in the previous paragraph.

For $\tau \in [b + c, \Delta + c]$, $M$ now recommends $S$’s product for all $p_m^* \leq b + c$ as long as $p_i^* > p_o^* + b$, due to the higher margin. This means if $S$ sets $p_i^* = c + \Delta + b$ and $p_o^* = c + \Delta$, its profit is $\mu \Delta + (\Delta + b - \tau) (1-\mu)$;
7.3.2 Proof of Proposition 11

We first formally state and prove the equilibrium in marketplace mode

**Lemma 10** (Marketplace with steering) In the unique equilibrium outcome, M sets \( \tau^{market} = b \), S sets \( p^*_m = c + \Delta + b \) and \( p^*_o = c + \Delta \). All regular consumers purchase from S through the marketplace. The equilibrium profits of M and S are \( \Pi^{market} = b(1 - \mu) \) and \( \pi^{market} = \Delta \) respectively.

**Proof.** Fix \( \tau \), and suppose S has joined the marketplace. Recall that fringe suppliers always set inside and outside prices at \( c + \tau \) and \( c \). For \( \tau > b \), M’s recommendation rule given in the main text means it always recommends S’s product over a fringe supplier’s product. If S sets prices to keep regular consumers in the marketplace, its highest profit is \( \Delta - (b - \tau)(1 - \mu) \) which it obtains by setting \( p_o = c + \Delta \) and \( p_i = c + \Delta + b \); if S sets prices to attract regular consumers to the direct channel, it earns \( \Delta \) by setting \( p_o = c + \Delta \) and \( p_i > c + \Delta + b \). The latter is clearly more profitable, implying \( \Pi = 0 \) in the equilibrium of the subgame following \( \tau > b \). For \( \tau \leq b \), S is recommended only when it set \( p_i \leq p_o + b \). Adhering to this constraint, S’s highest profit is \( \Delta \) which it obtains by setting \( p_o = c + \Delta \) and \( p_i = c + \Delta + \tau \). The deviation profit from setting \( p_i > p_o + b \) is at most \( \Delta \), and so is unprofitable. For all \( \tau \), S’s non-participation profit in the equilibrium of the subgame is at least \( \Delta \), so it always participates. Moving to the commission-setting stage, it is clear that M does best setting \( \tau = b \).

For the reseller mode, as stated in the main text the equilibrium has \( p^*_m = b + c \) and \( p^*_o = c + \Delta \), with profits \( \Pi^{resell} = (b + c)(1 - \mu) \) and \( \pi^{resell} = \mu \Delta \). Clearly, \( \Pi^{resell} \geq \Pi^{market} \), where the inequality holds whenever \( c > 0 \), so M chooses the reseller mode after the ban. The remaining results follow from a direct comparison with the equilibrium in Proposition 10.

7.3.3 Proof of Proposition 12

If objective recommendations are required (i.e. steering is banned), M always recommends the highest-surplus product to regular consumers in the marketplace and dual modes. Therefore, the demand is as if all regular consumers are fully informed of all available offers, i.e. the baseline model. As for the reseller mode, given that M does not recommend S as it does not host S, the equilibrium stated in the main text continues to hold, i.e. \( p^*_m = b + c \) and \( p^*_o = c + \Delta \). Letting the subscript obj denote the objective recommendation, the following lemma summarizes the equilibrium in each mode:

- The equilibrium characterization of the marketplace mode (Proposition 1) and dual mode (Proposition 3) carry over.

- The equilibrium of the reseller mode: \( p^*_m = c + b \), \( p^*_o = c + \Delta \), \( \Pi^{resell} = (1 - \mu)(b + c) \), and \( \pi^{resell} = \mu \Delta \).

Clearly, \( \Pi^{market}_{obj} = b(1 - \mu) \leq \Pi^{resell}_{obj} \), so M never chooses the marketplace mode after steering is banned. If \( \min \left\{ \frac{(b+c)\mu}{1-\mu}, \mu \Delta \right\} \leq c \), then

\[
\Pi^{dual}_{obj} = \left( b + \mu \min \left\{ \frac{b+c}{1-\mu}, \Delta \right\} \right) (1 - \mu) \leq \Pi^{resell}_{obj},
\]

so M switches to the reseller mode post-intervention. The comparison is the same as in Proposition 11. If \( \min \left\{ \frac{(b+c)\mu}{1-\mu}, \mu \Delta \right\} \geq c \), then \( \Pi^{dual}_{obj} \geq \Pi^{resell}_{obj} \). Post-intervention, M switches to dual mode without
steering, where $CS_{\text{regular}}$ increases from $v$ pre-intervention to $v + \Delta - \mu \min \left\{ \frac{b + c}{1 - \mu} , \Delta \right\}$ post-intervention, $\pi$ increases from $\mu \Delta$ to $\max \{ \mu \Delta, \Delta - b - c \}$, and $\Pi$ decreases from $(b + \Delta)(1 - \mu)$ to $b + \mu \min \left\{ \frac{b + c}{1 - \mu} , \Delta \right\} (1 - \mu)$. Meanwhile $W$ and $CS_{\text{direct}}$ are unchanged.

References


