Exclusion via non-exclusive contracts*

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Abstract

We establish that non-linear vertical contracts can allow an incumbent to exclude an upstream rival in a setting that does not rely on the exclusivity of the incumbent’s contracts with downstream firms or any limits on distribution channels available to the incumbent or rival. The optimal contract we describe is a three-part quantity discounting contract that involves the payment of an allowance to a downstream distributor and a marginal wholesale price below the incumbent’s marginal cost for sufficiently large quantities. The optimal contract is robust to allowing parties to renegotiate contracts in case of entry.

Keywords Exclusion, Entry, Quantity discounting, Slotting allowances

JEL Classification L42, C72

1 Introduction

Intel announced on November 12, 2009 that it will pay AMD $1.25 billion towards the settlement of all pending legal issues between them. The settlement put an end to antitrust disputes between the two companies during which AMD persistently alleged that Intel had attempted to foreclose competition and broaden its monopoly power by engaging in illegal pricing strategies. One of AMD’s main complaints was that Intel offered computer-makers

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substantial discounts for large purchases through an all-units quantity discounting scheme, so that in some cases the price of incremental purchases to computer-makers was below Intel’s own marginal cost and that “Intel’s practices exacerbate normal impediments to entry and expansion”. As an example, Intel allegedly paid Dell billions of dollars in a collaboration which involved Intel submitting below-cost bids to Dell in strategic contests against AMD’s products. Intel’s behavior raises the question of how an all-units quantity discounting scheme (or other similar non-linear contracts involving below-cost wholesale prices) can be profitably used by a dominant upstream firm with downstream firms to foreclose upstream competition. This paper offers an answer.

We consider a Bertrand environment in which the incumbent offers a contract with a downstream firm to keep out a more efficient entrant. A key feature of the optimal vertical contract we describe is that the wholesale tariff involves quantity discounting or declining marginal prices with the lowest wholesale price below the incumbent’s marginal costs. For low levels of purchases, the downstream firm purchases at a wholesale price set above the incumbent’s marginal cost, thereby providing a way for the incumbent to extract the profit of the downstream firm in case of no-entry. For purchases beyond some break-even quantity, the downstream firm purchases at a wholesale price set below the incumbent’s marginal cost, thereby ensuring that in the face of entry the downstream firm will want to compete aggressively, constraining the rival’s price without actually needing to sell anything itself. We show a three-part all-units quantity discounting contract (two linear parts and an allowance) is the simplest optimal contract for exclusion.

Our contract does away with the need for the incumbent to write an exclusive contract with the downstream firm or a contract that explicitly conditions on entry. Such contracts would raise standard antitrust concerns. Since we assume the entrant has a lower cost than the incumbent, the optimal contract has to leave the downstream firm with a rent sufficient to ensure the entrant cannot profitably attract it away from the dominant upstream firm. To deter entry, this rent has to be paid to the downstream firm irrespective of the quantity purchased, i.e., it represents an allowance, known as a slotting allowance in the context of retailing (Foros and Kind 2008). Another nice feature of the proposed optimal contract is that it is renegotiation-proof,

\footnote{Civil Action No. 05-441-JJF, US District Court (Delaware), filed 27 June 2005, available at http://www.ded.uscourts.gov/sites/default/files/Chambers/OtherOpinions/JJF/Opinions/Recent/Sep2006/05-441.pdf.}

\footnote{There are two theories of slotting allowances (Bloom et al. 2000). One considers them a tool for improving distribution efficiency. The other, which is consistent with our theory, proposes that slotting allowances enhance market power and damage competition.}
thereby ensuring the incumbent can credibly exclude the rival even when the contract can be renegotiated.

From a policy viewpoint, our theory provides a particular setting which supports the use of a predatory pricing standard for dealing with wholesale price discounts combined with allowances. In our theory, marginal wholesale prices must fall below a firm’s own marginal cost for sufficiently large quantities in order to deter entry. Also, there must be evidence of payments from the incumbent to downstream firms. Where there is no efficiency justification for below-cost wholesale prices, such contracts are therefore anticompetitive. Forcing the incumbent to raise its marginal wholesale price to be no less than marginal cost will encourage efficient entry and increase welfare in our setting. More generally, our paper provides a rationale for competition authorities to be concerned about vertical contracts which involve declining marginal wholesale prices which become very low for high quantities, especially when they are accompanied by certain types of allowances.

The rest of the paper proceeds as follows. Section 2 discusses the relationship between our work and the existing literature. An illustrative example is given in Section 3. The basic model setup is given in Section 4. Our main findings are derived in Section 5. Section 6 then considers several extensions, highlighting the role played by the various key assumptions. Section 7 briefly concludes.

2 Literature review

Our paper shows that more sophisticated tariffs may lead to implicit exclusivity in a vertical setting which in turn leads to credible entry deterrence of a more efficient entrant. Thus, the paper contributes to two rapidly growing strands in the literature: on the use of sophisticated pricing to mimic exclusivity and the use of exclusive contracts between upstream and downstream firms to exclude potential entrants.

The first strand of the literature has recently explored the ability of sophisticated pricing: bundled rebates, market-share discounts, and non-linear pricing such as all-units discounts, to have exclusionary effects. As Tom et al. (2000, p.615) notes “The traditional analysis governing exclusive dealing

Note however, that it might be difficult to detect such practices. In a complaint filed by the New York Attorney General (page 30 of http://www.intel.com/pressroom/legal/docs/NY_AG_v_Intel_COMPLAINT.pdf) it is said that: “...throughout this period, top executives at both companies took care that the dealings between them were kept secret. Although billions of dollars in rebate payments flowed from Intel to Dell during the period 2002-2006, there was no formal documentation of the secret agreements which led to them.”
arrangements has focused on a manufacturer’s requirement that its distributors deal exclusively with it. In recent years, however, some manufacturers have begun to use subtler arrangements in which incentives replace requirements ...”.

The papers closest to ours have been concerned with whether contracts with quantity discounts or fixed payments can deter entry by achieving similar exclusive ends as pure exclusive deals. Aghion and Bolton (1987) and Erutku (2006) show how fixed payments can be profitably used by an incumbent manufacturer to indirectly achieve exclusivity and deter a more efficient rival. Their mechanisms are similar to slotting allowances we use in our paper. However, both papers use some form of commitment from the part of downstream firm. In Aghion and Bolton (1987) the downstream firm has to pay liquidated damages to the incumbent in case of switching to the entrant. Erutku (2006) assumes that the downstream firm commits to exclusivity when accepting the contract. In our paper the downstream firm does not commit to exclusivity; the implicit commitment comes from the form of the contract ex-post, i.e., in the market game stage. Feess and Wohlschlegel (2010) show that all-unit discounts do not fully allow the incumbent seller to disentangle the implementation of the desired quantity from the surplus division with a buyer. They show that this leads to inefficiencies if and only if the buyer has a high outside option. Both these recent papers use some forms of two-part contracts. Our paper adds to this body of work by explaining how quantity discounts combined with an allowance allow an incumbent to exclude a rival, although without making its contract exclusive the incumbent must still sacrifice some monopoly profit to keep the downstream firm from contracting with the rival.

Majumdar and Shaffer (2009) consider a dominant firm and competitive fringe that supply substitute goods to a retailer that has private information about demand. They show that it is profitable for the dominant firm to condition its payment on how much the retailer buys from the fringe. The dominant firm thereby creates countervailing incentives for the retailer and, in some cases, is able to obtain the full-information outcome (unlike in standard screening models, where the agent earns an information rent in the high-demand state and output is distorted in the low-demand state). Also in a private information environment, Calzolari and Denicolò (2011a) show that market-share discounts may reduce information rents. Marx and Shaffer (2007) show that three-part tariffs lead to exclusion in a setting with one upstream firm and two retailers. Chao (2011) considers the welfare and competition effects of using three-part tariffs when retailers are local monopolies. Calzolari and Denicolò (2011b) consider the competition and welfare effects of non-linear contracts with end users who are privately informed about de-
mand. Compared to these existing works, our paper differs in two respects. First, much of this literature has focused on contracts with end-users (or a retailer representing end-users’ interests) which is not the case in reality. The contracts we consider are between upstream and downstream firms. Second, our paper provides a predatory-type purpose for quantity discounting. Whether or not volume discounts also include exclusivity provisions, their purpose in our theory is to commit the incumbent to price below cost where this is necessary to drive out the (potential) rival. They are not simply replicating exclusive deals that are designed to prevent distributors sourcing inputs from competing manufacturers so as to block (or soften the effect of) the manufacturers’ entry.

The second strand of the literature has studied exclusive dealing between upstream and downstream firms in which the exclusive contract involves a price commitment. In this literature, the incumbent uses exclusive contracts as a barrier to entry. Building on Rasmusen et al. (1991) and Segal and Whinston (2000), Simpson and Wickelgren (2001) and Fumagalli and Motta (2006) show that the incumbent manufacturer can commit to a low wholesale price (to exclude a rival), extracting the surplus enjoyed by downstream firms paying this low wholesale price through an upfront fee which it receives when the exclusive deal is signed. This enables the incumbent to exclude a rival although at a low price, meaning renegotiation would always be profitable. Our paper shows that through the use of more sophisticated contracts, the reliance on exclusive contracts or contracts that cannot be renegotiated is no longer needed. Moreover, our exclusion result is obtained despite there being no limitation on distribution options for the upstream firms. To do this the incumbent gives up some of its monopoly profit. We show in the extensions’ section that allowing the incumbent to also use exclusive deals along with all-units quantity discounting tariffs, the incumbent can restore its full monopoly profits in the face of a more efficient rival.

Another mechanism to deter entry that has been studied in the literature is the use of divisionalization, following the work of Schwartz and Thompson (1986). They establish that an incumbent may deter an equally efficient rival by (costlessly) creating independent competing divisions that emulate the behavior of the rival and therefore do not allow it to recover its fixed cost of entry. Their mechanism is akin to delegating production to competing downstream firms with a vertical contract in which the wholesale price is fixed at the incumbent’s marginal cost of production (and profits recovered through a profit sharing agreement). In our setting, such an approach would not work given we assume the rival is more efficient. Nevertheless, the idea of committing downstream divisions or firms to be more aggressive to deter entry is similar.
Finally, our theory relates to the literature on contingent contracts and delegation. Katz (2006) provides a nice analysis of the power of contingent vertical contracts in delegation games. In a framework where contracts are directly contingent on the rival’s contract he obtains a “folk theorem” result.\footnote{Similarly if we allow contracts that depend explicitly on entrant’s decision, i.e., to be entry contingent, then as Judd, Fershtman and Kalai (1991) proved, any individually rational outcome can be implemented.} The mechanism at work in our paper, that the optimal non-linear contract allows the incumbent to indirectly condition its contract on entry, is similar to the taxation principle in common agency (see Martimort and Stole, 2002) where two principals compete for one agent through non-linear schedules. As in our entry deterrence framework, the “punishment” to the other principal (e.g. the entrant) is carried out through the agent (e.g. the incumbent’s retailer). Similarly, Fershtman and Judd (1987) introduced the idea of strategic manipulation of agents’ incentives in delegation games. In our setting the incumbent strategically chooses to delegate pricing decision to the downstream firm.\footnote{For recent developments see Gerratana and Koçkesen (2011).}

3 An example

In this section we provide an example to illustrate the entry-deterring effect of non-linear contracts. We will also show that the parties do not want to renegotiate the contract.

Consider market demand $Q(P) = 2 - P$, where $P$ is the market price. The incumbent $I$ has a linear technology with unit cost $c_I = 1$. There is a potential entrant $E$ which produces the same good with the unit cost $c_E = 0.75$. The fixed cost of entry $F = 0.06$. The upstream firms $I$ and $E$ can sell directly to consumers if they wish. There is also a firm $D$ in the market, which in order to sell the good has to buy it from either $I$ or $E$. Assume that competition is in prices; the firm with the lowest posted price takes the whole market.

Assume that before the entrant enters the market, the incumbent offers $D$ the following contract $T^*(Q)$: in the market stage if $D$ decides to stay with $I$ it receives a lump-sum payment (an allowance) $L = 0.19$. After that $D$ buys any $Q \geq 0$ units from $I$ at the wholesale price 1.5 if $Q < 1.2$ and at a price 0.8 if $Q \geq 1.2$. The contract $T^*(Q)$ has a discontinuity at $Q = 1.2$; it is linear for $Q \in [0, 1.2)$ with wholesale price 1.5 per unit and wholesale price 0.8 for all units if 1.2 or more units are purchased. The wholesale part of contract (after lump-sum payment is sunk) is depicted in Figure 1. Note that $P_M = 1.5$ is the monopoly price for the incumbent and $P = 0.8$ is the

\[ Q(P) = 2 - P, \]
break-even price for the entrant (which is discussed below).

\[ \text{Figure 1: Optimal contract - example} \]

**Entry-deterrence:** Suppose that $D$ accepts the contract $T^*(Q)$ and $E$ enters and pays the cost $F$. Assume first that $D$ stays with $I$. In the subgame following entry, when $F$ is sunk, the entrant is ready to set any price equal or above $c_E = 0.75$. Since the minimal cost for $D$ (with $L$ sunk) is $P = 0.8 > 0.75$, there will be asymmetric Bertrand competition with the equilibrium price being $P = 0.8$ and $E$ (the lower cost competitor) taking the whole market. With this price $E$’s profit is $(0.8 - 0.75)Q(0.8) = 0.06 = F$, so the entry is not profitable ex-ante.\(^6\)

**Exclusion:** If $E$ enters and offers $D$ a bribe in return for revoking the contract $T^*$, then $D$ will accept this offer if it receives not less than $L = 0.19$ because it expects to obtain zero profit given the wholesale prices. Note however, that because $I$ still competes with $E$ and $D$ the maximum what $E$ can offer to $D$ is its profit under Bertrand competition with $I$ minus the cost of entry: $(c_I - c_E)Q(c_I) - F = (1 - 0.75)(1) - 0.06 = 0.19$. Therefore, if we assume that there is a very small cost involved in signing a new contract, $D$ will prefer in favor of $I$ in case of indifference. Thus, the lump-sum payment of $0.19$ prevents $D$ from switching to $E$.

**Renegotiation-proofness:** Consider the subgame in which $E$ enters. Following such entry, $I$ and $D$ do not have an incentive to renegotiate the contract. Since they can always sell nothing (if they are undercut), any new contract must lead to strictly positive profit to the pair $(I, D)$ for it be worthwhile for them to renegotiate. However, this is impossible because in order to sell a positive quantity in competition with $E$, either $I$ or $D$ has to set a price $P$ below $c_E$ which is below $I$’s cost. Thus, this contract is renegotiation-proof.

\(^6\)As usual, we assume that $E$ enters if it makes a strictly positive profit.
No-entry: When there is no entry, there are only two firms in the market: I and D. This is again a case of asymmetric Bertrand competition with I having cost $c_I$ and D having cost $P_M > c_I$. The equilibrium price in this case is $P_M$ with I obtaining the full monopoly profit (minus the allowance $L$) and D obtaining zero profit (plus the allowance $L$). Note setting a price below $P_M$ leads to the same zero profit for D. Indeed, if the price is between 0.8 and 1.5, then D’s profit is negative because it still has to pay $P_M$ for each unit. If the price is below 0.8, then D’s profit is also negative given its cost is 0.8 per unit in this case.

Even though it does not directly condition on entry, the contract $T^*$ implicitly stipulates the purchasing decision of D in case of entry. In this case D will be ready to buy the break-even quantity and pay less than the break-even price so as to make entry unprofitable. Since the offer is observable, the potential entrant will abstain from entering the market. In this case I’s profit is $(0.5)(1.5) - 0.19 = 0.56$.

4 Benchmark model

In this section we generalize the previous example. We focus on a benchmark model in which firms sell identical goods and set prices, i.e., homogenous Bertrand competition. The incumbent I faces constant marginal costs $c_I$. The potential entrant E faces lower marginal costs $c_E < c_I$ but some fixed cost of entry $F$. We assume that E enters only if it makes positive profit. Downstream firms $\{D_1, D_2, \ldots\}$ are assumed to be all identical: all with zero costs other than those arising from contracts, and all adding no additional value. To fix ideas we assume first that each firm I or E can sell by itself or through one or more downstream firm. Whichever firm sets the lower price obtains the entire market demand at that price. If firms set the same price, we assume that there is some exogenous profit-sharing rule to ensure equilibria are well defined, for example, the firm facing the lower marginal cost obtains the entire market.

Market demand $Q(P)$ is assumed to be continuous, non-negative and decreasing in price $P$. The inverse demand function is denoted $P(Q)$. We assume that the revenue function $R(Q) = P(Q)Q$ is strictly concave in $Q$. The monopoly price given any constant marginal cost $w$ is denoted

$$P_M(w) = \arg \max_P (P - w)Q(P).$$

For notational convenience, define $Q_M(w) = Q(P_M(w))$. The incumbent’s monopoly price and quantity are defined as $P_M = P_M(c_I)$ and $Q_M =$

\footnote{Several possible generalizations are discussed in Section 5.}
$Q_M(c_I)$, with corresponding monopoly profit $\Pi_M = (P_M - c_I) Q_M$. Assume $P(0) > c_I$ which ensures that if $I$ is a monopolist it will produce a positive output (and so can obtain a positive profit).

Our first key assumption is that the fixed cost of entry is not too large. 

A1. $F$ satisfies

$$0 \leq F < (c_I - c_E) Q(c_I).$$

If the cost of entry is too large, i.e. when $F \geq (c_I - c_E) Q(c_I)$, then $I$ will be able to exclude $E$ by competing directly with $E$. Thus, (1) allows us to consider the interesting case when it will always be profitable for $E$ to enter if it competes directly with $I$.

The next essential assumption states that $E$ is not too efficient. 

A2. $P_M(c_E) > c_I$ and

$$\Pi_M = (P_M - c_I) Q_M > (c_I - c_E) Q(c_I) - F.$$ 

The first part of A2 states that $E$’s cost advantage is not drastic. The second part states that its efficiency profit $(c_I - c_E) Q(c_I) - F$, the profit $E$ obtains when it competes directly with $I$ (after taking into account its entry cost), is less than $I$’s monopoly profit. In Section 5, we show both assumptions are required for exclusion.

The timing of the game is as follows:

- Stage 1 (Incumbent’s contracting) $I$ offers a contract (or contracts) to one or more downstream firms (stage 1a), which accept or not (stage 1b).

- Stage 2 (Entry) After observing $I$’s contract(s) and the acceptance decisions, $E$ can decide whether to enter the market (incurring the cost $F$).

- Stage 3 (Post-entry contracting / renegotiation) $I$ and $E$ (if it enters) simultaneously offer contracts with any subsets of downstream firms (stage 3a). Each downstream firm accepts or not its contract (stage 3b).

- Stage 4 (Market competition) In the last stage, all final contracts are observed and all firms (if they wish) set prices, and the terms of contracts are executed.

The contracting process at stage 3 is similar to one in Bernheim and Whinston (1985). We allow $E$ to offer a contract to the firm(s) which contracted with $I$ at stage 1. We assume $I$ and $E$ can commit to their vertical
contracts whereas downstream firms cannot. For example, we allow that a downstream firm can walk away from any contract which it finds unprofitable ex-post, i.e., after observing entry and even after observing the rival’s contract, by not buying anything from I. In this case a downstream firm pay nothing and receive nothing from I.

We assume upstream firms face some arbitrarily small cost of contracting and/or renegotiating contracts, so that contracts will only be offered or renegotiated if they strictly increase joint profits. In our setup, I and E cannot negotiate directly with each other, which would violate standard antitrust laws on horizontal agreements. We start with the assumption that I cannot write an exclusive contract in stage 1, but E (and I) can write exclusive contracts in stage 3. This represents the most challenging setting in which to consider exclusion of the rival. We also allow for free-disposal, that the downstream firm may buy some quantity from the upstream firm and freely dispose of it.

**Contract space.** We consider the general contract space $T$ which consists of contracts $T(Q) = L + W(Q)$, where $W(Q)$ is a total amount paid for $Q > 0$, and $L \in \mathbb{R}$ is a possible unavoidable lump-sum payment paid at stage 4. The feasible contracts depend only on the quantity downstream firms buy from respective upstream firms.\(^8\) We allow for a negative payment or allowance $L < 0$, known as a slotting allowance in the literature. The other possibility, that $L$ is an up-front fee paid at stage 1 will be discussed in Section 6. We do not impose any restriction on the total amount paid $W(Q)$.

This part of the contract represents variable cost for the downstream firm. We only require that there exist an equilibrium (possibly in mixed strategies) in the pricing game.\(^9\)

**Example 1**

a) The two-part contract $T(Q) = f + wQ$, where $f > 0$ is a fixed fee, is a special case of the class of contracts we consider. In this case if $f$ is paid irrespective of how many units of $Q$ are bought (an unavoidable fixed fee) then $L = f$ and $W(Q) = wQ$. If $f$ is paid only if $Q > 0$ (an optional fixed fee) then $L = 0$ and $W(Q) = f + wQ$.

b) Note that our definition of contracts is consistent with three-part contracts $T(Q) = S + f + wQ$ with a slotting allowance $S < 0$ and a fixed fee $f$, paid when the downstream firm buys a strictly positive quantity, as used by

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\(^8\)Apart from $E$’s possible entry, this is the only thing $I$ can directly observe. We do not allow contracts that depend explicitly on $E$’s entry decision. Making wholesale prices an explicit function of whether the rival enters may violate antitrust law. One of the points of our paper is to show such explicit dependence on entry is not necessary for exclusion.

\(^9\)It is sufficient to require only that $W(Q)$ is the set of lower-semicontinuous functions, which allows us to consider discontinuities in $W(Q)$. As Reny (1999) established, there always exists an equilibrium of the pricing subgame.
Marx and Sheffer (2007) and Rey and Whinston (2011). In our notation, \( L = S \) and \( W(Q) = f + wQ \).

Our set-up allows for \( I \) to offer a vector of contracts \( T_I \) to some subset of downstream firms.

**Definition:** An optimal contract \( T_I \) is a (vector) contract which leads to the highest payoff for the incumbent among the class of contracts \( T \).

The incumbent will contract with a downstream firm only if the contract deters entry and leads to some positive profit. Otherwise, \( I \) can save on contracting cost and achieve the same result of zero profit given the more efficient entrant will take the whole market in direct competition. An optimal contract can be a very complicated function from \( T \). An important focus of our analysis will be to find the simplest optimal contract among simple piece-wise linear marginal price schedules. The class \( T_A \) of all-units contracts consists of contracts in which marginal prices change at each increment, but the new marginal price applies to all units purchased rather than just marginal units. The widely used all-units quantity discounting contracts are just a special case of such contracts in which the marginal price declines at each increment. In Section 5, we show a similar analysis can be done with incremental-units contracts in which the marginal price applies only to the incremental units at each step.\(^{10}\)

Formally, the \( m \)-part contract \( T(Q) = L + W(Q; w, S) \in T_A^{(m)} \) is characterized by the lump-sum fee \( L \neq 0 \), the vector of marginal prices \( w = (w_1, w_2, ..., w_{m-1}) \) and the vector of price-breaks \( S = (S_1, S_2, ..., S_{m-1}) \), where \( S_1 = 0 \), such that \( T(Q) = L + w_iQ \) if \( Q \in [S_i, S_{i+1}) \). For purposes of consistency with the literature, we define all-units contracts with \( L = 0 \) and the vector of marginal prices \( w = (w_1, w_2, ..., w_{m-1}) \) as \( m-1 \)-part contracts.

# 5 Optimal contract

We define two parameters \( P \) and \( r \) which are instrumental in constructing an optimal contract. The first parameter is \( E \)'s break-even price \( P \) defined by

\[
P = \min \{ P \text{ such that } (P - c_E) Q(P) = F \}.
\]

By assumption A1, this \( P \) exists and satisfies \( c_F < P < c_I < P_M \). Indeed, (1) implies \( (P - c_E) Q(P) > F \) when \( P = c_I \) and \( (P - c_E) Q(P) < F \) when \( P = c_E \). The second parameter \( r \), the entrant’s efficiency profit, is defined by

\[
r = (c_I - c_E) Q(c_I) - F.
\]

\(^{10}\)Incremental-units and all-units contracts are discussed in Munson and Rosenblatt (1998), Kolay et al. (2004) and Wong (2011).
By (1), \( r > 0 \).

Initially, we assume that the market revenue function is non-decreasing at \( E \)'s break-even price. This is always true for constant elasticity and logit demand where the revenue function \( R(Q) \) is always increasing in \( Q \), but also for linear and exponential demand specifications provided the price elasticity of the market demand \( Q(P) \) is greater than unity (in magnitude) at \( Q(P) \).

Our goal is to find the simplest contract from the set \( T_A^{(m)} \) (i.e. with minimal \( m \)) which is optimal among all contracts from the general contract space \( T \). The next proposition characterizes the specific three-part contract that we claim is optimal.

**Proposition 1** There exists an optimal (among all \( T \in T \)) three-part contract \( T^* \) such that a) it exhibits quantity discounting and the lowest marginal wholesale price is below the incumbent’s marginal cost. b) the downstream firm obtains \( r > 0 \); c) the incumbent’s profit is \( \Pi_M - r \).

**Proof.** See Appendix

All proofs are in the appendix. To give the intuition for the result we assume for simplicity that \( I \) contracts only with one downstream firm \( D \). That this is optimal is shown in Proposition 2. The variable part \( W^*(Q) \) of the optimal contract \( T^*(Q) = -r + W^*(Q) \) is depicted in Figure 2. A lump-sum payment \( r \) is paid to \( D \) in stage 4. This contract has two marginal wholesale prices \( P_M \) and \( P \), which play the role of linear costs for \( D \) when it buys the corresponding quantity from \( I \).

\[ W^*(Q) \]
\[ Q \]

Figure 2. The variable part of the optimal three-part contract

\[ ^{11} \text{In Section 5, we will discuss how to modify } I \text{'s optimal contract when this condition does not hold.} \]
Entry: Assume first that $D$ accepts $T^*$ and assume $E$ enters in stage 2. In a market subgame in stage 4, $D$ competes with $I$, $E$ and possibly with other downstream firms that $E$ contracts with. Consider an equilibrium in this subgame. Denote by $P'$ the equilibrium price. Assume first that $P' > P$. Then $D$ can obtain a strictly positive profit by deviating to the pure strategy $P_D = P' - \varepsilon$ for some small $\varepsilon > 0$ such that $P_D > P$ and $E$ sells nothing. Thus, in the subgame, it must be that $P' \leq P$. However, if the price set by $E$ is equal to $P'$, given (3) the profit of $E$ cannot be greater than $F$. Thus, entry is not profitable.

Exclusivity: Assume now that in stage 3 $E$ has entered and that it contracts with $D$. Then in stage 4, the equilibrium price cannot be greater or equal to $c_I$ (given that $I$ competes in stage 4 and is ready to undercut any price higher than $c_I$). Then the maximum that $E$ can promise to $D$ to make entry profitable is its efficiency profit $r = (c_I - c_E)Q(c_I) - F$ minus some $\varepsilon$. However, $D$ will not accept this contract because it can obtain $r$ for sure from $I$.

Renegotiation-proofness: By the same reasoning as in Section 3, $I$ and $D$ do not want to renegotiate the contract $T^*$ in stage 3; in case entry occurs, the cost of entry $F$ is sunk and $E$ is ready to price down to its marginal cost $c_E$. Since $P > c_E$, in equilibrium $E$ must take the whole market. In this case, the joint profit of the pair $(I, D)$ in this subgame is zero. Any re-contracting between $I$ and $D$ will lead to a loss either to $I$ or to $D$ or to both.

No entry: Finally, consider the market subgame where there is no entry. It must be that the equilibrium price is $P_M$. Note that without competition from $I$ that $D$ has incentives to set the price above $P_M$. However, $I$ will always undercut any such price. Therefore, the equilibrium price is $P_M$ and $I$ appropriates the whole surplus through the contract except for the allowance $r$.

There are three instruments in the optimal contract $T^*$: two marginal prices $(P_M, P)$ and the rent $r$ paid to $D$. No instrument in the contract is redundant. The lower marginal price of $P < c_I$ that applies if at least $Q(P)$ units are purchased, ensures that $E$ does not find entry profitable when it competes by itself or through any other downstream firm(s) different from $D$. The first marginal price of $P_M$ ensures the optimal choice of quantity and price in equilibrium when there is no entry. Finally, to avoid the possibility of contracting with the entrant given the lack of exclusivity in the original contract, $D$ has to obtain a positive rent $r$.

Example 2 Consider the same setting as in Section 3 except we add the lump-sum payment $r = 0.19$ to the contract. The variable part of the contract
\( T^*(Q) \) is as before

\[
W^*(Q) = \begin{cases} 
1.5Q & \text{if } 0 \leq Q < 1.2, \\
0.8Q & \text{if } 1.2 \leq Q.
\end{cases}
\]

Note that the setup satisfies assumptions 1 and 2: \( P = 0.8 < c_I = 1 \) and \( \Pi_M = 0.25 > r = 0.19. \)

In Proposition 1, we constructed one particular optimal contract. Combining the results, we establish that any optimal contract from the contract space \( T \) has similar properties. In particular, the optimal contract will involve only one downstream firm. This firm will be paid a strictly positive allowance.

**Proposition 2** Any optimal contract \( T = L + W(Q) \in T \) is such that:

a) it involves \( I \) only contracting with one downstream firm;

b) \( L = -r < 0; \)

c) \( W(Q) \geq R(Q), \) for \( Q \geq Q(c_I); \)

d) there exists at least one \( Q \in \mathcal{Q}(\mathcal{B}), Q(c_E) \) such that \( W(Q) = R(Q). \)

**Proof.** See Appendix. ■

Proposition 2 gives the general form of optimal contracts. Proposition 1 proposes an optimal three-part contract with an allowance. It is easy to see that an optimal contract with one downstream firm cannot have lower dimensionality than that of a three-part contract.

**Proposition 3** For any optimal contract \( T \in T_A^{(m)} \) it must be that \( m \geq 3. \)

**Proof.** See Appendix. ■

## 6 Extensions

In this section, we discuss what happens when some of our assumptions are relaxed or modified from the above benchmark model.

Incremental-units quantity discounting: One of AMD’s main complaints in the Intel case was Intel’s use of all-units quantity discounts and the related pricing rebates. The FTC settlement with Intel prohibited the use of all-units quantity discounts. Instead, Intel can use incremental-units quantity discounts.\(^\text{12}\)

Proposition 1 shows that all-units quantity discounting can be used by I to exclude the rival. We note that incremental-units quantity discounts, another commonly analyzed type of piece-wise linear contract, achieves the same goal. This is a continuous, block declining contract, in which the marginal prices decline at each increment. The $n$-part contract $T(Q) = L + W(Q; w, S)$ is characterized by the vector of marginal prices $w = (w_1, w_2, ..., w_{m-1})$, a lump-sum fee $L \neq 0$ and the vector of price-breaks $(S_1, S_2, ..., S_{m-1})$ such that $T^*(Q) = L + w_1 Q$ if $Q < S_1$, $T^*(Q) = L + w_1 S_1 + w_2 (Q - S_1)$ if $Q \in [S_1, S_2)$ etc. Incremental-units quantity discounting involves the declining marginal prices: $w_1 > w_2 > ... > w_{m-1}$.

The next proposition is a counter-part of Proposition 1. It shows that I can optimally exclude $E$ by using a three-part block declining contract which exhibits incremental-units quantity discounting.

**Proposition 4** There exists an optimal incremental-units three-part contract $T^* = L + W(Q; w, S) \in T^I_3$ such that (a) $L < 0$; (b) the incumbent’s profit is $\Pi_M - r$; (c) the lowest marginal wholesale price is below the incumbent’s marginal cost.

The profit obtained is identical to that obtained with the three-part all-units contract characterized in Proposition 1. The contract has the form $T^*(Q) = L + W(Q; w, S)$, where $w = (P_M, R'(Q(p_M))), S = (0, \frac{d}{dP}(Q(p_M))) Q(p_M)$ and $L = -r$.

Note that the lowest marginal wholesale price under incremental-units quantity discounting is smaller than the lowest marginal wholesale price under all-units discounting scheme. The incremental-units quantity discounting contract makes it more transparent that wholesale prices are being set below cost. Therefore, even though banning all-units quantity discount contracts but still allowing incremental-units quantity discounts contracts will not prevent the predatory behavior this paper highlights, it may make it easier for regulatory authorities to detect below-cost pricing.

**The efficiency of the entrant:** If the efficiency profit of $E$ is larger than the monopoly profit of $I$, then the rent $r$ in Proposition 1 will be greater than $\Pi_M$. $I$ will not be able to prevent $D$ from contracting with $E$. Thus, it is critical for our result that the cost advantage of $E$ cannot be too large. Similarly, the assumption that the cost advantage of $E$ be non-drastic is also critical. If $E$ has a drastic cost advantage, this means the rent that $I$ must offer $D$ to prevent it contracting with $E$ will be equal to $(p_M(c_E) - c_E) Q(p_M(c_E))$ since this is the amount $E$ can offer $D$ in stage 3. Since this is necessarily more than $\Pi_M$, $E$ cannot be excluded.

**The entrant cannot write exclusive contracts:** In order to consider the most difficult environment in which to consider exclusion, in our
benchmark setting we assumed that $E$ could write exclusive contracts upon entry. Given $E$ is more efficient, this gave it considerable power in attracting downstream firms in stage 3 and meant that $I$ had to offer $D$ a non-trivial rent $r$ to prevent entry. If instead $E$ cannot write exclusive deals in stage 3, then $E$ will no longer obtain the same advantage from attracting downstream firms. The three-part contract $T^*$ described in Proposition 1 will continue to lead to exclusion. Moreover, $I$ can do better, offering $D$ an arbitrarily small allowance $-L = \varepsilon > 0$. The downstream firm $D$ will always accept such a contract since if it does not, then $E$ will enter and $D$ will be left with no surplus. Due to the structure of the contract $T^*$, $D$ will continue to constrain the pricing decision of $E$, in this case even if $E$ also contracts with $D$. In particular, $E$ cannot sell anything at a price above $P$ if it competes with $D$ in the retail market (as before). If instead it sells through $D$, it will still not be able to obtain a price above $P$ given that $D$ can buy at this price through $I$ and since $E$ is willing to undercut any retail price of $D$ that exceeds the wholesale price it charges $D$. Thus, $E$ is again excluded, with $I$ now obtaining almost full monopoly profits.

**The incumbent can write an exclusive contract:** In the main section, $E$ is allowed to contract with any downstream firms in stage 3 if $E$ decides to enter. This possibility leads to a strictly positive allowance for $D$ and less than the monopoly profit for $I$. Suppose now $I$ can offer an exclusive contract in stage 1 to prevent such contracting between $E$ and $D$ in stage 3 as was the case in the example of Section 3. The timing of the game is unchanged except that in stage 3 the entrant cannot contract with $D$, i.e., there is exclusive dealing between $I$ and $D$. We consider both cases when $I$ can sell directly in the downstream market and when it cannot.

**Proposition 5** Under exclusive contracting the incumbent will obtain full monopoly profits, excluding $E$ in the process.

a) If $I$ competes in the downstream market this can be achieved by using a two-part all-units contract.

b) If $I$ cannot compete in the downstream market this can be achieved by using a three-part all-units contract.

**Proof.** The proof follows from Proposition 1.

a) $I$ offers $D$ the contract $T^*(Q) = L + W(Q; w, S)$, where $w = (P_M, P)$, $S = (0, Q(P))$ and $L = 0$. This contract is depicted on Figure 3a).

b) $I$ offers $D$ the contract $T^*(Q) = L + W(Q; w, S)$, where $w = (R(0), P_M, P)$, $S = (0, Q_M, Q(P))$ and $L = 0$. This contract is depicted on Figure 3b).

In both cases entry deterrence follows from Lemma 3. The analysis of pricing in the entry subgames is the same as in Lemma 1. 

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Note also that in the case when upstream firms can sell in the downstream market, then even when $D$ is the only downstream firm available to upstream firms, it is still optimal for $D$ to accept the exclusive contract proposed by $I$. Indeed, suppose $D$ decides to reject this contract and contract with $E$ in order to try to extract some rent from it. This will not work since in stage 3 when entry occurs, $D$ does not bring any value to the upstream firms given that they can both sell directly to consumers.

The “disposal-rent”: When the revenue function is strictly decreasing at $Q(P)$, $I$ must leave some additional rent to $D$. If there is no entry (as will be the case in equilibrium), $D$ can buy $Q(P)$ units for $T'(Q(P))$ but then sell fewer units so as to obtain a higher revenue by setting a higher retail price. Indeed, since $W(Q(P)) = R(Q(P))$ and $R(Q(P)) < R(Q_R)$, where $Q_R = \arg\max_Q R(Q)$, $D$ freely disposes $Q(P) - Q_R$ additional units and obtains the extra profit $R(Q_R) - R(Q(P))$.

To avoid $D$ ordering $Q(P)$ units in equilibrium, $I$ will offer $D$ an extra rent $r_d = R(Q_R) - R(Q(P))$. We call this rent the “disposal-rent”, the extra-rent $D$ can obtain in equilibrium given it can freely dispose of the good. The same amount has to be added to the allowance and rent $D$ obtains when there is no entry. Thus, $I$ may still exclude $E$, but its profit will be reduced by the size of this rent.$^{13}$ The resulting total rent that must be left to $D$ is $r' = r + r_d$.

Upfront fees: Upfront fees can make it easier for $I$ to exclude since they provide a further first-mover advantage to $I$. In particular, they provide a mechanism for $I$ to capture any rent $r$ (or $r + r_d$) that must be offered to $D$ in

\[\text{as a result, the assumption in A2 needs to be tightened so that } \Pi_M > (c_I - c_E)Q(c_I) - F + r_d.\]
stage 4. Thus, they allow $I$ to capture the full monopoly profit $\Pi_M$. In case there is entry and $D$ does face competition, this upfront fee is a sunk cost for $D$, and does not affect the incentives facing $D$ to undercut competitors as is required to prevent entry. This also means, with upfront fees, $D$ may regret signing its contract with $I$, in the off-equilibrium case that there is entry. Other than this difference, the existing optimal contract continues to work as in Proposition 1.

**Upstream firms cannot sell directly to consumers.** In the benchmark model we assumed that $I$ can sell directly in the downstream market. Having upstream firms sell directly is not critical but it is a convenient way of capturing the idea that the upstream firm could always sell through other identical downstream firms ex-post if this was desirable given our focus on non-exclusive contracts and unrestricted competing downstream channels.

There are cases of vertical channels when it is not reasonable to assume upstream firms can sell directly in the downstream market (for example it would presumably be quite costly for Microsoft to start selling computers). In such cases, we assume that there are multiple identical distributors in the final stage of the game, which each upstream firm can sell to through the spot market, and which due to Bertrand competition price at the wholesale prices they face. In this case, everything is as though each upstream firm can also sell directly in the final market at their chosen price in the last stage of the game. The analysis would therefore continue to apply.

## 7 Conclusions

The key new idea developed in this paper is that contracts involving quantity discounting and allowances can have exclusion effects. Quantity discounting contracts with allowances are commonly used in practice, but the possible anticompetitive effects of these contracts have only recently come into focus. An upstream incumbent can use such contracts to commit its downstream distributor to be more aggressive in the face of competition. For low levels of purchases, the downstream firm purchases at a wholesale price set above the incumbent’s marginal cost, thereby providing a way for the incumbent to extract the downstream firm’s profit. For purchases beyond some higher level, the downstream firm purchases at a wholesale price set below the incumbent’s marginal cost, thereby ensuring that in the face of competition, the downstream firm will want to compete aggressively, in such a way that the rival will not want to enter. A third instrument in the optimal contract includes an allowance paid to the downstream firm. This rent ensures that the downstream firm is not willing to contract with the rival instead, in case
it enters. The amount of rent that needs to be paid is limited to the entrant’s efficiency profit. The proposed optimal contract is also renegotiation-proof, thereby ensuring the incumbent can profitably exclude the rival even when its contract can be renegotiated for an arbitrarily small cost. Thus, we provide a new explanation of how efficient entry can be excluded based on vertical contracts, one that avoids making the usual assumptions of exclusivity or commitment without renegotiation.

The benchmark model we have provided can be extended in numerous directions. Several natural modifications have been analyzed in this paper, including to the cases in which the incumbent can use exclusive deals or upfront fees. In the former case, we showed exclusive deals eliminate the rent that has to be paid to the downstream firm so the incumbent can obtain full monopoly profit. In the latter case, the rent must still be paid ex-post but it can be fully extracted in the initial contract through an upfront fee.

One can think of the vertical contracts we consider as a type of vertical limit pricing or predation given that the incumbent offers to sell below its own cost, for sufficiently large purchases. Therefore, our theory supports the use of a predatory pricing standard for dealing with wholesale price discounts. In our theory, there are two testable features of contracts: marginal wholesale prices must fall below a firm’s own marginal cost for sufficiently large quantities and it must either rely on allowances paid to the downstream firm or exclusive contracts.

8 Appendix

8.1 Proof of Proposition 1

First we establish the properties which the optimal contract should have.

Suppose that $I$ proposes contracts $T_I = \{T_1, ..., T_n\}$, $T_i(Q) = L_i + W_i(Q) \in T$ to $n$ downstream firms $\{D_1, ..., D_n\} \subset \{D_1, D_2, ...\}$ in stage 1.\footnote{We can re-numerate downstream firms $\{D_1, D_2, ...\}$ if necessary.}

Assume that these contracts are all accepted by the respective downstream firms.

**No-entry subgame:** If $E$ does not enter at stage 2 and $T_I$ is optimal, then $I$ and $\{D_1, ..., D_n\}$ compete at stage 4. Note that at that stage the actions of downstream firms are determined only by variable parts of the contracts $W_i(Q)$. Denote by $\pi_i$ the equilibrium profit of $D_i$ and by $\pi^W_i$ the equilibrium profit of $D_i$ net of payment $L_i : \pi_i = \pi^W_i - L_i$. The total profit of the incumbent and downstream firms $\{D_1, ..., D_n\}$ is $\Pi(P) = R(Q(P)) - c_I Q(P)$.
Lemma 1  a) $\pi_i^W = 0, \forall i \in \{1, 2, ..., n\}$.
   b) $L_i \leq 0, \forall i \in \{1, 2, ..., n\}$.
   c) The equilibrium price in stage 4 is $P_M$ and the equilibrium quantity is $Q_M$.

Proof.  a) Note first that for all $i \in \{1, 2, ..., n\}$, $\pi_i^W \geq 0$. Indeed, after paying (receiving) $L_i$, $D_i$ has an option to walk away with $L_i$ by buying nothing from $I$. Assume now that $P_n = W_i > 0$. Let $P^*$ be an equilibrium price set by a subset $\{D_1, ..., D_k\} \subset \{D_1, ..., D_n\}$ and $I$.

Since the competition is in prices, $\pi_i^W = 0$ for $i > k$. Therefore, accounting for the possibility that $I$ sets $P^*$,

$$\pi^W \leq R(Q(P^*)) - \sum_{i=1}^{k} W_i(Q(P^*)). \quad (5)$$

By continuity of the total profit function $\Pi(P)$ there exists $\varepsilon > 0$ such that

$$\Pi(P^*) - \Pi(P^* - \varepsilon) < \pi^W. \quad (6)$$

If $I$ deviates and sets the price $P^* - \varepsilon$ then it takes the whole market and its profit is $\Pi(P^* - \varepsilon)$. By (5) and (6) we have

$$\Pi(P^* - \varepsilon) > -\pi^W + \Pi(P^*) \geq \sum_{i=1}^{k} W_i(Q(P^*)) - c_I Q(P^*).$$

The right-hand side represents $I$’s equilibrium profit. Thus, there is a profitable deviation and therefore, $\pi^W = 0$ and $\pi_i^W = 0, \forall i \in \{1, 2, ..., n\}$.

b) Follows from a). Indeed, if for some $i$, $L_i > 0$ then $\pi_i = \pi_i^W - L_i = -L_i < 0$ so we get a contradiction.

c) Suppose the equilibrium price $P^* \neq P_M$. Then there exists $\varepsilon > 0$ such that

$$\varepsilon < \Pi_M - \Pi(P^*).$$

Assume at stage 3, $I$ offers each $D_i$ the contract $\tilde{T}_i(Q) = \tilde{L}_i + wQ$ where $\tilde{L}_i = L_i - \frac{\varepsilon}{n}$ and $w$ is high enough so that each downstream firm does not buy from $I$ at stage 4. By a) and subgame perfection each $D_i$ accepts $\tilde{T}_i(Q)$. $I$’s profit at stage 4 is

$$\Pi_M - \varepsilon > \Pi(P^*)$$

implying a contradiction. Thus, $P^* = P_M$ and $Q(P^*) = Q_M$.  

\footnote{We re-numerate $D_i$ if necessary.}
The incumbent does not leave any (net) profit to downstream firms. Indeed, the (net) profit cannot be negative because $D_i$ are not obliged to buy anything from $I$. On the other hand because it competes directly in the market it can undercut any profitable price for downstream firms. From this Lemma we conclude that in the no-entry subgame, the profit of the each downstream firm $D_i \in \{D_1, ..., D_n\}$ is equal to $-L_i$.

**Entry subgame:**
Assume now that entry occurs at stage 2.

**Lemma 2** If $E$ does not contract with any of $\{D_1, ..., D_n\}$, then $E$ does not contract with any other downstream firm in stage 3.

**Proof.** Suppose $E$ contracts in stage 3 with downstream firms $D'_1, D'_2, ..., D'_k$ (other than $\{D_1, ..., D_n\}$) and in stage 4

$$\Pi_E (P, P_E, P'_1, ..., P'_k) + \sum_{i=1}^{k} \pi'_i (P, P_E, P'_1, ..., P'_k) > F,$$

(7)

where $(P, P_E, P'_1, ..., P'_k)$ is the equilibrium in stage 4, $P$ is the equilibrium price vector for $(I, D_1, ..., D_n)$ and $\pi'_i$ is the profit of $D'_i$. Consider the following deviation of $E$: in stage 3 $E$ does not contract with any downstream firm, in stage 4 $E$'s strategy is $\min \{P_E, P'_1, ..., P'_k\}$. The best responses of $\{D_1, ..., D_n\}$ and $I$ to such a strategy of $E$ is the price vector $P$. With this deviation the profit of $E$ in stage 4 is as in (7). However, $E$ is strictly better off since it saves on the costs of contracting. 16

From this lemma, we conclude that in the stage 4 the price competition is between $I, E$ and $\{D_1, ..., D_n\}$.

**Lemma 3** a) (Renegotiation-proofness). Neither of firms $D_i$ sets the price below $c_E$: $\forall i \in \{1, 2, ..., n\}$

$$\max_{Q \geq Q(c_E)} R(Q) - W_i(Q) < 0,$$

(8)

b) (Exclusion). At least one firm $D_i$ sets the price below or equal to the break-even price $P$: $\exists i \in \{1, 2, ..., n\}$ such that

$$\max_{Q \in [Q(E), Q(c_E)]} R(Q) - W_i(Q) \geq 0.$$

(9)

16Note that by Reny (1999), an equilibrium exists for any final subgame (possibly in mixed strategies). If the equilibrium is in mixed strategies then $E$ replicates the outcome of the original strategy profile by playing the minimum price that would have arisen for each possible realization of the mixed strategies adopted by $E, D'_1, D'_2, ..., D'_k$ with adjusted probabilities.
Proof.  a) Consider an equilibrium in pure strategies of the pricing game $(P_1, P_1, ... P_n, P_E)$.\footnote{The case of mixed strategies is similar.  We can take $P_1$, the lower bound of retail prices chosen with positive probability by competing downstream firms, and show that $P_1 \leq P$.} If for some $i$ we have $\max_{Q \geq Q(e_i)} R(Q) - W_i(Q) > 0$ then the equilibrium price $P^* = \min (P_1, P_1, ... P_n, P_E) \leq c_E$. Consider the subset $\{D_1, ..., D_k\}$ of $\{D_1, ..., D_n\}$ such that $P_j = P^*$ for $j \in \{1, ..., k\}$. In this case, $\{D_1, ..., D_k\}$ takes the whole market and the joint profit of $(I, D_1, ..., D_k)$ in this subgame is $R(Q(P^*)) - c_I Q(P^*) \leq c_E Q(P^*) - c_I Q(P^*) < 0$. Assume at stage 3, $I$ offers each $D_i$, $i = 1, ..., k$ the contract $\widetilde{T}_i(Q) = \widetilde{L}_i + wQ$, where $\widetilde{L}_i = L_i - \xi$ and $w$ is high enough so that each downstream firm does not buy from $I$ at stage 4. Each $D_i$ accepts $\widetilde{T}_i(Q)$. In competition with $E$, $I$’s profit at stage 4 is zero implying a contradiction.

b) If for all $i$ we have $\max_{Q \in \{Q(P), Q(c_E)\}} R(Q) - W_i(Q) < 0$, then by a), $\max_{Q \geq Q(E)} R(Q) - W_i(Q) < 0$ for all $i$. Thus, $E$ can take the whole market by posting the price $P + \varepsilon$. By (3), this will lead to strictly positive net profit for $E$. \hfill \blacksquare

This Lemma shows that if (8) and (9) are satisfied for $T_i$ and $E$ does not contract with $\{D_1, ..., D_n\}$, then $E$ does not cover the cost of entry.

Note that if entry occurs, any downstream firm $D_i \in \{D_1, ..., D_n\}$ that does not contract with $E$ obtains profit $-L_i$. Note also that if for some $i' \in \{1, 2, ..., n\}$ we have $\max_{Q \geq Q(E)} R(Q) - W_{i'}(Q) < 0$, then $L_{i'} = 0$.\footnote{Note that the set $\{D_1, ..., D_n\}$ of firms contracting with $I$ can be split in two groups. The firms in the first group are ready by Lemma 3 a) to price between $(c_E; P)$ in stage 4. The equilibrium price for firms in the second group is above $P$. The firms in group 1 play an active role in entry deterrence whereas the firms in group 2 are “phantom” players.} Now we assume that $E$ is free to approach any of $\{D_1, ..., D_n\}$.

**Lemma 4** $- \sum_{i=1}^{n} L_i \geq r$

**Proof.** In stage 4, the equilibrium price cannot be greater than or equal to $c_I$ (given that $I$ competes in stage 4). By A2 and concavity of the revenue function, we have

$$Q(c_I) = \arg \max_{Q \geq Q(c_I)} (R(Q) - c_E Q - F)$$

and

$$\max_{Q \geq Q(c_I)} (R(Q) - c_E Q - F) = r.$$
Thus, the maximum that \( E \) can promise to \( \{D_1, ..., D_n\} \) is \( r \) which leads by (4) to a profit less than or equal to \( F \).

By Lemma 3, the downstream firms contracting with \( I \) receive their profit only through allowances: \(-L_i = \pi_i\).\(^{19}\) Therefore, if \(-\sum_{i=1}^{n} L_i < r\), then \( E \) proposes to each \( D_i \) the contracts \( T'_i = L_i - \varepsilon/n + W_i \), where \( \varepsilon > 0 \) is such that \(-\sum_{i=1}^{n} L_i + \varepsilon < r\) and \( W_i \) is a linear contract with \( w_i \) high enough so that \( D_i \) does not buy (say \( w_i > c_E \)). The downstream firms accept these contracts. Thus, the total rent paid to the downstream firms must be greater than or equal to \( r \) to ensure that \( E \) cannot profitably contract with downstream firms in this way.

Suppose \( E \) offers contracts to a proper subset \( \{D_1, ..., D_k\} \subset \{D_1, ..., D_n\} \) so that \( \sum_{i=1}^{k} \pi_i < r \). In this case there exists at least one downstream firm from \( \{D_1, ..., D_n\} \setminus \{D_1, ..., D_k\} \) for which (9) is satisfied. Thus the entry will not be profitable. \( \blacksquare \)

We are now ready to prove Proposition 1. The proof is by construction. \( I \) offers a single downstream firm which we denote by \( D \) the contract \( T_I(Q) = L + W(Q; w, S) \in \mathcal{T}^{(3)}_A \), where \( w = (P_M, P) \), \( S = (0, Q(P)) \) and \( L = -r \). This contract has two marginal wholesale prices \( P_M \) and \( P \), which play the role of linear costs for \( D \). A lump-sum payment \( r \) is paid to \( D \) in stage 4. By Lemma 3, \( T_I \) deters entry, and \( I \) and \( D \) do not renegotiate the contract. By Lemma 1, the equilibrium retail price is \( P_M \). This implies \( I \)'s profit is \( \Pi_M - r \). By Lemma 4, \( I \) cannot decrease the rent by contracting with other downstream firms in stage 3.

8.2 Proof of Proposition 2

Property (a) follows because there is an arbitrarily small cost of contracting, dealing with multiple downstream firms leads to higher costs of contracting than dealing with only one downstream firm given that Proposition 1 guarantees the same final allocation for \( I \). Property (b) follows from Lemma 4. Properties (c) and (d) follow from Lemma 3 respectively.

8.3 Proof of Proposition 3

Proof. By Proposition 2 we have \( L < 0 \) and \( w^* \geq P^* = P_M \). Since in the case of entry the marginal wholesale price has to be less than or equal to \( P \), the optimal all-units contract must have at least two marginal prices. \( \blacksquare \)

\(^{19}\)Note that \( L_i = 0 \) for “phantom” firms.
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